Synthesis of Transfer Wave Matrix Polynomials for Digital Structure of Microstrip Ultra-Wideband Filter utilizing Short-circuited Stubs

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Abstract – **An efficient method based on transfer wave matrix polynomials is proposed to obtain the scattering parameters in *z*-domain of planar microstrip structure utilizing short-circuited stubs. A wave digital network (WDN) which represents a digital model of a microstrip stub-line structure is observed here. WDN is composed of cascaded unit elements and three-port adaptors. An ultra-wideband filter realized in microstrip technique, proving the response accuracy of the new technique, is given.**

Keywords – **Wave digital approach, wave digital networks, microwave circuits, short-circuited stubs, ultra-wideband filter.**

I. INTRODUCTION

The wave digital concept has been introduced in order to obtain digital filter structures that, due to their inherent passivity, possess many advantageous properties such as stability. Wave digital filters (WDFs) represent a class of digital filters with a particular interest. A detailed discussion of WDF theory is given in references [1-5]. Well known theory of WDFs is used for modeling of the planar structures by wave digital elements [5-14].

The basic idea of the 1D wave digital approach is to treat the complex structure as a typical connection of several uniform segments. The delays of uniform segments vary from one another, and because of this each segment has to be represented as cascade of several unit elements (UEs).

The wave digital model of a short-circuited transmission line, which represents the background to the modeling strategies used here, is given in [14].

This paper is devoted to the synthesis of polynomials of transfer wave matrix elements in *z*-domain using wave digital approach. Microstrip structures, such as ultra-wideband filter utilizing short-circuited stubs, can be modeled by use of the wave digital networks (WDNs) in MATLAB environment [15]. WDN response can be calculated in the frequency or in the time domain directly from known network function in *z*-domain that is going to be presented in this paper.

The paper is organized as follows. The WDN of the observed structure is given in Section II. Moreover, the calculating of the scattering parameters of the known WDNs is described.

Finally, to show the validity of the proposed modeling and analysis approaches of a microstrip ultra-wideband filter utilizing short-circuited stubs, the simulated results of wave digital approach are presented and discussed in Section III.

II. NETWORK TRANSFER WAVE MATRIX

The calculating of the wave transfer matrix polynomilas of the known WDNs is discussed here.

A planar microstrip stub-line structure can be represented as M uniform segments connected in a typical way. The ladder wave digital network of the planar microstrip stub-line structure is composed of several building blocks: UEs, multipliers, and two-port and three-port adaptors.

A WDN with n_t UEs and $M-1$ three-port parallel adaptors, pictured in Fig. 1, is analyzed here. This WDN is a two-port circuit having at each port an input and an output wave variable. Each uniform segment of the planar structure – so-called UTL segment (uniform transmission lines and stubs) - is modeled by n_k cascaded sections, $k=1,2,\ldots,M$ (assigned as $n_k \times T$ blocks in the Fig. 1). Each UE is associated with its delay T, and port resistances R_k at either port. The port impedances of the UEs in those blocks are equal, which means that they can be directly cascade connected (coefficients of two-port adaptors are zeros). The simulation of connections between the three models of UTL segments (one of them is a short-circuited stub connected at dependent port) is achieved by three-port parallel adaptor. The incident wave A_0 is equal to voltage U_S of the source, and reflected wave B_m is equal to voltage $2U_L$ on the load. The first and the last two-port series adaptors are used for matching source and load resistances to the rest of the WDN.

The transfer wave matrix for a single UE is

$$\boldsymbol{T}_{UE} = \begin{bmatrix} 1 & 0 \\ 0 & z \end{bmatrix} = \frac{1}{z^{-1}} \cdot \begin{bmatrix} z^{-1} & 0 \\ 0 & 1 \end{bmatrix}. \qquad (1)$$

Finally, the transfer wave matrix for one uniform segment which is modeled by n_k cascaded UEs is

$$\boldsymbol{T}_{UE}^{n_k} = \underbrace{\boldsymbol{T}_{UE} \times \boldsymbol{T}_{UE} \times \cdots \times \boldsymbol{T}_{UE}}_{n_k} = \frac{1}{z^{-n_k}} \cdot \begin{bmatrix} z^{-n_k} & 0 \\ 0 & 1 \end{bmatrix} \qquad (2)$$

