The Application of OSTBC with Alamouti Scheme in Spectrum-Sharing Cognitive Radio

Vesna Blagojević[1] and Predrag Ivaniš[1]

***Abstract* – In this paper the application of Alamouti scheme on the performance of secondary network in spectrum-sharing cognitive radio with limited average interference power at the primary user is analyzed. We considered the case when fading in all branches follows Nakagami-*m* distribution. Analytical results are confirmed using an independent simulation method.**

***Keywords* – Spectrum sharing, Cognitive radio, Orthogonal Space Time Block Codes, Alamouti scheme, Nakagami fading.**

I. Introduction

Radio spectrum is one of the most limited resources, so its efficient use is of highest importance for the further development of wireless communications. Regulatory bodies grant license for exclusive access to allocated spectrum bands with the strict regulations concerning interference protection. On the other hand, recent measurements have shown that during significant periods of time many allocated portions of spectrum are not occupied. As the majority of the spectrum has already been allocated, new approaches for its utilization have to be found [1].

The concept of cognitive radio (CR) was proposed by D. J. Mitola [2], as the further extension of software radio. These techniques are attracting more attention recently, as they can improve spectrum utilization through adaptive, dynamic and intelligent processes. One way of improving spectrum utilization is by permitting an unlicensed secondary user (SU) to access a spectrum hole unoccupied by the licensed primary user (PU). This function is often referred to as spectrum sensing. On the other hand, another important function of CR is spectrum sharing, where SUs can share spectrum with the PUs in a controlled fashion. SU is allowed to transmit under the condition that the level of interference that is caused to the primary user is not harmful [1], [3].

Furthermore, it is is well known that multiple input multiple output (MIMO) systems can achieve significant improvement in spectral efficiency and reliability of wireless communications over fading channels [4]. Orthogonal space-time block coding (OSTBC) represents a low-complexity technique, that although suboptimal, results in a maximum diversity gain. Within this class of codes, simple scheme proposed by Alamouti [5] has very important advantage of implementation simplicity.

Channel capacity for spectrum-sharing communications is analyzed in [1], for various fading environments, under both scenarios of average and peak interference power constraints at the PU receiver. In [3] closed form expressions for ergodic, outage and minimum rate capacity are derived for the case of Rayleigh fading, under both average and peak interference power constraints caused at the PU. In [6] the secondary channel capacity is analyzed for the case of Rayleigh fading channel with maximal ratio combining (MRC) diversity on the secondary link and average interference power constraint.

In this paper we extend analysis to the case of both transmit and receive diversity. To the best of author's knowledge, the secondary link ergodic capacity of the spectrum-sharing system that employs OSTBC has not been analyzed.

We derive closed-form secondary link capacity expressions for spectrum-sharing communications, under the primary user interference power constraint. Fading on all links is assumed to be Nakagami distributed. The Alamouti scheme is employed in SU network. The analytical results are confirmed using an independent simulation method.

In Section II, system and channel model are presented. The analysis of secondary link ergodic capacity under the average interference constraint at the primary user side is given in Section III. Numerical results are presented in Section IV. Finally, conclusions are given.

II. Channel and System Models

We consider radio network where PUs and SUs coexist in spectrum-sharing system. SUs are equipped with two antennas and PU receiver uses one antenna, as it is shown in Fig. 1. Further, it is assumed that channel fading is slow and perfect channel side information (CSI) of links to the both PU and SU receiver are available to the secondary user transmitter.

At the SUs, Alamouti code scheme is employed, presented in Fig. 2. At the given symbol period, two signals are simultaneously transmitted from the two antennas. The signal transmitted from first antenna (Tx_0) is denoted by x_0 and from the second one (Tx_1) by x_1. During the next symbol period signal $-x_1^*$ is transmitted from first antenna, and x_0^* from the second antenna. Complex conjugate operation is denoted by *. The complex channel fading envelope between transmit and receive antennas are denoted as h_i, i=0,1,2,3.

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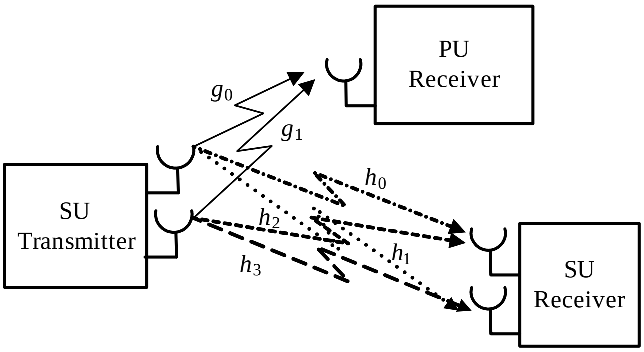

Fig. 1. Secondary user is sharing spectrum with the primary user.

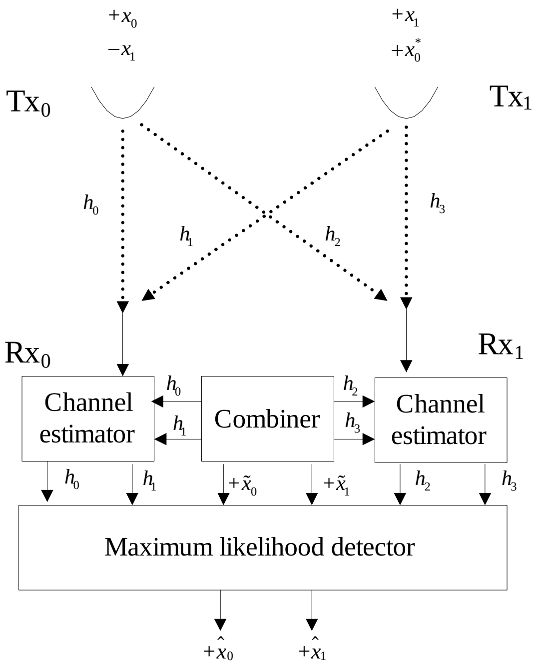

Fig. 2. The application of Alamouti scheme with two transmit and two receive antennas at SUs transmitter and receiver.

