Outage Probability of AF System with Interference-Limited Relay over Rayleigh/Rician Fading Channels

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***Abstract* – This paper studies the performance of a dual-hop amplify-and-forward system where the source-relay and the relay-destination channels experience Rayleigh and Rician fading, respectively. The relay node is corrupted by Rayleigh faded multiple co-channel interferences. New closed-form expressions for outage probability of the end-to-end signal-to-interference and noise ratio (SINR) are derived. An influence of various parameters, such as Rician *K* factor, number of cochannel interferences and outage threshold on outage probability is considered.**

***Keywords* – amplify-and-forward relay, co-channel interference, outage probability**

I. Introduction

Multihop relaying technology in communication networks is often used as a reasonable solution in a case of deep faded direct link. Relayed transmission is based on several terminals between source and destination providing better communication and reliable signal traversing without using large transmitting power [1, 2]. This type of transmission has gained enormous interest in the context of user-cooperative communications. Regarding the nature and complexity of relays used in cooperative communication, there are two dominant categories of multihop transmission systems: regenerative and nonregenerative systems. In regenerative systems, referred as decode-and-forward (DF), relays decode the signal and retransmitting it to the next node [1]. On the other hand, in nonregenerative systems or amplify-and-forward (AF) systems, relays just amplify and forward the incoming signal without performing any decoding at all [2-4]. AF relaying mechanism imposes less power consumption and can be preferable in practice. Variable gain requires knowledge of the instantaneous channel realization at the relays. Those are so-called channel state information (CSI) - assisted relays, which use the instantaneous CSI from the previous hop to produce their gain leading to a power control of the retransmitted signal [4-7].

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In many published papers, the research goal on this subject has been to derive the outage probability expression of dual-hop wireless communication systems in noise-limited environment [1-7]. Nevertheless, in many practical scenarios co-channel interference (CCI) is also an important issue and should be taken in consideration. Consideration of CCI is necessary because of the aggressive reuse of frequency channels for high spectrum utilization in cellular systems [8-10]. In few published works, the impact of interference on the AF and the DF relaying performance have been investigated either at the relay(s) or destination(s) [11-12]. In [11], the performance of a two hop CSI-assisted amplify-and-forward system, with co-channel interference at the relay was analyzed. The system's outage probability and the average bit error rate (ABER) in the presence of Rayleigh faded multiple interferers were investigated. In the paper [12], analytical closed-form expressions for outage probability of AF and DF relays in interference-limited environment were derived. In that paper, the performance of a Rayleigh dual-hop relay channel effected by an additive white Gaussian noise (AWGN) at the relay and multiple co-channel interferences at the destination was investigated. The paper [13] studies the outage probability of the non-regenerative relays over Rayleigh fading channels in an interference-limited environment (the relay and destination nodes are corrupted by co-channel interference).

In this paper, we focus on dual-hop AF relay transmission systems and study their performance over mixed Rayleigh and Rician fading channels in the presence of co-channel interferences. In practice, different links in relay networks can experience separate fading conditions. For example, WINNER II project [14] confirms the existence of different (mixed Rayleigh/Rician) fading conditions. Also in [15-16], a base station-relay link is considered as Rayleigh, while the relay-mobile link is observed as Rician link because of a strong line-of-sight (LoS) component. In the following analysis, we assume that there are multiple interferers at the relay node independent of the desired signal. In the observed scenario, co-channel interferences are subjected to Rayleigh fading.

Closed-form expressions for the outage probability of the end-to-end signal-to-interference and noise ratio (SINR) for CSI-assisted relayed systems are derived. In particular, we derive exact expressions in terms of an infinite series, which numerically converges quickly for a finite number of terms.

II. SYSTEM AND CHANNEL MODEL

The system observed in this paper is depicted in Fig. 1. We consider the wireless communication system, from the source terminal S to the destination terminal D assisted by a nonregenerative gain relay transmission. The relay terminal is corrupted by co-channel interferences and AWGN while destination terminal is only perturbed by an AWGN.