The k^{th} three-port parallel adaptor with port 2 chosen as dependent port (Fig. 1) is described by set of equations

$$B_{k-1} = B_k + A_k - A_{k-1}, \qquad (3a)$$

$$B_{k+1} = B_k + A_k - A_{k+1}, \qquad (3b)$$

$$B_k = A_k + \alpha_{k-1} \cdot (A_{k-1} - A_k) + \alpha_k \cdot (A_{k+1} - A_k), \qquad (3c)$$

where the multiplier coefficients are

$$\alpha_{k-1} = \frac{2G_{k-1}}{G_{k-1} + G_k + G_{k+1}}, \quad \alpha_k = \frac{2G_{k+1}}{G_{k-1} + G_k + G_{k+1}}, \qquad (4)$$

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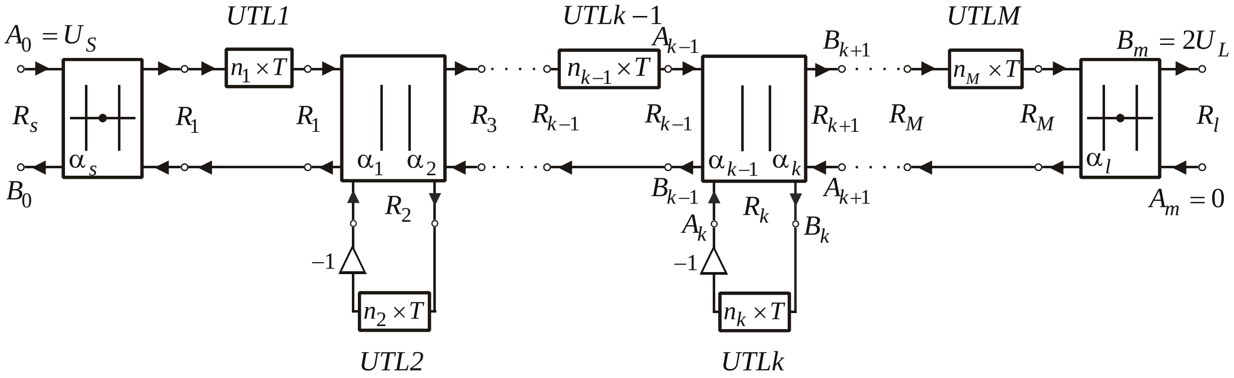

Fig. 1. Wave digital network of a planar microstrip structure with short-circuited stubs

and G_{k-1}, G_k and G_{k+1} are the port conductances. A_{k-1}, A_k and A_{k+1} are incident, and B_{k-1}, B_k and B_{k+1} are reflected waves at ports (*k*-1)-(*k*-1)’, *k*-*k*’ and (*k*+1)-(*k*+1)’, respectively.

A short-circuited stub is connected to the port 2 being dependent and for this port the wave variables can be written

$$A_k = -z^{-n_k} \cdot B_k \,. \tag{5}$$

If dependent port 2 of three-port adaptor is terminated by a model of a short-circuited stub, its network is reduced to a two-port network, and for wave transfer matrix can be written

$$\begin{bmatrix} B_{k-1} \\ A_{k-1} \end{bmatrix} = \boldsymbol{T}^{n_k}_{\alpha_{k-1}\alpha_k} \cdot \begin{bmatrix} A_{k+1} \\ B_{k+1} \end{bmatrix} = \begin{bmatrix} T^{n_k}_{11} & T^{n_k}_{12} \\ T^{n_k}_{21} & T^{n_k}_{22} \end{bmatrix} \cdot \begin{bmatrix} A_{k+1} \\ B_{k+1} \end{bmatrix} \tag{6}$$

The wave transfer matrix for the k^{th} three-port adaptor, with a short-circuited stub connected on the dependent port 2, $\boldsymbol{T}^{n_k}_{\alpha_{k-1}\alpha_k}$ is obtained by use of relations (2)-(5) and its elements are

$$T^{n_k}_{11} = \frac{-\alpha - z^{-n_k}}{\alpha_{k-1} \cdot (1 - z^{-n_k})}\,, \tag{7a}$$

$$T^{n_k}_{12} = \frac{(\alpha_{k-1} - 1) + (\alpha_k - 1) \cdot z^{-n_k}}{\alpha_{k-1} \cdot (1 - z^{-n_k})}\,, \tag{7b}$$

$$T^{n_k}_{21} = \frac{(1 - \alpha_k) + (1 - \alpha_{k-1}) \cdot z^{-n_k}}{\alpha_{k-1} \cdot (1 - z^{-n_k})} \tag{7c}$$

and

$$T^{n_k}_{22} = \frac{1 + \alpha \cdot z^{-n_k}}{\alpha_{k-1} \cdot (1 - z^{-n_k})}\,, \tag{7d}$$

