Methods for Generation of Compact Lumped Element Model for Passive Microwave Circuits

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***Abstract* – The representation of distributed passive electromagnetic structures by lumped element circuits is considered in this paper. Network models are established by a combination of system identification and circuit synthesis methods and their subsequent application to data obtained by TLM numerical simulation. Two different methods for synthesis of compact lumped element models for linear lossy reciprocal multiports are discussed in the paper. Accuracy and efficiency of developed compact models are verified on the example of a low-pass microstrip filter.**

***Keywords* – Distributed passive electromagnetic structures, circuit synthesis methods, compact model.**

I. INTRODUCTION

As modern analog and digital electronics are operating at microwave and millimeter wave frequencies and at gigabit rates, full-wave electromagnetic (EM) tools for the design and modeling of distributed passive circuit structures are required [1]. However, a different approach to overall system analysis is possible, based on an extraction of an equivalent circuit model from a full-wave electromagnetic simulation of the structure under consideration. Such compact lumped element models can be embedded into conventional circuit simulators and treated by methods of network theory [2], [3]. Compared with field oriented simulation, the application of network oriented design methods yields considerable lower computational effort and time.

In general, the representation of distributed circuits by lumped element network models requires that the transfer function of a distributed circuit is realized by an equivalent circuit with an infinite number of lumped circuit elements. This equivalent network should give the same response for a required excitation, as the considered distributed structure (so-called synthesis problem [4]). As this description needs to be valid within a certain frequency range only and within a certain accuracy margin, one can find an equivalent circuit with a limited number of circuit elements. This lumped element model provides a compact description of the distributed circuit. Such representation of distributed circuit sections by a lumped element models can be very useful especially in the case of modeling complex circuit containing also nonlinear and active lumped elements.

In order to generate compact model, the combination of system identification (SI) of microwave structures and subsequent lumped element model synthesis has to be performed by full-wave simulation or measuring of the input and output signals of the device in the time or in the frequency domain [5-8]. To establish the network model of a distributed circuit a three-step procedure is performed:

1. Determination of the transfer functions by numerical EM full-wave analysis or by measurement,
2. Determination of the rational functions representing the transfer functions by system identification using e.g. vector fitting (VF) method,
3. Synthesis of a lumped element equivalent circuit realizing the transfer function.

In this paper, two different methods are discussed in order to perform step 3 - synthesis of compact lumped element models for linear lossy reciprocal multiports. The first method is based on Foster expansion for lossless circuit and its extension to account for lossy structures as suggested in [9]. The second method is Brune's synthesis procedure that ultimately provides network synthesis by positive lumped elements [10-13] with minimum number of elements. Accuracy and efficiency of compact models, developed by these two methods, are verified on the example of a low-pass microstrip filter. The first two steps in the above mentioned procedure to create the network model of a distributed circuit are subsequently performed by the TLM (Transmission-Line Matrix), electromagnetically based, numerical method in the time-domain [14] and VF method originally introduced in [15-17].

II. CIRCUIT SYNTHESIS METHODS

In this section, two systematic synthesis procedures for the generation of lumped element equivalent circuit models for passive microwave structures are described. Initial data to develop a compact model for the structure under consideration can be generated either by numerical full wave analysis or by measurement. Then, for the impedance data, obtained in this way, an approximation by a rational function is performed. The rational fit is computed at discrete frequencies over the bandwidth of interest. This yields the following closed form expression:

$$Z(s) = E \cdot s + D + \sum_{k=1}^{K} \frac{B_k}{s - s_k} . \qquad (1)$$

A. Extended Foster Equivalent Circuit Synthesis

The Foster realization approach starts with the characterization of the given impedance parameters z_{ij} by partial fraction expansion:

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$$z_{ij}(s) = \frac{k_{ij}^{(0)}}{s} + \sum_m \frac{2k_{ij}^{(m)} s}{s^2 + \omega_m^2} + \ldots + k_{ij}^{(\infty)} s\,. \qquad (2)$$

For example, for a two-port device, the Foster impedance realization can be obtained if all Z parameters are known:

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \left(\begin{bmatrix} k_{11}^{(0)} & k_{12}^{(0)} \\ k_{21}^{(0)} & k_{22}^{(0)} \end{bmatrix} + \sum_m \begin{bmatrix} k_{11}^{(m)} & k_{12}^{(m)} \\ k_{21}^{(m)} & k_{22}^{(m)} \end{bmatrix} + \begin{bmatrix} k_{11}^{(\infty)} & k_{12}^{(\infty)} \\ k_{21}^{(\infty} & k_{22}^{(\infty)} \end{bmatrix} \right) \cdot f^{(m)} \quad (3)$$

where:

$$f^{(m)} = \begin{cases} \dfrac{1}{s}, & \text{for } m = 0 \\ \dfrac{2s}{s^2 + \omega_m^2}, & \text{for } m = 1,2,\ldots,\mathrm{M} \\ s, & \text{for } m = \infty \end{cases} \qquad (4)$$

and s is complex frequency; $k_{ij}^{(0)}$ and $k_{ij}^{(\infty)}$ are residues of the poles at zero and infinity, respectively; $k_{ij}^{(m)}$ is the residue at the intermediate pole of the complex frequency ω_m and M is a total number of intermediate poles.

