System of Square-shaped Electrodes as a Pillar Grounding System

Nenad N. Cvetković[1]

***Abstract* – The pillar grounding system formed of two horizontal square-shaped wire electrodes and an iron armature connected to the main grounding connector is analyzed in the paper. Recently proposed procedures for modeling the influence of a concrete cylinder on a grounding system, and for approximating a square-shaped electrode with a ring electrode, are used.**

***Keywords* – Ground inhomogeneity, grounding systems, method of moments, pillar foundation, quasi-stationary EM field.**

I. INTRODUCTION

Pillars used as a part of the overhead power, telecommunications, or lightning protection system necessarily include corresponding grounding systems. A basic electrode of such systems is usually formed of a basic star, rectangular or circularly shaped electrode [1-2] connected to an iron armature of the pillar foundation, which can be treated as a second part of the grounding system.

Official publications, as [1], usually neglect influence of concrete foundation. This influence can be modelled as a hemispherical, [3-4], inhomogeneity in a shape of a parallelepiped [5-7]. A simple procedure for approximate modelling of this influence was proposed in [5] and it provides simplification of the analyzed problem to a problem of a grounding sys tem in the homogeneous ground. The level of foundation's influence depends on electrical parameters of the concrete and the ground, which can have different values depending on the ground structure, humidity, etc.

In [7], a grounding system having one rectangular basic electrode is analyzed in the quasi-stationary regime. The same approach is applied in this paper to the analysis of the grounding system having main electrodes' system formed of two rectangular basic electrodes, Fig. 1. Leakage currents` distributions are assumed as constant while the earthing conductor's influence is neglected. The surrounding ground is modelled as a homogeneous semi-conducting media of known electrical parameters, while the feeding current is modelled by an ideal LF current generator. Calculations are carried out for various values of concrete's specific resistivity and different embedding depths, whose values correspond to those of grounding systems realized in practice [1-2, 8-11].

As in [9], the obtained results are also compared with the results for ring basic electrodes [2, 4], since such problem is simpler from the numerical point of view, and for some approximations it is possible to analyze circular instead of rectangular electrode making error which is acceptably small.

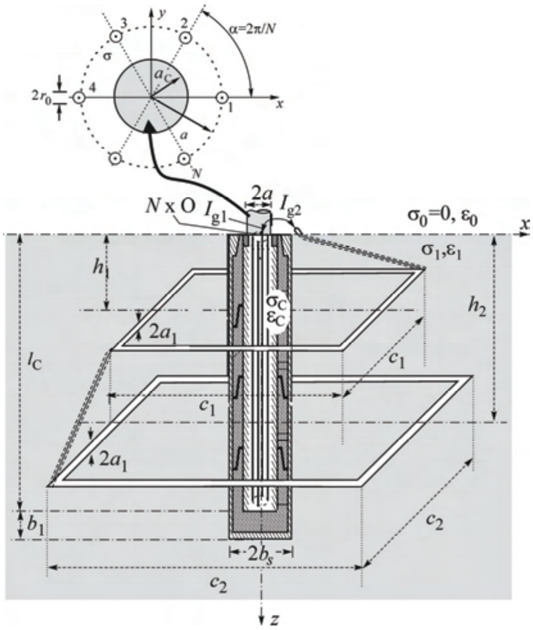

Fig. 1. Pillar grounding system.

II. THEORETICAL BACKGROUND

A. Description of the Grounding System

The pillar foundation grounding system formed of two square-shaped wire electrodes of sides c_1 and c_2, and embedded at depths h_1 and h_2, is observed (Fig. 1). The electrode is made of a strip conductor (usually FeZn strip) assumed to be a wire conductor having circular cross-section of an equivalent radius a_1, [4]. The structure of the observed grounding system also includes a single vertical electrode, which actually models the wire armature cage formed of N parallel conductors of length l_C, having a circular cross-section of radius r_0, (upper part of Fig. 1). The equivalent vertical wire electrode is of the same length l, and the circular cross-section of equivalent radius, $a_C = a\sqrt[N]{N\, r_0 / a}$, [5]. Armature (i.e. equivalent vertical wire electrode) and square-shaped ground electrodes' system are connected at the main ground point.

