Outage Probability of Correlated SC SIR-based Diversity System over *K* Fading Channels

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***Abstract* – The paper presents brief analysis of selection combining (SC) diversity over correlated *K*–fading channels. The proposed system is considered as interference-limited system in correlated fading environment. The cumulative distribution function (cdf) of signal-to-interference ratio (SIR) at the output of SC receiver is derived in a form of Meijer*G* functions. According to this new expression, the outage probability is considered and the effects of shaping parameters and correlation coefficients on the performance gain are analysed. The proposed analytical approach is accompanied by numerical results.**

***Keywords* – SC diversity system, interference, *K*-fading channels, outage probability**

I. Introduction

The statistical characterization of signal propagation, in performance evaluations of wireless communication system, is frequently based on analysis of composite fading: large-scale fading due to multipath propagation and small-scale fading due to shadowing phenomenon [1]. This destructive combination of obstacles often occurs simultaneously. The multipath fading is mostly modeled by Rice, Rayleigh and Nakagami-*m* distributions and shadowing by lognormal distribution. This lognormal based fading model is analyzed in few papers [1-3]. However, the analytical analysis of this model is very complicated, so in some other papers the Gamma distribution is employed as an useful solution for describing shadow fading phenomena [4]. Furthermore, based on Gamma distribution, several generalized distributions have been proposed as composite fading channel models, the generalized Gamma , the *G* [5] and *K* [6-9] fading channel models.

An efficient technique to diminish the influences of fading and cochannel interference is space diversity reception. The principle of space diversity reseption is to provide the receiver with multiple faded replicas of the same information-bearing signal. The goal is to upgrade transmission reliability without increasing transmission power and bandwidth, as well as to increase channel capacity.

Among the well-known diversity schemes, selection combining (SC) space-diversity technique is with lower complexity nature opposed to maximal-ratio combining (MRC) and equal-gain combining (EGC) techniques, which require the amount of the channel-state information of transmitted signal.

In an interference-limited system (the thermal noise is ignored), the effect of interference needs to be taken into consideration of performance analysis. The most effective performance criterion, in that case, is to select the signal-to-interference ratio (SIR) [10]. Independently of the channel condition, the transmitted signals as well as the interfered ones could be correlated due to small distance between the diversity antennas.

The presented evaluations in [6] affirm that the Rayleigh-lognormal and *K* distributions are similar, but the latter has a simpler form and thus is more appropriate for analysis and design of wireless communication systems. The moment generating function (mgf) based performance analysis of generalized selection combining (GSC) diversity operating over independently distributed *K* fading channels is presented in [7]. In [8] the brief performance evaluations of the output signal-to-noise ratio (SNR) for SC diversity operating over correlated *K* fading channels is presented. The performance analysis of multiple branches SC diversity combining technique, this time over exponentially correlated *K* fading channels is considered in [9]. Again, the environment corrupted by additive white Gaussian noise (AWGN) is considered.

In this paper, the performance of SC diversity receiver in interference-limited environment is analyzed. Considering the composite correlated fading channel condition, infinite series expressions for the performance evaluation are derived. The detailed analysis of outage probability is presented. Based on analytical results, required numerical results are also presented.

II. SC Performance over *K* Fading Channels

In general, the SC combiner selects the branch with the highest signal-to-noise ratio (SNR), actually the branch with the strongest signal assuming equal noise power among the branches. In environments where the level of the cochannel interference is sufficiently high as compared to the thermal noise, SC picks the signal at the branch with highest SIR

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(SIR-based selection diversity). The system model, we have analyzed in this paper is presented in Fig. 1.

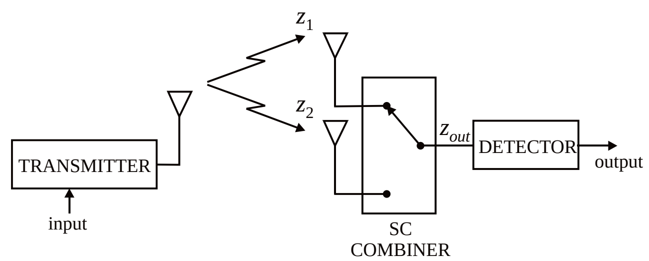

Fig. 1. System model

Assuming the composite fading channel model, the *K* distribution is convenient and mathematically tractable distribution for evaluating adequate performance criteria. The *K* distribution accurately approximates some of fading and/or shadowing models.