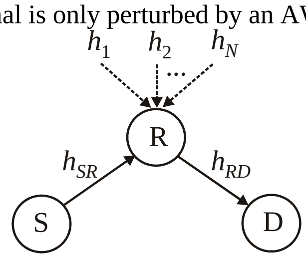

Fig. 1. System model

We assume that terminal S transmits a desired symbol, s_0, with an average power $\mathrm{E}\left[|s_0|^2\right]=P_0$ (E[.] is the expectation operator). The level of co-channel interference at the relay is high enough compared to the level of thermal noise, so the thermal noise can be neglected as in all interference-limited fading environments. The received signal at relay terminal R, in interference-limited fading environment, can be written as

$$r_R = h_{SR}s_0 + \sum_{i=1}^{N} h_i s_i \tag{1}$$

where h_{SR} is the fading amplitude of the channel between terminals S and R, $\{h_i\}_{i=1}^N$ are amplitudes of the interferers at the input of R and $\{s_i\}_{i=1}^N$ are interfering symbols with an average power P_i each of them. In nonregenerative systems, the signal r_R is then multiplied by the gain G of terminal R and retransmitted to terminal D effected by an AWGN. The received signal at terminal D can be presented as

$$r_D = h_{RD}G\left(h_{SR}s_0 + \sum_{i=1}^{N} h_i s_i\right) + n_D \tag{2}$$

where h_{RD} is the fading amplitude of the channel between terminals R and D, and n_D is the AWGN at the input of D with $\mathrm{E}\left[|n_D|^2\right]=\sigma_D^2$.

The overall SINR at the receiving end can be expressed as [11]

$$\gamma_{eq} = \frac{|h_{SR}|^2|h_{RD}|^2G^2P_0}{|h_{RD}|^2G^2\sum_{i=1}^{N}|h_i|^2P_i+\sigma_D^2} = \frac{|h_{SR}|^2P_0\dfrac{|h_{RD}|^2P_R}{\sigma_D^2}}{\dfrac{|h_{RD}|^2P_R}{\sigma_D^2}\sum_{i=1}^{N}|h_i|^2P_i+\dfrac{P_R}{G^2}} \tag{3}$$

where P_R is the power of the transmitted signal at the output of the relay.

When terminal R has available instantaneous CSI from the first hop, the gain G is given by

$$G_1^2 = \frac{P_R}{|h_{SR}|^2P_0+\sum_{i=1}^{N}|h_i|^2P_i} \tag{4}$$

Therefore, the instantaneous equivalent end-to-end SINR in this case can be obtained by substituting (4) in (3) and has a form [11]

$$\gamma_{eq1} = \frac{\gamma_1\gamma_2}{\gamma_3(\gamma_2+1)+\gamma_1} \tag{5}$$

where $\gamma_1=|h_{SR}|^2P_0$, $\gamma_2=\dfrac{|h_{RD}|^2P_R}{\sigma_D^2}$ and $\gamma_3=\sum_{i=1}^{N}|h_i|^2P_i$. A CSI-assisted nonregenerative relay requires a continuous estimation of the channel fading amplitude from the first hop and inverts the fading effect in order to limit the output power of the relay [5].

We consider an asymmetric fading scenario of the $S-R$ and $R-D$ links. Namely, the $S-R$ link is subjected to Rayleigh fading and the $R-D$ link is subjected to Rician fading. If a link experiences Rayleigh fading, γ_1 is exponentially distributed by probability density function (PDF) given by

$$p_{\gamma_1}(\gamma)=\frac{1}{\bar{\gamma}_1}\exp\left(-\frac{\gamma}{\bar{\gamma}_1}\right) \tag{6}$$

where $\bar{\gamma}_1=\mathrm{E}\left[|h_{SR}|^2\right]P_0$ is the average signal power of $S-R$ channel.