where

$$\alpha = 1 - \alpha_{k-1} - \alpha_k \,. \tag{7e}$$

For WDN depicted in Fig. 1, the elements of the wave transfer matrix satisfy relations

$$B_0 = T_{11} \cdot A_m + T_{12} \cdot B_m \,, \tag{8}$$

$$A_0 = T_{21} \cdot A_m + T_{22} \cdot B_m \,. \tag{9}$$

The complete transfer wave matrix *T* corresponding to the analyzed WDN is a product of the wave matrices of network building blocks as

$$\boldsymbol{T} = \boldsymbol{T}_{\alpha_s} \times \boldsymbol{T}^{n_1}_{UE} \times \boldsymbol{T}^{n_2}_{\alpha_1\alpha_2} \times \boldsymbol{T}^{n_3}_{UE} \times \ldots \times \boldsymbol{T}^{n_{k-1}}_{UE} \times \boldsymbol{T}^{n_k}_{\alpha_{k-1}\alpha_k} \times \\ \times \boldsymbol{T}^{n_{k+1}}_{UE} \times \ldots \times \boldsymbol{T}_{\alpha_l} \,. \tag{10}$$

Consider now the matrices of two-port series adaptors as follows

$$\boldsymbol{T}_{\alpha_s} = \frac{1}{1 - \alpha_s} \cdot \begin{bmatrix} -1 & \alpha_s \\ \alpha_s & -1 \end{bmatrix} = \frac{1}{1 - \alpha_s} \cdot \boldsymbol{Q}_s \tag{11}$$

and

$$\boldsymbol{T}_{\alpha_l} = \frac{1}{1 - \alpha_l} \cdot \begin{bmatrix} -1 & \alpha_l \\ \alpha_l & -1 \end{bmatrix} = \frac{1}{1 - \alpha_l} \cdot \boldsymbol{Q}_l \,, \tag{12}$$

where adaptors’ coefficients are

$$\alpha_s = \frac{R_s - R_1}{R_s + R_1}, \text{ and } \alpha_l = \frac{R_M - R_l}{R_M + R_l}, \tag{13}$$

with the port resistances R_s, R_1, R_M, and R_l assigned as shown in Fig. 1.

The matrix of one uniform segment modeled with n_{2j-1} UEs can be written in the form

$$\boldsymbol{T}^{n_{2j-1}}_{UE} = \frac{1}{z^{-n_{2j-1}}} \cdot \begin{bmatrix} z^{-n_{2j-1}} & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{z^{-n_{2j-1}}} \cdot \boldsymbol{Q}_{n_{2j-1}} \,, \tag{14}$$

where is $j = 1, 2, \ldots, N_1$.

The matrix of three-port parallel adaptor with stub on dependent port 2 which is modeled by n_{2j} UEs is given in the form

$$\boldsymbol{T}^{n_{2j}}_{\alpha_{2j-1}\alpha_{2j}} = \frac{1}{\alpha_{2j-1} \cdot (1 - z^{-n_{2j}})} \cdot \boldsymbol{Q}_{n_{2j}} \tag{15a}$$

where the matrix $\boldsymbol{Q}_{n_{2j}}$ elements are

$$Q^{n_{2j}}_{11} = -\alpha - z^{-n_{2j}} \,, \tag{15b}$$

$$Q^{n_{2j}}_{12} = (\alpha_{2j-1} - 1) + (\alpha_{2j} - 1) \cdot z^{-n_{2j}} \,, \tag{15c}$$

$$Q^{n_{2j}}_{21} = (1 - \alpha_{2j}) + (1 - \alpha_{2j-1}) \cdot z^{-n_{2j}} \,, \tag{15d}$$

$$Q^{n_{2j}}_{22} = 1 + \alpha \cdot z^{-n_{2j}} \,, \tag{15e}$$

and $j = 1, 2, \ldots, N_1$. The analyzed planar microstrip structures can have different number of UTL segments. The number N_1 depends on the number of segments in structure as

$$N_1 = \begin{cases} M/2 & \text{for } M \text{ being even,} \\ (M-1)/2 & \text{for } M \text{ being odd.} \end{cases} \tag{16}$$