Foundation is parallelepiped concrete domain of a square cross-section having side length b_s, while concrete is of specific conductivity σ_C and permittivity ε_C. The surrounding ground is assumed as linear, isotropic and homogeneous semi-conducting media of specific conductivity σ_1 and permittivity ε_1. For the assumed quasi-stationary regime, complex con-

---

[1] Nenad N. Cvetković is with the University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, P.O. Box 73, 18000 Niš, Serbia, E-mail: nenad.cvetkovic@elfak.ni.ac.rs.

ductivities are $\underline{\sigma}_k = \sigma_k + \mathrm{j}\omega\varepsilon_k \approx \sigma_k$, $k = 0, 1, \mathrm{C}$, (for air, surrounding ground and concrete, respectively), while ω is angular frequency. Also, since for air $\sigma_0 = 0$, the value of the complex reflection coefficient can be considered: $R_{i0} = (\underline{\sigma}_i - \underline{\sigma}_0)/(\underline{\sigma}_i + \underline{\sigma}_0) \approx 1$, $i = 1, \mathrm{C}$.

Applying the procedure described in [5] the vertical electrode system and the concrete foundation can be replaced by a vertical electrode of equivalent length $l_1 = \mathrm{K_e}\, l_\mathrm{C}$ and cross-section radius $a_\mathrm{e} = \mathrm{K_e}\, a_\mathrm{C}$, placed in a homogeneous ground of specific conductivity σ_1, Fig. 2. Parameter $\mathrm{K_e}$ is determined from the expression:

$$\mathrm{K}_\mathrm{e}^{-1} = \sigma_1/\sigma_\mathrm{C} + \left(1 - \sigma_1/\sigma_\mathrm{C}\right)\ln\left(1 + l_\mathrm{C}/b\right)/\ln\left(1 + l_\mathrm{C}/a_\mathrm{C}\right), \qquad (1)$$

where is $b = b_s(1 + \sqrt{2})/4$, [5].

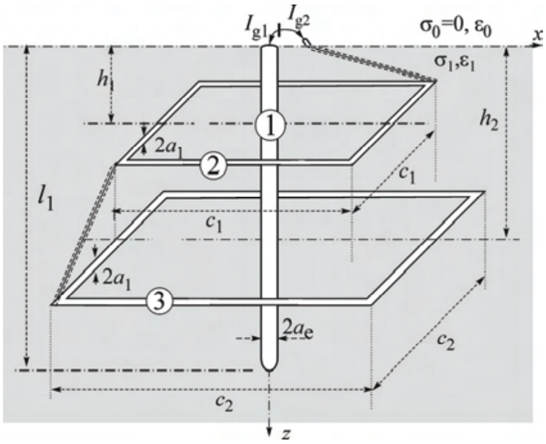

Fig.2. Equivalent grounding system in homogeneous ground.

This way, the problem is reduced to analysis of the equivalent grounding system in the homogeneous ground shown in Fig. 2. Equivalent vertical electrode (labeled by 1) and square-shaped wire electrodes' system (electrodes labeled by 2 and 3) are fed by very low frequency currents $I_{\mathrm{g}1}$ and $I_{\mathrm{g}2}$, respectively. For practical values it is reasonable to assume that $a_\mathrm{e} << l_1$, $a_1 << l_2, l_3$ and $a_1, a_\mathrm{e} << \lambda_1$ ($l_2 = 4c_1$ and $l_3 = 4c_2$ are length of electrodes 2 and 3, while λ_1 is wavelength in the surrounding ground). Unknown longitudinal current distributions along the vertical electrode (1) and square wire electrodes (2 and 3) are labeled by $I_k(s'_k)$ $s'_k \in [0, l_k]$, $k = 1, 2, 3$. Generally, for the purpose of obtaining the grounding resistance, the current distribution in the grounding network is not critical for the calculation of an accurate resistance value. Therefore, leakage currents` distributions are assumed as constant,

$$I_{\mathrm{leak}\,k}(s'_k) = -\partial I_k(s'_k)/\partial s'_k = -I'_k(s'_k) = I_{\mathrm{L}k}/l_k,\ k = 1, 2, 3 .(2)$$

where $I_{\mathrm{L}k}$, $k = 1, 2, 3$ are total leakage currents from the corresponding conductors. Since earthing conductors influence has not been taken into consideration, and $I_1(l_1) = 0$, the following conditions are satisfied

$$I_{\mathrm{L}1} = I_{\mathrm{g}1},\ I_{\mathrm{L}2} + I_{\mathrm{L}3} = I_{\mathrm{g}2}. \qquad (3)$$