If a link experiences Rician fading, γ_2 is a noncentral χ^2 chi-squared distributed

$$p_{\gamma_2}(\gamma)=\frac{(K+1)\exp(-K)}{\bar{\gamma}_2}\exp\left(-\frac{(K+1)\gamma}{\bar{\gamma}_2}\right)I_0\left(2\sqrt{\frac{K(K+1)\gamma}{\bar{\gamma}_2}}\right) \tag{7}$$

where $\bar{\gamma}_2=\mathrm{E}\left[|h_{RD}|^2\right]P_2/\sigma_D^2$ is the average SNR per hop of $S-R$ channel, K is Rician factor and $I_0(.)$ is the zeroth order modified Bessel function of the first kind [17, eq. 8.406]. We assume that co-channel interference fading amplitudes are also modeled as Rayleigh random processes. When all N interferers are identical, γ_3 becomes a central χ^2 random variable with $2N$ degrees of freedom [12]

$$p_{\gamma_3}(z)=\frac{1}{\bar{\gamma}_3^N\Gamma(N)}\exp\left(-\frac{z}{\bar{\gamma}_3}\right) \tag{8}$$

where $\bar{\gamma}_3=\mathrm{E}\left[|h_i|^2\right]P_i$, i=1,2,...N, is the average power of each CCI.

III. OUTAGE PROBABILITY

One of the accepted performance measures, involving dual-hop systems in fading environments, is the outage probability. Roundly speaking for this case, outage probability is defined as probability that the instantaneous equivalent SINR, γ_{eq}, falls below a predetermined protection ratio, γ_{th} [18]-[19].

When terminal R has available CSI from the first hop, the outage probability of the instantaneous equivalent SINR can be expressed as [6], [11]

$$\begin{aligned} P_{eq1}(\gamma_{th}) &= \Pr(\gamma_{eq1}<\gamma_{th}) \\ &= \int_0^{\infty}\int_0^{\infty} P_{\gamma_1}\left(\frac{\gamma_1\gamma_2}{\gamma_3(\gamma_2+1)+\gamma_1}\le \gamma_{th}\middle|\gamma_2,\gamma_3\right)p_{\gamma_2}(\gamma_2)p_{\gamma_3}(\gamma_3)\mathrm{d}\gamma_2\mathrm{d}\gamma_3 \\ &= \int_0^{\infty}\int_0^{\infty} P_{\gamma_1}\left(\gamma_1\le\frac{\gamma_{th}(y+1)z}{y-\gamma_{th}}\right)p_{\gamma_2}(y)p_{\gamma_3}(z)\mathrm{d}y\mathrm{d}z \end{aligned} \quad (9)$$

After some mathematical manipulations and simplifications, (11) can be rewritten as

$$P_{eq1}(\gamma_{th}) = 1-\int_0^{\infty}\int_0^{\infty} P_{\gamma_1}\left(\gamma_1\ge\frac{\gamma_{th}(x+\gamma_{th}+1)z}{x}\right)p_{\gamma_2}(x+\gamma_{th})p_{\gamma_3}(z)\mathrm{d}x\mathrm{d}z \quad (10)$$

In order to evaluate (10), the complementary cumulative distribution function (CDF) of γ_1 and PDFs of the γ_2 and γ_3 are needed. The complementary CDF of γ_1 can be expressed as $P_{\gamma_1}(\gamma)=\exp(-\gamma/\bar{\gamma}_1)$ and PDFs of γ_2 and γ_3 are given by (7) and (8), respectively.

After applying these formulae, the outage probability has the following form

$$\begin{aligned} P_{eq1}(\gamma_{th}) = 1 &- \frac{(K+1)\exp(-K)}{\bar{\gamma}_2}\exp\left(-\frac{K+1}{\bar{\gamma}_2}\gamma_{th}\right)\frac{1}{\bar{\gamma}_3^{N}\Gamma(N)} \\ &\cdot\int_0^{\infty}\int_0^{\infty}\exp\left(-\frac{\gamma_{th}(x+\gamma_{th}+1)z}{\bar{\gamma}_1 x}\right)\exp\left(-\frac{K+1}{\bar{\gamma}_2}x\right) \\ &I_0\left(2\sqrt{\frac{K(K+1)(x+\gamma_{th})}{\bar{\gamma}_2}}\right)z^{N-1}\exp\left(-\frac{z}{\bar{\gamma}_3}\right)\mathrm{d}x\mathrm{d}z \end{aligned} \quad (11)$$

