The influence of multiple co-channel interferers on the Selection Diversity System Performance over Weibull fading channels

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[1]***Abstract*** **— Selection combining (SC) diversity receiver experiencing an arbitrary number of multiple, independent, equal power co-channel interferers, in the presence of Rayleigh fading channels was analyzed. Closed form expressions are obtained for the output SIR's probability density function (PDF) and cumulative distribution function (CDF). In order to show the effects of the number of multiple interferers, diversity order and input SIR unbalance to the system performances, an outage probability (OP) analysis is derived. Another important measure of the system's performances, an average bit error probability (ABER) is efficiently evaluated for non-coherent modulation schemes such as binary frequency-shift keying (BFSK) and binary differentially phase-shift keying (BDPSK).**

Keywords **— co-channel interference, correlated fading channels, probability distribution, Weibull fading, selection diversity combining**

I. Introduction

In cellular communication systems, usually a large number of low-power transmitters broadcast a signal in relatively small geographic areas - cells. Commercial and military cellular systems tend to conserve the available spectrum by reusing allocated frequency channels in areas that are geographically located as close to each other as possible. Unfortunately, due to frequency reuse, signals from two or more channels operating at the same frequency, but from different locations, interfere. Amount of co-channel interference determines limitation in distance for reusing frequency channels.

In general, the power of any interfering signal diminishes with increasing distance between interfering users. A carrier frequency can be reused if the interference level is reduced sufficiently by separation between the co-channel calls.

In order to determine the practical system implementation which satisfies the predetermined minimum performance levels, it is necessary to analyze how the interference as a general distortion affects well-accepted criterions of performance of wireless systems, such as outage probability and average bit error probability [17]. The interference level can be measured through the signal-to-interference power ratio (SIR). The SIR ratio is the primary criteria used in designing frequency reuse plans.

Multipath fading can also seriously degrade performances of wireless communication systems. In cellular radio systems, various techniques for reducing fading effects and influence of co-channel interference are used. Space diversity reception is an effective remedy that exploits the principle of providing the receiver with multiple faded replicas of the same information-bearing signal. When multiple receiver antennas are used space diversity is an efficient method for amelioration system's quality of service (QoS). Two criteria are necessary for obtaining high degree of improvement from a space diversity system. First, the fading between the branches should have low cross-correlation. Second, the mean power available from each branch should be almost equal.

In fading environments as in cellular systems where the level of the co-channel interference is sufficiently high as compared to the thermal noise, SC selects the branch with the highest signal-to-interference ratio (SIR-based selection diversity) [5].

This type of SC in which the branch with the highest SIR is selected, can be measured in real time both in base stations and in mobile stations using specific SIR estimators as well as those for both analog and digital wireless systems (e.g., GSM, IS-54) [6-7]. Most of the recently the published papers assume independent fading between the diversity branches and also between the co-channel interferers.

The effect of co-channel interference on the performance metrics of wireless communication system has been extensively analyzed [8-12]. In [8-9] performance analysis of optimum combining with multiple co-channel interferers over Rayleigh fading channels were presented. In [10] closed-form expressions for outage probabilities of mobile radio channels experiencing multiple co-channel, independent Nakagami-*m* interferers are derived. SIR based analysis of dual branch SC was presented in [11-14], but for the case of single interferer over each channel.

In this paper an analytical study of SIR based selection combining involving assumed Weibull fading channels experiencing an arbitrary number of multiple, independent, equal power Weibull co-channel interferers, will be presented.

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For proposed system model, closed form expressions for cumulative distribution function (CDF) and probability distribution function (PDF) of the output SIR will be derived. Infinite-series expressions for important performance measures such as the outage probability will be obtained. Outage probability will be shown graphically for different system parameters. Effects of the number of multiple interferers, diversity order and input SIR unbalance to the system performances will be discussed. In designing a cellular mobile system, one may wish to determine optimal values of system parameters in order to achieve reasonable influence of interferers on the outage, so this discussion could have high level of significance. An average error probability will be also efficiently evaluated for several non-coherent modulation schemes such as binary frequency-shift keying (BFSK) and binary differentially phase-shift keying (BDPSK).