According to the relations (10)-(16), the polynomials can be written in the form

$$W_e(z) = (1-\alpha_s)\cdot(1-\alpha_l)\cdot\prod_{j=1}^{N_1}\left(\alpha_{2j-1}\cdot(1-z^{-n_{2j}})\cdot z^{-n_{2j-1}}\right)$$

$$\text{for even } M, \tag{17}$$

or $$W_o(z) = W_e(z)\cdot z^{-n_M}, \text{ for odd } M. \tag{18}$$

The complete matrix can be represented in the form

$$\boldsymbol{Q}_e(z) = \boldsymbol{Q}_s \times \prod_{j=1}^{N_1}\left(\boldsymbol{Q}_{n_{2j-1}} \times \boldsymbol{Q}_{n_{2j}}\right) \times \boldsymbol{Q}_l, \text{ for even } M, \tag{19}$$

or

$$\boldsymbol{Q}_o(z) = \boldsymbol{Q}_s \times \prod_{j=1}^{N_1}\left(\boldsymbol{Q}_{n_{2j-1}} \times \boldsymbol{Q}_{n_{2j}}\right) \times \boldsymbol{Q}_{n_M} \times \boldsymbol{Q}_l, \text{ for odd } M \tag{20}$$

where the matrix $\boldsymbol{Q}_{n_M}$ corresponds to the last segment in the series branch.

Finally, the complete wave transfer matrix $\boldsymbol{T}$ due to the number of segment in the structure can be written in one of two forms

$$\boldsymbol{T} = \frac{1}{W_e(z)}\cdot\boldsymbol{Q}_e(z), \text{ for even } M \tag{21}$$

or $$\boldsymbol{T} = \frac{1}{W_o(z)}\cdot\boldsymbol{Q}_o(z), \text{ for odd } M. \tag{22}$$

The wave matrix elements are the rational polynomial functions of z^{-1}. In other words, the complete wave transfer matrix can be written in the form of polynomials

$$\boldsymbol{T} = \begin{bmatrix} T_{11}(z) & T_{12}(z) \\ T_{21}(z) & T_{22}(z) \end{bmatrix} = \frac{1}{W_{e/o}(z)}\cdot\begin{bmatrix} Q_{e/o11}(z) & Q_{e/o12}(z) \\ Q_{e/o21}(z) & Q_{e/o22}(z) \end{bmatrix} \tag{23}$$

where index e/o corresponds to even (e) or odd (o) M. Only two elements, $T_{12}(z)$, and $T_{22}(z)$, have to be calculated. If necessary, two other elements, $T_{11}(z)$, and $T_{21}(z)$, can be derived from the previously ones.

Providing $A_m = 0$, the output response (forward voltage transmission coefficient) is

$$S_{21} = \left.\frac{B_m}{A_0}\right|_{A_m=0} = \frac{W_{e/o}(z)}{Q_{e/o22}(z)}, \tag{24}$$

and the input response (input reflection coefficient) is

$$\Gamma_0 = S_{11} = \left.\frac{B_0}{A_0}\right|_{A_m=0} = \frac{Q_{e/o12}(z)}{Q_{e/o22}(z)}. \tag{25}$$

III. RESULTS

The objective of this section is to prove the accuracy of the proposed modeling and analyzing approaches. To demonstrate the main idea and approach, a microstrip ultra wideband filter with a central frequency of 2.1 GHz [16] is depicted. The layout is shown in Fig. 2. It is analyzed on FR-4 substrate with dielectric constant $\varepsilon_r = 4.6$, and the board thickness $h = 0.6\,mm$. Metalisation is cooper and the metal thickness is $t = 17.5\,\mu m$. Its wave-based model is implemented in MATLAB/Simulink in paper [14]. The filter is symmetrical and approximated by connection of 19 uniform segments with parameters given in the Table I.

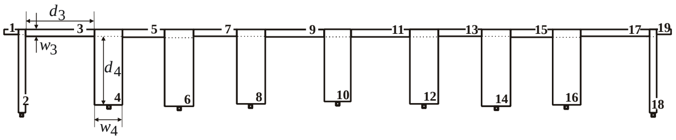

Fig. 2. Layout of ultra-wideband filter with short-circuited stubs

TABLE I. PARAMETERS OF UNIFORM SEGMENTS

nv	d [mm]	w [mm]	Zc [Ohm]	Tv [ps]
1,19	3.1000	1.1000	49.5082	19.2338
2,18	18.0000	1.5000	40.9808	113.1258
3,17	14.8000	1.5000	40.9808	93.0145
4,16	16.3000	6.0000	14.3797	108.8778
5,15	9.0000	1.7000	37.7671	56.8691
6,14	16.6000	6.2000	13.9839	111.0299
7,13	9.3000	1.5000	40.9808	58.4483
8,12	16.1000	6.1000	14.1790	107.6145
9,11	12.6000	1.6000	39.3056	79.4078
10	15.6000	5.7000	15.0181	103.9822