We are unaware of a closed-form analytical solution to this integral. Nevertheless, using the infinite-series representation of I_0(.) [17, Eq. (8.447.1)], the integral in (11) can be solved

$$\begin{aligned} P_{eq1}(\gamma_{th}) = 1 &- \frac{(K+1)\exp(-K)}{\bar{\gamma}_2}\exp\left(-\frac{K+1}{\bar{\gamma}_2}\gamma_{th}\right)\left(\frac{\rho}{\gamma_{th}(\gamma_{th}+1)}\right)^{N} \\ &\cdot\sum_{i=0}^{\infty}\sum_{j=0}^{i}\frac{\gamma_{th}^{i-j}}{i!\,j!(j-1)!}\left(\frac{K(K+1)}{\bar{\gamma}_2}\right)^{i}\left(\frac{\gamma_{th}(\gamma_{th}+1)}{\gamma_{th}+\rho}\right)^{N+j+1} \\ &\Gamma(N+j+1)U\left(N+j+1,\ j+2;\ \frac{K+1}{\bar{\gamma}_2}\frac{\gamma_{th}(\gamma_{th}+1)}{\gamma_{th}+\rho}\right) \end{aligned} \quad (12)$$

where $\rho=\bar{\gamma}_1/\bar{\gamma}_3$ and $U(a,b;z)$ is the confluent hypergeometric function of the second kind defined as $U(a,b;z)=\frac{1}{\Gamma(a)}\int_0^{\infty}e^{-zt}t^{a-1}(1+t)^{b-a-1}\mathrm{d}t$ [17, Eq. (9.211.4)].

Outage probability as a function of average SNR of $S-R$ channel, $\bar{\gamma}_2$, for various values of ρ is presented in Fig. 2. Numerical results are presented for the case of Rayleigh fading (K=0) on both hops and obtained curves are consistent with those reported in [11, Fig. 1]. Also, the results for the considered model in this paper and K=10 dB are shown. As expected, when ρ increases, the outage probability decreases.

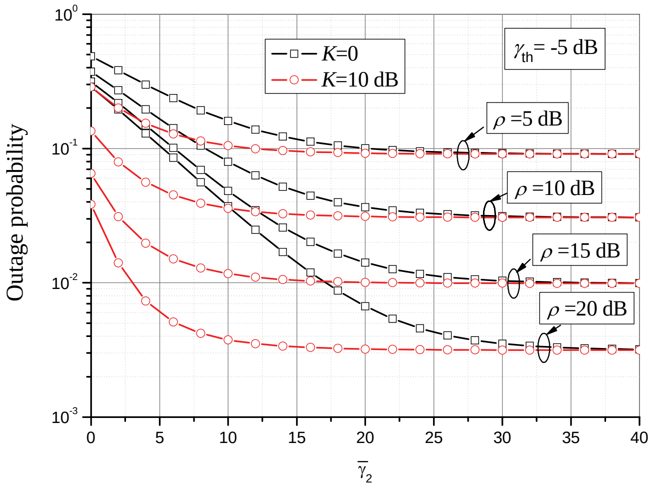

Fig. 2. Outage probability for various values of ρ

In Fig. 3. outage probability is represented for various values of Rician K factor. Rician K factor significantly affects the outage performance i.e. outage probability decreases as K increases. It is noticeable that for particular values of average SNR, values of outage probability tend to irreducible outage floor. We can also see that outage floor does not depend on K, but only on ρ.