II. SYSTEM MODEL

Let us assume M_i independent equal power Weibull distributed interferers over i-th branch of the SC diversity system with arbitrary number of branches. This assumption is a reasonable one when all interfering signals are at approximately the same distance from the mobile station. The independent instantaneous interfering signals are added together to produce the resultant instantaneous interfering signal at the i-th branch of diversity system can be written as:

$$I_i = I_{1i} + I_{2i} + I_{3i} + \cdots I_{M_i} \tag{1}$$

with the total probability density function given by [10] as:

$$p_{y_i}(y_i) = \frac{\alpha_i M_i^{M_i} y_i^{\alpha_i M_i - 1}}{M_i^{M_i} \Omega_{yi}^{M_i} \Gamma(M_i)} \exp\left(-\frac{y_i^{\alpha_i}}{\Omega_{yi}}\right) \tag{2}$$

where Ω_{yi} is the average power of each interferer

The desired signal envelopes on the i-th diversity branch also follow the Weibull fading distribution, whose probability density function is given by:

$$p_{x_i}(x_i) = \frac{\alpha_i x_i^{\alpha_i - 1}}{\Omega_{xi}} \exp\left(-\frac{x_i^{\alpha_i}}{\Omega_{xi}}\right) \tag{3}$$

In previous equations parameters α_i denote Weibull-fading parameters ($\alpha_i > 0$) which represent fading intensity measure.

Let $\lambda_i = x_i^2/y_i^2$ be the SIR at the i-th (i=1,2…..N) diversity branch of the SC receiver. The joint probability density function of independent random variables $\lambda_1, \lambda_2 \ldots \lambda_N$ (since branches are not correlated), can be written as:

$$\begin{aligned} &p_{\lambda_1,\lambda_2\ldots\lambda_N}(\lambda_1,\lambda_2\ldots,\lambda_N) = p_{\lambda_1}(\lambda_1)\, p_{\lambda_2}(\lambda_2)\cdots p_{\lambda_N}(\lambda_N); \\ &p_{\lambda_i}(\lambda_i) = \int_0^\infty y_i\, p_{x_i}(y_i\lambda_i)\, p_{y_i}(y_i)\, dy_i \end{aligned} \tag{4}$$

After substituting (2) and (3) into (4) and some mathematical manipulation pervious expression can be written in the form of:

$$p_{\lambda_i}(\lambda_i) = \frac{1}{2} \frac{\alpha_i \lambda_i^{\frac{\alpha_i}{2}-1} M_i^{M_i} S_i^{M_i} \Gamma(M_i+1)}{\Gamma(M_i)\left(\lambda_i^{\frac{\alpha_i}{2}} + S_i M_i\right)^{M_i+1}} \tag{5}$$

with $S_i = \dfrac{\Omega_{xi}}{M_i \Omega_{yi}}$ being the average SIR's at the i-th input branch of the selection combiner system.

Similarly, joint cumulative distribution function can be written in the form of:

$$\begin{aligned} &F_{\lambda_1,\lambda_2\ldots\lambda_N}(\lambda_1,\lambda_2\ldots,\lambda_N) = F_{\lambda_1}(\lambda_1)\, F_{\lambda_2}(\lambda_2)\cdots F_{\lambda_N}(\lambda_N); \\ &F_{\lambda_i}(\lambda_i) = \int_0^{\lambda_i} p_{t_i}(t_i)\, dt_i \end{aligned} \tag{6}$$

After substituting (5) into (6) and some mathematical manipulation pervious expression can be written in the form of:

$$F_{\lambda_i}(\lambda_i) = \left(\frac{\lambda_i^{\alpha_i}}{\lambda_i^{\alpha_i} + S_i M_i}\right) \frac{\Gamma(M_i+1)}{\Gamma(M_i)}\, {}_2F_1\left(1,2;1-M_i;\frac{\lambda_i^{\alpha_i}}{\lambda_i^{\alpha_i} + S_i M_i}\right) \tag{7}$$

with ${}_2F_1$ ($u_1,u_2;u_3;x$), being the Gaussian hyper-geometric function [11].