For given error of $n\_er = 0.01\%$, a total minimal number of sections in *WDN* is $n_t = \sum_{k=1}^{19} n_k = 582$. The numbers of sections in individual segments $n_k = round\left[q\cdot T_k / T_{\min}\right]$ are 7, 41, 34, 40, 21, 40, 21, 39, 29, 38, 29, 39, 21, 40, 21, 40, 34, 41, and 7, respectively. A total delay for the digital model of the structure is $T_t = n_t\cdot T_{\min}/q = 1599.1561\,ps$ where a multiple factor is $q = 7$ and a minimum delay is $T_{\min} = \min\{T_1, T_2, ..., T_{19}\} = 19.2338\,ps$. A total real delay of the structure is $T_\Sigma = \sum_{k=1}^{19} T_k = 1599.2257\,ps$. A sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_s = n_t / T_t = 363.9420\,GHz$. In this case, a relative error of delay is $er = \frac{T_\Sigma - T_t}{T_\Sigma}\cdot 100\,\% = 0.004352\,\%$. According to the relation (4), the three-port adaptor coefficients are $\alpha_1 = \alpha_{18} = 0.5855$, $\alpha_2 = \alpha_{17} = 0.7073$, $\alpha_3 = \alpha_{16} = 0.4053$, $\alpha_4 = \alpha_{15} = 0.4398$, $\alpha_5 = \alpha_{14} = 0.4327$, $\alpha_6 = \alpha_{13} = 0.3988$, $\alpha_7 = \alpha_{12} = 0.4054$, $\alpha_8 = \alpha_{11} = 0.4227$, $\alpha_9 = \alpha_{10} = 0.4332$. The two-port adaptor coefficients are $\alpha_S = -\alpha_L = 0.004942$.

Fig. 3 shows responses both simulated in MATLAB by a new proposed approach and obtained in ADS simulator [17]. Microstrip circuits inevitably incorporate transmission line discontinuities, such as T-junction. A complete understanding of microstrip circuits requires characterization of various discontinuities included in the circuit. Only in that case, WDN curve is going to have better agreement with other results. That will be considered in the future.

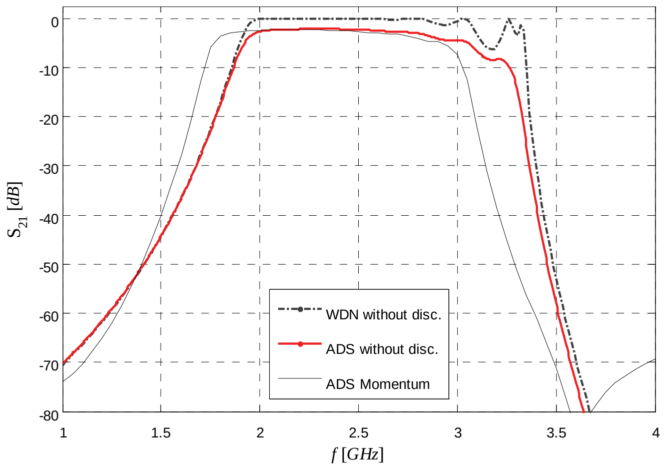

Fig. 3. Frequency response

IV. CONCLUSION

There are several main conclusions that can be drawn here:

1) A synthesis method is presented for evaluating scattering parameters of microwave circuits formed by connecting several multiport networks together. The synthesis is quite general and can be employed to analyze structure with any number of segments.

2) The analysis of wave digital structures is efficiently automated, which is inevitable when structures with larger numbers of building blocks are to be dealt with.

3) As has been told previously, response can be calculated in the frequency or in the time domain directly from known network function in *z*-domain. Known network functions in *z*-domain can be used as input data in some other simulations.

4) A great advantage of this method is its computational efficiency. The proposed approach is implemented on a processor Intel® Pentium® Dual CPU E2220 @ $2.4\,GHz$. A time for a response calculation in the frequency domain directly is $0.0753\,s$. This approach provides the fast structure simulation versus complex and time consuming 3D models.

5) In order to prove the accuracy of the proposed modeling and analyzing approaches, the computer simulated results obtained by WDN are compared to those of linear and momentum simulations obtained in ADS. One can observe that results obtained by described approach have good agreement with ADS data in whole frequency band.

6) Implementation of WDS in analysis of microwave structures can be used by microwave engineers because of the associated simplicity and accuracy.

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