The received resultant of desired and interfering signal, considering *K* fading environment, can be presented as [11]

$$y(t) = x(t)\exp\left(j\left(2\pi f_c t + \psi(t) + \theta(t)\right)\right) \tag{1}$$

where f_c is a carrier frequency, $\psi(t)$ is the information signal, $\theta(t)$ is the random phase and $x(t)$ is *K* distributed random envelope process, given by [7, eq. (1)]

$$p(x) = \frac{4}{\Gamma(k)\Omega^{k/2}} x^k K_{k-1}\left(\frac{2x}{\sqrt{\Omega}}\right) \tag{2}$$

where $K_{k-1}(.)$ is the $(k$-1)th order modified Bessel function of the second kind [12, eq. (8.407)], $\Gamma(.)$ is the Gamma function [12, eq. (8.310[7].1)] and $\Omega = \mathrm{E}(x^2)/k$ with E(.) being expectation operator. The parameter k is a shadowing severity parameter $k \in (0,\infty)$.

The proposed model refers to the case of a single cochannel interferer. The performance of SC is carried out by considering the effect of only the strongest interferer. We assume that the remaining interferers are combined and considered as uncorrelated interference between antennas.

There is a need to derive the joint statistics for two *K* variables. Assuming [8, eq. (4)], the joint pdfs for both desired and interfering signal envelopes can be, respectively, expressed as

$$p_{R_1R_2}(R_1,R_2) = \frac{16}{\Gamma(k_d)} \sum_{a,b=0}^{\infty} \frac{\rho_{nd}^a \rho_{gd}^b}{\Gamma(1+a)\Gamma(k_d+b)} \times \frac{\prod_{l=1}^{2}\left(R_l/\sqrt{\Omega_{dl}}\right)^{\varepsilon_d} K_\psi\left(2R_l\sqrt{1/(1-\rho_{nd})(1-\rho_{gd})\Omega_{dl}}\right)}{a!b!(1-\rho_{nd})^{k_d+a+b}(1-\rho_{gd})^{1+a+b} R_1R_2} \tag{3}$$

and

$$p_{r_1r_2}(r_1,r_2) = \frac{16}{\Gamma(k_c)} \sum_{c,d=0}^{\infty} \frac{\rho_{nc}^c \rho_{gc}^d}{\Gamma(m_c+c)\Gamma(k_c+d)} \frac{\prod_{l=1}^{2}\left(r_l/\sqrt{\Omega_{cl}}\right)^{\varepsilon_c} K_\psi\left(2r_l\sqrt{1/(1-\rho_{nc})(1-\rho_{gc})\Omega_{cl}}\right)}{c!d!(1-\rho_{nc})^{k_c+c+d}(1-\rho_{gc})^{1+c+d} r_1r_2} \tag{4}$$

where, $\varepsilon_d = k_d + 1 + a + b$, $\varepsilon_c = k_c + 1 + c + d$, $\psi_d = k_d + b - 1 - a$, $\psi_c = k_c + d - 1 - c$; ρ_{nd} and ρ_{nc} are the power correlation coefficients between the envelopes of the desired signals and interfering signals, respectively; and ρ_{gd} and ρ_{gc} are correlation coefficients between the average fading power of the desired signals and interfering signals, respectively. Parameters k_d and k_c are shadowing severity parameters of desired and interfering signals, respectively.

Instantaneous values of SIR at the *l*-th input diversity branch can be defined as $z_l = R_l^2/r_l^2$, l=1, 2. The selection combiner chooses and outputs the branch with the highest SIR

$$z = z_{out} = \max(z_1, z_2). \tag{5}$$

Let $S_l = \hat{R}_l^2/\hat{r}_l^2$ be the average SIR at the *l*-th input branch of the selection combiner.

Furthermore, due to a scenario with closely placed diversity antennas, both desired and interfering signal envelopes experience correlated *K* fading. The joint pdf for input SIRs, regarding $\Omega_{dl} = \mathrm{E}\left(R_l^2\right)/k_d$, $\Omega_{c_l} = \mathrm{E}\left(r_l^2\right)/k_c$, can be evaluated as [13]

$$p_{z_1z_2}(z_1,z_2) = \frac{1}{4\sqrt{z_1z_2}} \int_0^\infty\int_0^\infty p_{R_1R_2}\left(r_1\sqrt{z_1}, r_2\sqrt{z_2}\right) p_{r_1r_2}(r_1,r_2) r_1r_2 dr_1dr_2 \tag{6}$$