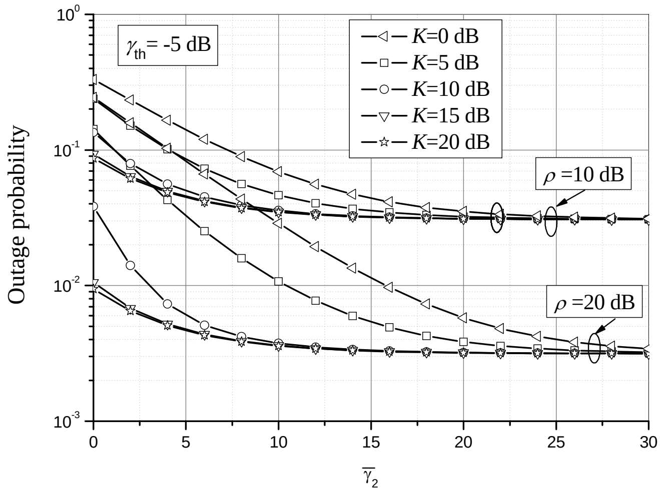

Fig. 3. Outage probability for various K factors

Fig. 4. depicts the outage performance for different outage threshold of CSI-assisted relay. It is observed that as γ_{th} increases, the outage probability also increases. When γ_{th} increases from -5dB to 5dB, outage probability changes about 10 times.

Fig. 5. shows the outage performance for different number of interferences. As the number of interference increases, outage probability increases, which degrades the system performance. The largest performance degradation is present when the number of CCI increase from one to two. It is evident that outage floors depend on number of CCI.

Figs. from 2 to 5 cleary show the existence of outage probability floor. It is evident that outage floors strongly depend on values of ρ.

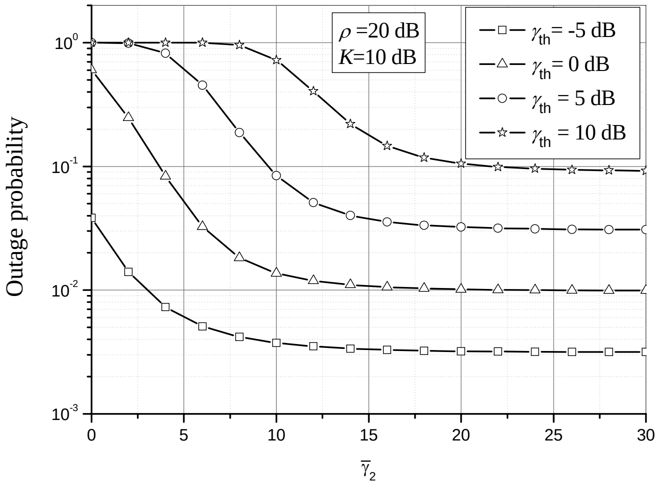

Fig. 4. Outage probability for various values of outage threshold

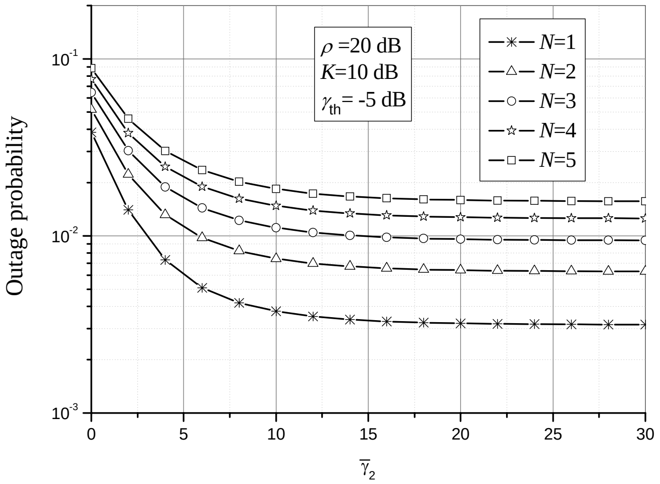

Fig. 5. Outage probability for various number of co-channel interferences

IV. CONCLUSION

The performance of dual-hop systems with amplify-and-forward relays under assumption that relay is affected by multiple co-channel interferences has been studied. We have derived closed-form expression for the outage probability of the instantaneous SINR for dual-hop transmission with CSI-based relays operating over asymmetric fading channels. The effects of Rician K factor, number of co-channel interferences, outage threshold on the system's performance, were interpreted. After all, proposed analysis can be used in design of a cellular mobile system to determine optimal values of system parameters to provide a reasonable indication of signal outage.

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