Once the Foster expansions have been obtained for each impedance parameter, the poles and residues are used to determine the equivalent circuit component values. For each column in the Foster expansion, there will be a corresponding sub T-network [9]. For the equivalent circuit to be realizable, the transformer turns ratio, $a^{(m)}$, for an arbitrary pole m must comply within the following equations

$$\frac{\left|k_{12}^{(m)}\right|}{k_{11}^{(m)}} \le a^{(m)} \le \frac{k_{22}^{(m)}}{\left|k_{12}^{(m)}\right|}, \text{ and } \frac{k_{12}^{m)}}{a^{(m)}} \ge 0\,. \qquad (5)$$

The equations outlined above are for lossless networks. Therefore, all residues k_{ij} are real and poles are approximated to lie along the imaginary axis. For a lossy structure, the Foster expansion term for an intermediate pole ω_m for all z_{ij} has to be modified as suggested in [9]:

$$\frac{2k_{ij}^{(m)} s}{s^2 + \omega_m^2 + js / B^{(m)}}, \qquad (6)$$

where $B^{(m)}$ in the linear term added to the denominator is obtained directly from the real part of impedance calculated either from EM simulation or measurement.

B. Brune's Equivalent Circuit Synthesis

Equivalent lumped element circuits for general lossy or lossless two-ports, such as considered in this paper, can be obtained from Brune's circuit synthesis procedure [10-13]. Cauer or Foster representations of lossless circuits, explained in subsection *A*, can be extended to lossy circuits but negative elements even for passive circuits would result. Brune synthesis yields the realization of passive circuits with minimum number of elements.

A positive real (PR) character is required for the impedance function to be synthesized. The impedance (or admittance) function is of the form:

$$W = \frac{a_n \cdot s^n + a_{n-1} \cdot s^{n-1} + a_{n-2} \cdot s^{n-2} + \cdots + a_0}{b_n \cdot s^n + b_{n-1} \cdot s^{n-1} + b_{n-2} \cdot s^{n-2} + \cdots + b_0} = \frac{P(s)}{Q(s)}\,. (7)$$

For a PR function, all poles and zeros are located in the left half of the complex frequency plane, or on the imaginary axis respectively. Poles or zeros lying on the imaginary axis can be separated from the rational function without disturbing the function's PR character. In Brune's equivalent network synthesis procedure, the impedance function is analyzed and poles and zeros on the imaginary axis can be separated from the impedance (or admittance) function (7), and can be realized in a subcircuit in a straightforward manner. However, if all poles and zeros are strictly in the left half plane a special so called Brune's process is applied. The global minimum of the real part of the function on the imaginary axis is determined. The value of this global minimum is subtracted from the function. Depending on which frequency this minimum value is found at, we have to extract the real part of the impedance function (for $s = 0$ and $s = \infty$) or, if the minimum occurs at a finite frequency, we have to extract real and imaginary part. Possible subcircuit extractions for this Brune's process are shown in Fig.1.

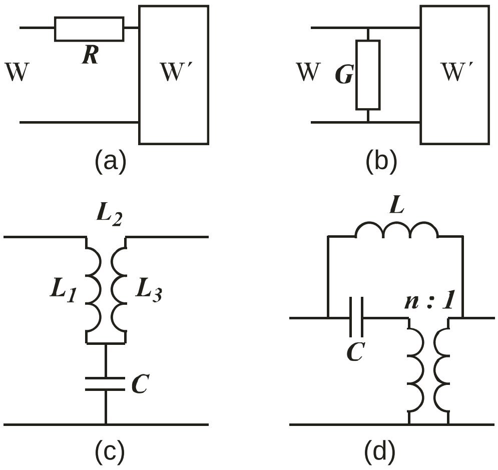


Fig.1. Extracted circuits from the Brune's process

III. NUMERICAL ANALYSIS

For the numerical study, we consider a low-pass microstrip filter, shown in Fig.2, to demonstrate the two methods for synthesis of compact lumped element models for linear lossy reciprocal two-port devices described above.