Substituting (3) and (4) in (6) and representing $K_\psi(.)$ in a form of Meijer*G* function as $K_\psi(x) = \frac{1}{2} G_{0,2}^{2,0}\left(\left.\begin{matrix} - \\ \frac{\psi}{2}, -\frac{\psi}{2}\end{matrix}\right| \frac{x^2}{4}\right)$, [14, eq. (03.04.26.0006.01)], we get the following double-fold integral form

$$I = \int_0^\infty\int_0^\infty \prod_{l=1}^{2} r_l^{\varepsilon_d+\varepsilon_c-1} G_{0,2}^{2,0}\left[\frac{r_l^2 z_l}{(1-\rho_{nd})(1-\rho_{gd})\Omega_{dl}}\left|\begin{matrix} - \\ \psi_d/2,\ -\psi_d/2\end{matrix}\right.\right] \times G_{0,2}^{2,0}\left[\frac{r_l^2}{(1-\rho_{nc})(1-\rho_{gc})\Omega_{cl}}\left|\begin{matrix} - \\ \psi_c/2,\ -\psi_c/2\end{matrix}\right.\right] dr_1dr_2 \tag{7}$$

Now, making change of variables $t = r_1^2/(1-\rho_{nc})(1-\rho_{gc})\Omega_{c1}$ and $u = r_2^2/(1-\rho_{nc})(1-\rho_{gc})\Omega_{c2}$ and using [14, eq. (07.34.21.0011.01)], we get

$$p_{z_1z_2}(z_1,z_2) = \sum_{a,b,c,d=0}^{+\infty} \mathrm{A} \times z_1^{\varepsilon_d/2-1} z_2^{\varepsilon_d/2-1} \times G_{2,2}^{2,2}\left[\frac{z_1\sigma}{S_1}\left|\begin{matrix} 1-(\varepsilon_d+\varepsilon_c+\psi_c)/2,\ 1-(\varepsilon_d+\varepsilon_c-\psi_c)/2 \\ \psi_d/2,\ -\psi_d/2\end{matrix}\right.\right] \times G_{2,2}^{2,2}\left[\frac{z_2\sigma}{S_2}\left|\begin{matrix} 1-(\varepsilon_d+\varepsilon_c+\psi_c)/2,\ 1-(\varepsilon_d+\varepsilon_c-\psi_c)/2 \\ \psi_d/2,\ -\psi_d/2\end{matrix}\right.\right] \tag{8}$$

with

$$\mathrm{A} = \frac{\rho_{nd}{}^{a}\rho_{gd}{}^{b}\rho_{nc}{}^{c}\rho_{gc}{}^{d}}{\Gamma(k_d)\Gamma(k_c)\Gamma(1+a)\Gamma(k_d+b)\Gamma(1+c)\Gamma(k_c+d)S_1^{\varepsilon_d/2}S_2^{\varepsilon_d/2}},$$
$$\times\frac{(1-\rho_{nc})^{k_d+a+b+2}(1-\rho_{gc})^{k_d+k_c+a+b+1}}{a!b!c!d!(1-\rho_{nd})^{k_d+a+b}(1-\rho_{gd})^{1+a+b}}$$

and $\sigma = (1-\rho_{nc})(1-\rho_{gc})/((1-\rho_{nd})(1-\rho_{gd}))$.

For this case, the joint cumulative distribution function (cdf) of z_l, l=1,2, can be evaluated as [13]

$$F_{z_1,z_2}(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} p_{z_1,z_2}(x_1,x_2)dx_1dx_2 \qquad (9)$$

Substituting (8) in (9) and following the same procedure and using the same equations as in solving double-fold integral in (6); the integral in (9) is solved in the form of infinite series. Finaly, equating arguments $t_1=t_2=t$, the derived cdf of output SIR has the following form

$$F(z) = \sum_{a,b,c,d=0}^{+\infty}\mathrm{A}\times z^{\varepsilon_d}$$
$$\times G_{3,3}^{2,3}\left[\frac{z\sigma}{S_1}\middle|\begin{matrix}1-(\varepsilon_d+\varepsilon_c+\psi_c)/2, 1-(\varepsilon_d+\varepsilon_c-\psi_c)/2, (1-\varepsilon_d)/2\\ \psi_d/2, -\psi_d/2, -\varepsilon_d/2\end{matrix}\right]$$
$$\times G_{3,3}^{2,3}\left[\frac{z\sigma}{S_2}\middle|\begin{matrix}1-(\varepsilon_d+\varepsilon_c+\psi_c)/2, 1-(\varepsilon_d+\varepsilon_c-\psi_c)/2, (1-\varepsilon_d)/2\\ \psi_d/2, -\psi_d/2, -\varepsilon_d/2\end{matrix}\right]$$
(10)

III. NUMERICAL RESULTS

The outage probability, P_{out}, is a standard measure of the communication system performance and is commonly used to control the noise or cochannel interference level in wireless communication systems.