The selection combiner chooses and outputs the branch with the largest SIR.

$$\lambda = \lambda_{out} = \left(\lambda_1, \lambda_2 \ldots \lambda_N\right) \tag{8}$$

The CDF of multibranch SIR-based SC output could be derived from (7) by equating the arguments $\lambda_1 = \lambda_2 = \ldots = \lambda_N$ as:

$$\begin{aligned} &F_\lambda(\lambda) = F_{\lambda_1}(\lambda) F_{\lambda_2}(\lambda) \cdots F_{\lambda_N}(\lambda) = \prod_{i=1}^N F_{\lambda_i}(\lambda); \\ &F_\lambda(\lambda) = \prod_{i=1}^N \left(\frac{\lambda^{\alpha_i}}{\lambda^{\alpha_i} + S_i M_i}\right) \frac{\Gamma(M_i+1)}{\Gamma(M_i)}\, {}_2F_1\left(1,2;1-M_i;\frac{\lambda^{\alpha_i}}{\lambda^{\alpha_i} + S_i M_i}\right) \end{aligned} \tag{9}$$

The probability density function at the output of the SC can be found as:

$$p_\lambda(\lambda) = \frac{dF_\lambda(\lambda)}{d\lambda} = \sum_{i=1}^N p_{\lambda_i}(\lambda) \prod_{\substack{j=1 \\ j\neq i}}^N F_{\lambda_j}(\lambda); \quad p_{\lambda_i}(\lambda) = \frac{dF_{\lambda_i}(\lambda)}{d\lambda};$$

$$\begin{aligned} p_\lambda(\lambda) = \sum_{i=1}^N &\left(\frac{1}{2} \frac{\alpha_i \lambda^{\frac{\alpha_i}{2}-1} M_i^{M_i} S_i^{M_i} \Gamma(M_i+1)}{\Gamma(M_i)\left(\lambda^{\frac{\alpha_i}{2}} + S_i M_i\right)^{M_i+1}} \right. \\ &\left. \prod_{\substack{j=1 \\ j\neq i}}^N \left(\frac{\lambda^{\alpha_j}}{\lambda^{\alpha_j} + S_j M_j}\right) \frac{\Gamma(M_j+1)}{\Gamma(M_j)}\, {}_2F_1\left(1,2;1-M_j;\frac{\lambda^{\alpha_j}}{\lambda^{\alpha_j} + S_j M_j}\right) \right) \end{aligned} \tag{10}$$

Fig. 1 shows the PDF of output SIR for various values of the number of multiple interferers and fading severity.

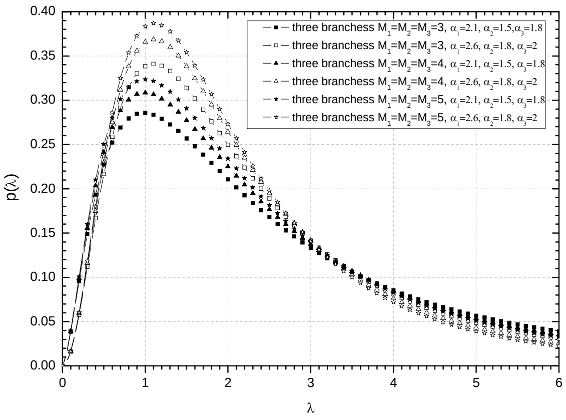

Fig. 1 PDF of output SIR for various values of the number of multiple interferers and diversity branches.

III. Outage Probability

Outage probability P_{out} is one of the accepted performance measure for diversity systems operating in fading environments. P_{out} is a measure of the system performance, used to control the co-channel interference level, helping the designers of wireless communications system's in order to meet the QoS and grade of service (GoS) demands. This performance measure is very useful in wireless communication systems design especially for the cases when co-channel interference is present.

In the interference limited environment, P_{out} is defined as the probability that the output SIR of the SC falls below a given outage threshold γ_{th} also known as a protection ratio. Protection ratio depends on modulation technique and expected QoS.

$$P_{out} = P_R(\xi < \gamma) = \int_0^{\gamma} p_{\xi}(t)\,dt = F_{\xi}(\gamma) \quad (12)$$

Outage probability versus normalized parameter S_1/γ for balanced and unbalanced ratio of SIR at the input of the branches and various values of the number of multiple interferers, diversity order is shown on Fig. 2.

From Fig. 2. we can see that for the constant number of co-channel interferers outage probability behavior improves as the diversity order (number of branches) increases. It can be observed that if we want to achieve the same quality of the transmission (for example outage probability of 10^{-4}), we need higher level of average input SIR for dual branch case than for the triple branch case (for example of above 6 dB).

IV. Average Error Probability

The average error probability at the SC output is derived for non-coherent and binary signaling according to following expressions:

$$P_e = \int_0^{\infty} p_{\zeta}(t)\frac{1}{2}e^{-gt}dt, \quad (13)$$

Where g denotes modulation constant, i.e., g = 1 for BDPSK and g = 1/2 for BFSK. Substituting (11) in (13) numerically obtained average error probability is shown on Figs. 3 and 4 for balanced and unbalanced ratio of SIR at the input of the branches and various values of diversity order and the number of multiple interferers.