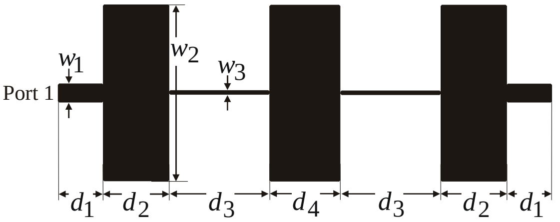


Fig.2. Layout of low-pass microstrip filter

The physical dimensions of symmetrical low-pass filter in millimeters are: line widths $w_1 = 0.217054$, $w_2 = 2.31921$ and $w_3 = 0.0248336$; line lengths $d_1 = 0.566318$, $d_2 = 0.84057$, $d_3 = 1.29201$ and $d_4 = 0.901333$. The substrate height is $h = 0.2$ mm and its relative permittivity is $\varepsilon_r = 12.9$.

The full wave EM analysis results are obtained from TLM simulations. In order to generate the equivalent circuit, impedance parameters of the full-wave analysis have to be de-embedded. Compact lumped element models obtained by Foster and Brune's synthesis procedure are shown in Figs.3 and 4, respectively. Before applying Brune's method, symmetric two-port device from Fig.2 is first transformed into a connection of one-ports using Bartlett's theorem. These one-ports are then synthesized by Brune's process which yields a minimum number of elements.

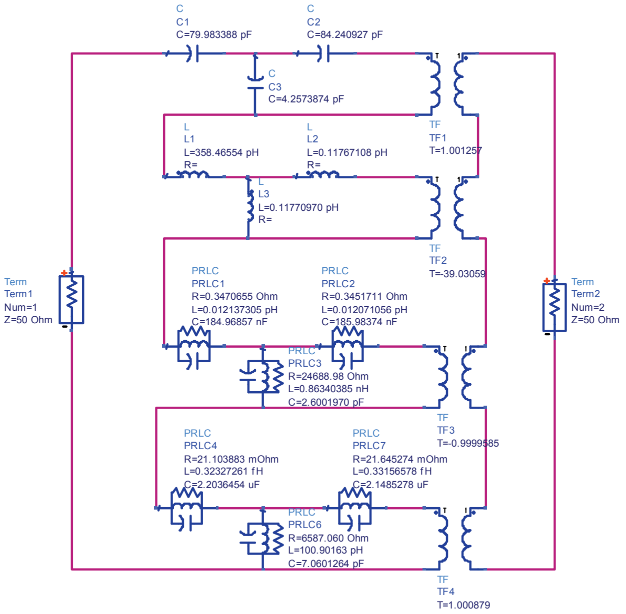
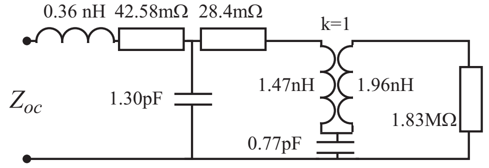
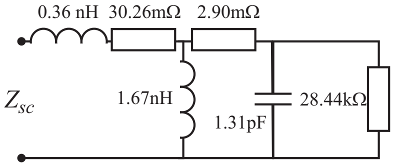


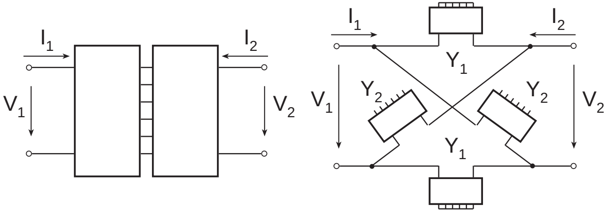
Fig.3. Equivalent Foster circuit



(a) Equivalent Brune's circuit of Y_2



(b) Equivalent Brune's circuit of Y_1



(c) Bartlett's transformation of symmetric two-port network

Fig.4. Equivalent Brune's circuit

Figs.5-8 compare the synthesized equivalent Foster and Brune's circuit scattering matrix results to the results obtained directly from EM simulation.