B. Electrical Scalar Potential and Determining of Unknown Current Distributions and "$\underline{Z}$" Parameters

Taking into consideration all presumptions pointed out in the previous text, electric scalar potential at the points defined by the field vector $\vec{r}$ in the vicinity of the equivalent grounding system shown in Fig. 2, can be determined using the expression

$$\varphi(\vec{r}) = \sum_{k=1}^{3} \frac{I_{\mathrm{L}k}}{4\pi\sigma_1 l_k} \int_{l_k} \left(\frac{1}{r_k} + \frac{1}{r_{k\mathrm{i}}}\right) \mathrm{d}s'_k, \qquad (4)$$

where r_k and $r_{k\mathrm{i}}$ denote distances between the current element, i.e. its image, and the field point, respectively. Since the quasi-stationary approach is applied, it is assumed that the surface of the electrode 1 is equipotential, and that the same is valid for square-shaped electrodes 2 and 3 that are of the same potential value. Applying the Method of Moments [12] and matching potential value on the n-th ($n = 1, 2, 3$) conductor`s surface defined by the field vector $\vec{r}_1 = (l_1/2)\,\hat{z}$ (vertical conductor), $\vec{r}_2 = 0.5c_1\,\hat{x}$ and $\vec{r}_3 = 0.5c_2\,\hat{x}$ (points on the square-shaped electrodes), the following system of integral equations is formed:

$$\varphi(\vec{r} = \vec{r}_1) \cong U_1,\ \varphi(\vec{r} = \vec{r}_k) \cong U_2,\ k = 2, 3. \qquad (5)$$

In order to solve the described system, it is needed to adopt potential values U_1 and U_2. The system is solved in two regimes, so-called symmetric ($U_1 = U_2 = 1\mathrm{V}$) and anti-symmetric ($U_1 = -U_2 = 1\mathrm{V}$) feeding regimes. Based on these solutions, the solution for "Z" parameters of the electrode system are obtained [4-5]. They are formulated as

$$U_1 = \underline{Z}_{11} I_{\mathrm{g}1} + \underline{Z}_{12} I_{\mathrm{g}2},\ U_2 = \underline{Z}_{21} I_{\mathrm{g}1} + \underline{Z}_{22} I_{\mathrm{g}2}, \qquad (6)$$

where $\underline{Z}_{11}$ and $\underline{Z}_{22}$ are self-impedances, while $\underline{Z}_{12}$ and $\underline{Z}_{21}$ are mutual-impedances of the wire electrode 1 and the square electrodes' system (electrodes 2 and 3). "Z" parameters represent integral grounding system characteristics and indicate level of mutual-influence between the two electrodes. For linear systems $\underline{Z}_{12} = \underline{Z}_{21}$. If electrodes are connected, i.e. they form a unique grounding system, substituting $U_1 = U_2 = 1\,\mathrm{V}$ in (6), grounding impedance can be determined as

$$\underline{Z}_\mathrm{T} = 1/(I_{\mathrm{g}1} + I_{\mathrm{g}2}) = \frac{\underline{Z}_{11}\underline{Z}_{22} - \underline{Z}_{12}\underline{Z}_{21}}{\underline{Z}_{11} + \underline{Z}_{22} - \underline{Z}_{12} - \underline{Z}_{21}}. \qquad (7)$$

C. Approximation of Square-shaped Wire Electrode by an Equivalent Ring Electrode

As it has been already emphasized, the pillar grounding system with circular (instead of square) basic electrode is more simple for numerical solving. Because of that, it could be of interest to find some appropriate relationship between the square side and the radius of the equivalent circular wire electrode that would approximately model the square electrode. Such analysis has been already carried out for the case of a single square-shaped electrode as a main ground electrode

[7]. A case of the square electrodes being approximated by rings of the same length and same surface, as well as the case of ring electrodes having equivalent radii determined by estimation method [4], is analyzed in this paper. The mathematical model has the same form, and in that case l_2 and l_3 in (4) label contours of the ring electrode, instead of square wire electrodes. Matching points are chosen on ring surfaces, and presumption of constant leakage current also stays valid.