In the interference-limited environment, outage probability is defined as the probability that combined-SIR falls below a given outage threshold λ, also known as a protection ratio. The outage probability can be evaluated using (10) as

$$P_{out} = \int_0^{\lambda} p(z)dz = F(\lambda) \qquad (11)$$

Based on (11), we have analysed outage probability for different condition parameters. Fig. 2 shows the outage probability versus the normalized average SIR, S_1/λ, for various values of shaping parameter k_d. We have observed balanced $S_1=S_2$ and unbalanced $S_2=S_1/2$ case at the input branches. It is obvious that when parameter k_d increases, shadowing severity decreases and so the outage probability decreases. Also, it is noticable that the performance gain achieved when k_d increases from 1 to 2 is better compared to the performance gain achieved when k_d goes from 2 to 3. The better performance gain is achieved for balanced SIRs at the input for all randomly picked values of parameters.

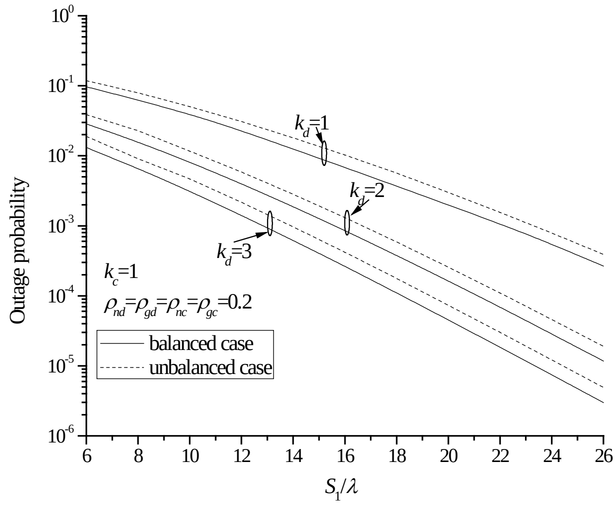

Fig. 2. Outage probability for various values of parameter k_d

The outage probability, as a function of correlation coefficients $\rho_{nd}=\rho_{nc}$ for different values of shaping parameters, k_d and k_c, is shown in Fig. 3. We observed the balanced SIRs at the input $S_1=S_2=S$ and different values for normalized average SIR S/λ. When shaping parameters increases, the shadowing severity decreases and performance is better. When correlation coefficients increase, the outage performance is degraded. The better outage performance is achieved in the case of S/λ=20dB.

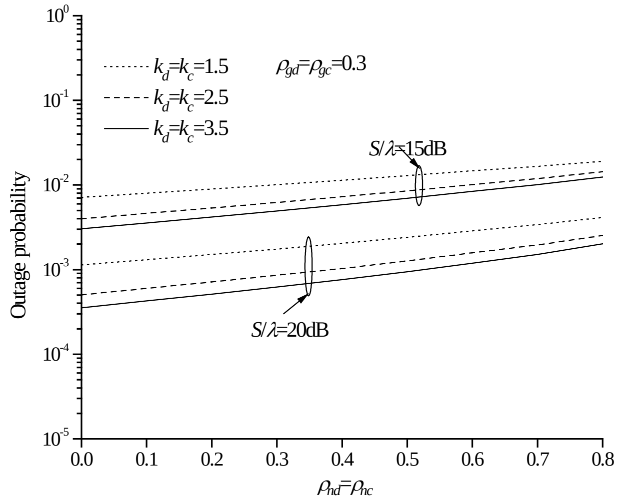

Fig. 3. Outage probability as a function of correlation coefficients

IV. CONCLUSION

In this paper, the performance analysis of system with SC based on SIR, over correlated composite fading channels, was carried out. The K distribution was observed for evaluating outage performances of proposed diversity system. The new expression for cdf of SIR at the output of SC receiver was derived. Using this new formula, the outage probability was efficiently evaluated. The effects of various parameters, such as the shadowing severity parameters and level of correlation of desired signal and interference to the system's performance were also examined.

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