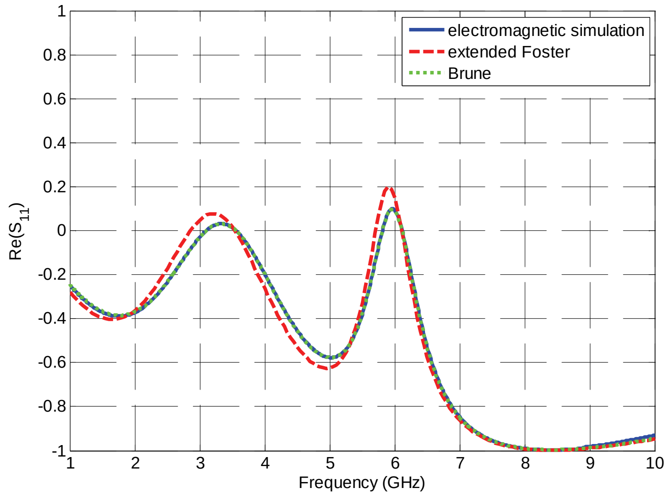


Fig.5. Real part of S_{11} parameter

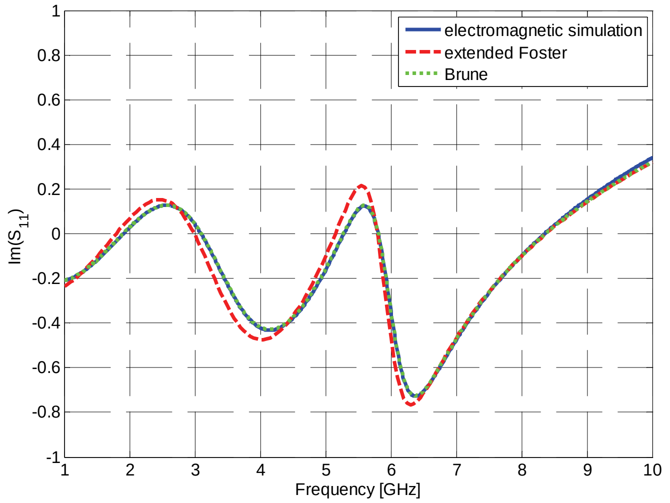


Fig.6. Imaginary part of S_{11} parameter

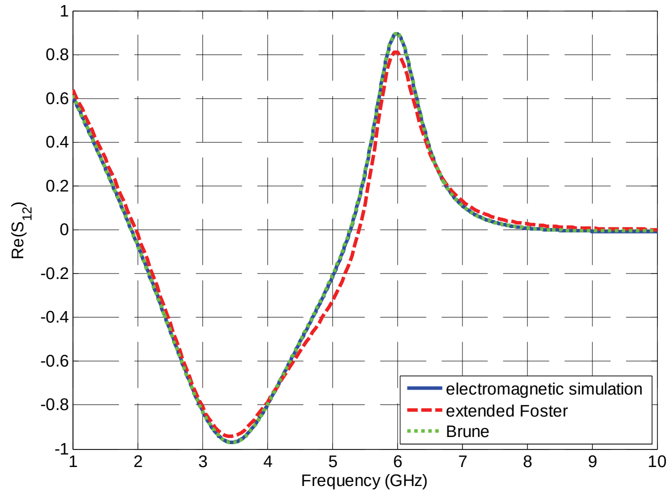


Fig.7. Real part of S_{21} parameter

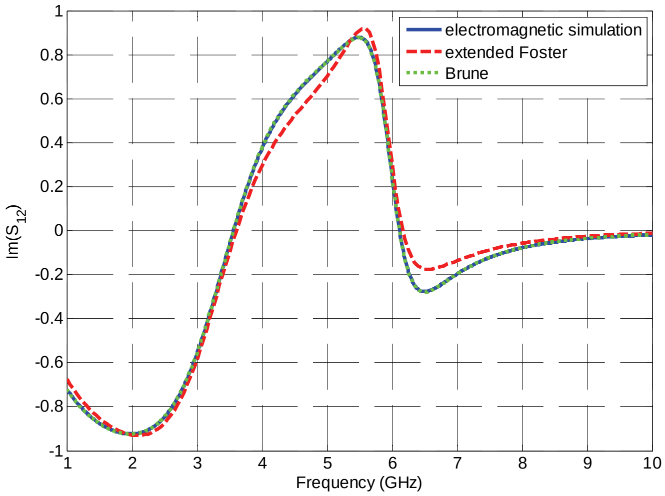


Fig.8. Imaginary part of S_{21} parameter

IV. CONCLUSION

Two methods for synthesis of compact lumped element models for linear lossy reciprocal multiports are discussed in the paper. Brune’s method has provided a better agreement approximation of the EM simulation data, it is more suitable for the lossy structures as it provides only positive lumped elements in the equivalent circuit. However, it is applicable at the moment to the one-port device; symmetrical two-port devices can be transformed into one-ports by using Bartlett’s theory. In the extended Foster approach, the agreement of the approximation to the EM data was slightly reduced, and this approach could result in negative elements in the case of lossy structures, but it can be apllied easily to multiport devices.

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