Related to the square side *c*, which in this case takes values c_1 or c_2, equivalent ring radius *R* is now, [9],

$$R = 0.25c\left(1+\sqrt{2}\right) \text{ (estimation method)}, \tag{8a}$$

$$2R\pi = 4c \Rightarrow R = 2c/\pi \text{ (same length)}, \tag{8b}$$

$$R^2\pi = c^2 \Rightarrow R = c/\sqrt{\pi} \text{ (same surface)}. \tag{8c}$$

III. NUMERICAL RESULTS

Based on the presented model, the corresponding program packages are developed and applied to approximate solving of the pillar foundation grounding system formed of a vertical conductor in a concrete cylinder and two square-shaped electrodes, Fig. 1. Values of geometry parameters and concrete specific conductivity are selected according to the values from [1, 8-11]. Used values of all parameters in Figs. 1 and 2 are: $\rho_1 = 1/\sigma_1 = 100\,\Omega\,\text{m}$, $a = 0.25\,\text{m}$, $b_s = 0.4\,\text{m}$, $b_1 = 0.2\,\text{m}$, $l_C = 2\,\text{m}$, $c_1 = 1.5\,\text{m}$, $c_2 = 2.5\,\text{m}$, $h_1 = 0.5\,\text{m}$, $a_1 = 9.7\,\text{mm}$, $r_0 = 0.007\,\text{m}$ (radius of armature conductors` cross-section), $N = 10$, $b = 0.483\,\text{m}$, and $a_C = 0.18\,\text{m}$.

Mutual-resistance $R_{12} = \text{Re}\{\underline{Z}_{12}\}$ of the system from Fig. 1 versus embedding depth of the lower square electrode h_2, having concrete specific resistivity $\rho_C = 1/\sigma_C$ taken as parameter, is shown in Fig. 3. It is noticeable that the position of the lower square electrode related to vertical electrode's system influences the mutual resistance, and this influence differs for different specific resistivity concrete/ground ratio (ρ_C/ρ_1).

Mutual resistance R_{12} of the system from Fig. 1, versus the embedding depth h_2 when $\rho_1 = \rho_C$, for the square electrodes and different radii of equivalent ring electrodes (8a-c) is presented in Fig. 5. It is obvious that best approximations of the square electrodes are the ring electrodes of radii (8c).

In Fig. 6, graphs of the self-resistance $R_{22} = \text{Re}\{\underline{Z}_{22}\}$ versus the embedding depth h_2 when $\rho_1 = \rho_C$, for the square electrodes and different radii of equivalent ring electrodes (8a-c), are presented. In this case, rings having radii given by the expression (8a) are the best approximation of the square electrodes.

Finally, in Fig. 7, the total resistance R_T of the grounding system from Fig. 1 versus the embedding depth h_2 when $\rho_1 = \rho_C$, for square electrodes and different radii of the equivalent ring electrode (8a-c) is shown. The best approximation of the square electrode is using ring electrodes of radii (8c) and (8a), depending on the embedding depth h_2.

Total resistance $R_T = \text{Re}\{\underline{Z}_T\}$ of the system from Fig. 1, versus embedding depth h_2, and concrete specific resistivity $\rho_C = 1/\sigma_C$ taken as parameter, is shown in Fig. 4. It is expected that the total resistance will decrease while increasing depth h_2, and also, that the curves corresponding to different values of specific resistivity of concrete are mutually shifted.

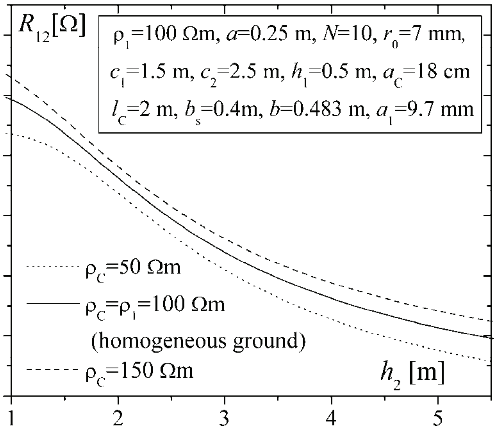

Fig.3. Mutual resistance of the grounding system shown in Fig. 1 versus embedding depth h_2, when ρ_c is taken as a parameter.