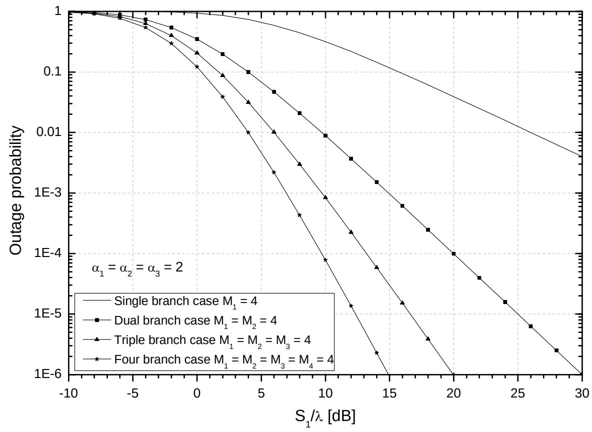

Figure 2. Outage probability versus normalized parameter S_1/γ for various numbers of diversity order

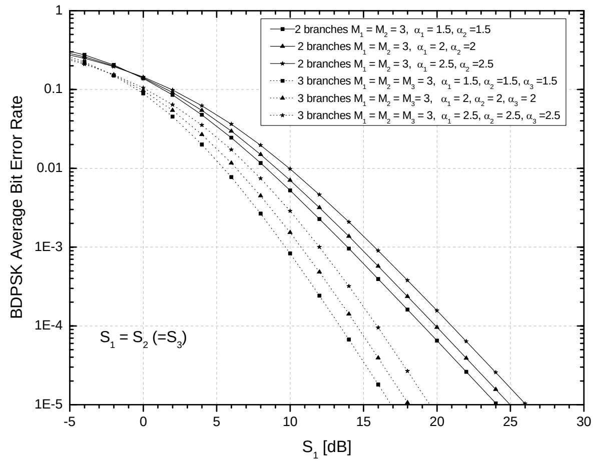

Figure 3. Average BER versus S_1 in non-coherent BDPSK versus SIR at the input of the branches and various numbers of diversity order and fading severity

It is evident how ABER increases at both figures when the number of multiple independent co-channel interferers increases due to growth of interference domination. ABER performance behavior improves for the constant number of co-channel interferers as the diversity order increases. If we want to achieve the same quality of the transmission (for example outage probability of 10^{-4}), we need higher level of average input SIR for dual branch case then for the triple branch case (for example of above 6 dB for BDPSK).

Also comparison of curves shows better performance of BDPSK modulation scheme against BFSK modulation scheme.

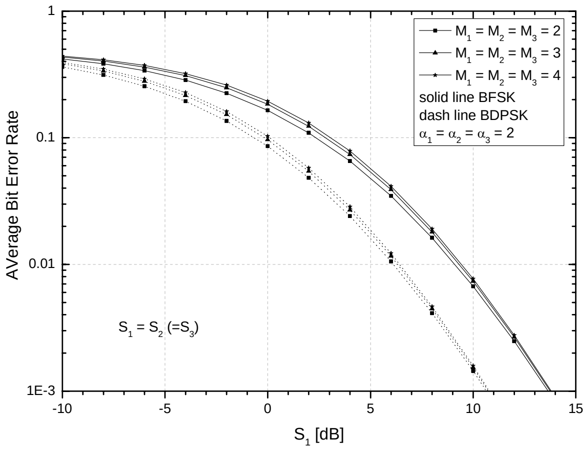

Figure 4. Average BER versus S_1 in BFSK and BDPSK at triple branch diversity for various numbers of multiple interferers

V. CONCLUSION

Multibranch SIR-based SC diversity receiver operating over Weibull fading channels where each channel experiences an arbitrary number of multiple, independent, Weibull co-channel interferers with equal average powers, was analyzed in this paper. Expressions for the first order statistics of the combiner output, namely SIR's probability density function (PDF) and cumulative distribution function (CDF) are presented in the closed form. Standard performance measures, OP and ABER for some non-coherent modulation techniques, are graphically analyzed, in order to show the effects of the number of fading severity, multiple interferers, diversity order. Obtained expressions, analysis and discussions, could find application in designing cellular mobile systems.

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