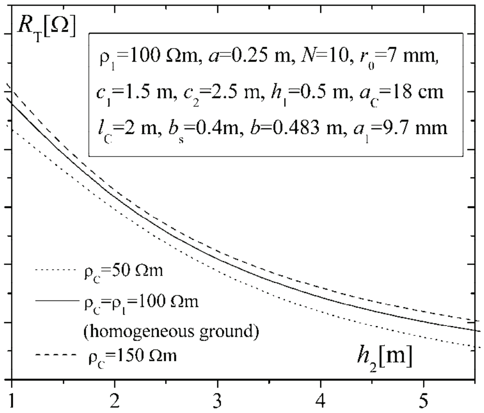

Fig.4. Total resistance of the grounding system shown in Fig. 1 versus embedding depth h_2, when ρ_c is taken as a parameter.

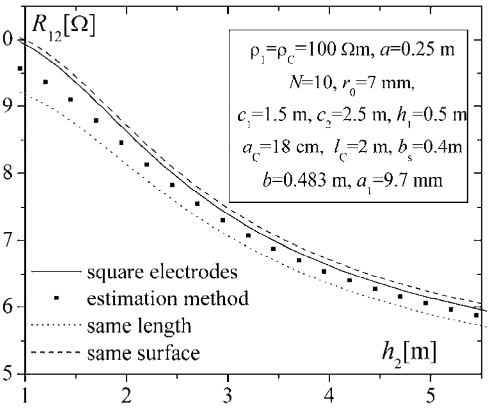

Fig.5. Mutual-resistance of the grounding system shown in Fig. 1 versus embedding depth h_2 for ρ_1=ρ_c and different dimensions of the equivalent ring electrodes.

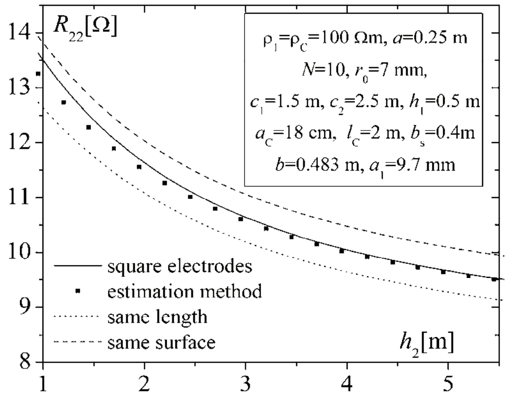

Fig.6. Self- resistance R_{22} corresponding to the basic electrode of the grounding system shown in Fig. 1 versus embedding depth h_2 for $\rho_1=\rho_c$ and different dimensions of the equivalent ring electrodes.

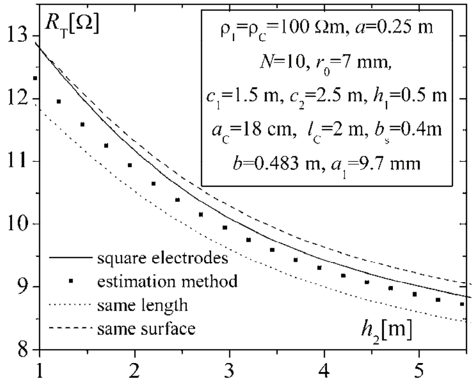

Fig.7. Total resistance of the grounding system shown in Fig. 1 versus embedding depth h_2 for $\rho_1=\rho_c$ and different dimensions of the equivalent ring electrodes.

IV. CONCLUSIONS

The analysis of the influence of the pillar foundation on characteristics of the pillar grounding system is carried out in this paper and the results are obtained in the quasi-stationary regime assuming the surrounding ground as a linear homogeneous semi-conducting media of known electrical parameters. A grounding system realization applied in practice, with a rectangular wire conductors' system as a basic electrode, is analyzed. The foundation influence is modeled using a recently proposed procedure [7], based on approximating the pillar foundation by a concrete domain having parallelepiped shape of square cross-section.

Based on the obtained results one can conclude that the foundation`s influence on grounding system`s impedance can be significant for real values of concrete`s specific resistivity and it should be taken into consideration during the design of such systems. Since from the numerical point of view, it is easier to solve an analogue grounding system with a ring basic electrodes' system, different approaches for approximating a square electrode with an equivalent ring electrode are also presented in the paper.

Presented results indicate a possibility to approximate well the square electrode using an equivalent ring electrode, but the most appropriate approximations differ depending on the embedding depth of the basic electrode. Another collateral conclusion that follows the presented analysis is the significance of the influence of the shape of the electrodes, which form basic electrodes' system, on the self-, mutual- and consequently, the total impedance. The obtained results show that selection of the approximating method given by Eqs. (8) depends on the number of the electrodes in the basic electrodes' system, since the results presented in the paper differ a little bit from those ones given in [7].

ACKNOWLEDGEMENTS

This paper is partly supported by the project grant III44004 (2011-2014) financed by the Ministry of Education and Science of Republic of Serbia.

The results presented in the paper were obtained by the joint research of the paper author with prof. Predrag D. Rančić (1947-2011) from the University of Niš, Faculty of Electronic Engineering. Author is grateful to prof. Rančić for his support and help during the years of successful collaboration.

REFERENCES

[1] *Technical Recommendation of Electric Power Industry of Serbia: Grounding of the Power Network Pillars, TR No .9*, Belgrade, Serbia, 2000. [in Serbian]. Available at: http://www.eps.rs/publikacije/interni_standardi/preporuke/tp9.zip.

[2] J. Nahman, *Neutral Grounding of Distribution Networks*, Naučna knjiga, Belgrade, Serbia, pp. 281-288, 1980. [in Serbian]

[3] N. N. Cvetković and P. D. Rančić, "A Simple Model for a Numerical Determination of Electrical Characteristics of a Pillar Foundation Grounding System", *Engineering Analysis with Boundary Elements*, Elsevier, vol 33, pp. 555-560, 2009.

[4] N. N. Cvetković and P. D. Rančić, "Influence of Foundation on Pillar Grounding System's Characteristics", *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, Emerald Group Publishing Limited, vol. 28, no 2, pp. 471-492, 2009.

[5] N. N. Cvetković and P. D. Rančić, "Model for Analyzing a Pillar Grounding System with Cylindrically-Shaped Concrete Foundation", *Electromagnetics*, vol. 29, no. 2, pp. 151–164, 2009.

[6] P. D. Rančić and N. N. Cvetković, "Pillar Star-shaped Grounding System", *Przeglad Elektrotechniczny (Electrical Review)*, R. 86 NR 5/2010, pp.100-104, 2010.

[7] P. D. Rančić and N. N. Cvetković, "Square shaped Electrode as Pillar Grounding System", *TELSIKS`09, Conference Proceedings*, pp. 351-354, Niš, Serbia, 2009.

[8] T. Radojičić, *Concrete Constructions*, Prosveta, Niš, Serbia, 1981. [in Serbian]

[9] "Ferro Concrete Montage Foundation EBB-MT1", *Web presentation of ELEKTROIZGRADNJA Bajina Bašta Company*, 2009. [In Serbian]. Available at: http://www.ebb.rs.

[10] M. Chouteau and S. Beaulieu, "An Iinvestigation on Application of the Electrical Resistivity Tomography Method to Concrete Structures", *Geophysics*, Los Angeles, CA, USA, 2002. Available at: http://www.dot.ca.gov/hq/esc/geotech/gg/geophysics2002/050chouteau_resistivity_tomography.pdf.

[11] J. Ribič and J. Podlipnik, "Grounding and Negative Effects of Using Zinc Ground Electrodes in Objects Foundation", *Web presentation of Hermi Company*, 2008. [In Serbian]. Available at: http://www.hermi.si/sr/Projektanti/Uzemljenje/tabid/523/Default.aspx

[12] R. F. Harrington, *Field computation by Moment Methods*, New York: The Macmillan Company, 1969.