Then, the input signals to the maximum likelihood detector (MLLD) are equal [5]

$$\widetilde{x_0} = \sum_{i=0}^{3} |h_i|^2 \cdot x_0 + h_0^* n_0 + h_1 n_1^* + h_2^* n_2 + h_3 n_3^*$$
$$\widetilde{x_1} = \sum_{i=0}^{3} |h_i|^2 \cdot x_1 - h_0 n_1^* + h_1^* n_0 - h_2 n_3^* + h_3^* n_2, \quad (1)$$

where n_i, i=0,1,2,3 are complex random variables (RVs) representing additive white Gaussian noise(AWGN).

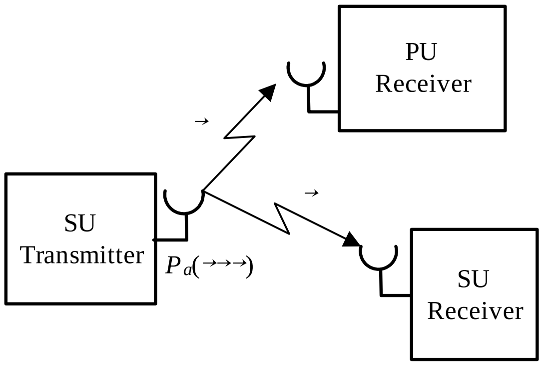

Fig. 3. Equivalent channel power gains for the spectrum-sharing scheme.

It is assumed that fading on links between SU transmitter and reciever antennas is Nakagami distributed with the same fading parameter m_s. The squared fading envelope α_i can be represented as the sum of squares of independent Gaussian zero-mean RVs, according to the expression [7, Eq. 7]

$$\alpha_i = |h_i|^2 = \begin{cases} \sum_{k=1}^{m_s} (c_{i,k}^2 + s_{i,k}^2), & 2m_s \text{ even}, \\ c_{i,0}^2 + \sum_{k=1}^{m_s-0.5} (c_{i,k}^2 + s_{i,k}^2), & 2m_s \text{ odd}. \end{cases} \quad (2)$$

Each pair $c_{i,k}$ and $s_{i,k}$ represents a scattered wave, consisting of an in-phase and quadrature component of a complex RV $c_{i,k} + js_{i,k}$ ($j = \sqrt{-1}$), with a Rayleigh distributed magnitude.

The equivalent channel power gain α between SU transmitter and the receiver is given with [8]

$$\alpha = \sum_{i=0}^{3} \alpha_i, \quad (3)$$

and its probability density function (PDF) is equal

$$f_\alpha(\alpha) = \frac{1}{(4m_s - 1)!} \alpha^{4m_s - 1} e^{-\alpha}, \ \alpha > 0. \quad (4)$$

We further assume that fading on the links between SU transmitter antennas and PU receive antenna follows Nakagami distribution with the fading parameter m_p. As the signals coming from SUs antennas are orthogonal, we can consider incoming interference signals mutually independent. Therefore, the channel power gain β between SU transmitter and PU receiver is given with

$$\beta = |g_0|^2 + |g_1|^2, \quad (5)$$

and its PDF is given with

$$f_\beta(\beta) = \frac{1}{(2m_p - 1)!} \beta^{2m_p - 1} e^{-\beta}, \ \beta > 0. \quad (6)$$

III. Capacity Analysis

We evaluate the secondary link capacity, for a SU operating within licensed band of PU subject to the received interference power constraint. In this paper we suppose that the PU receiver's operation is limited by the average interference level.

Let us denote the transmit power of SUs antennas by $P_{a.}$ In the fading environment, in order to maximize capacity, transmitted power is changing over time so the received interference constraint is fulfilled and $P_a = P_a(\alpha,\beta)$. Therefore, the following maximization problem should be solved

$$\frac{C}{B} = \max_{P(\alpha,\beta)} \int_\beta \int_\alpha \log_2\left(1+\frac{\alpha P_a(\alpha,\beta)}{N_0 B}\right) f_\alpha(\alpha) f_\beta(\beta)\, d\alpha\, d\beta, \quad (7)$$

where B is total available bandwidth and N_0 the power spectral density of the AWGN at the SU receiver.

As the links between SU transmit and PU receive antennas are independent, the received power at the PU equals

$$\left(|g_0|^2+|g_1|^2\right)\cdot P_a = \beta P_a. \quad (8)$$

The average interference power constrained is given by

$$\int_\beta \int_\alpha \beta P_a(\alpha,\beta) f_\alpha(\alpha) f_\beta(\beta)\, d\alpha\, d\beta \le Q, \quad (9)$$

where Q is maximum average power, permitted at the primary user's receiver [1].

The optimal power allocation $P_a(\alpha,\beta)$ is given by [1]

$$P(\alpha,\beta) = \left(\frac{1}{\lambda_0 \beta} - \frac{N_0 B}{\alpha}\right)^+, \quad (10)$$

where $(x)^+$ denotes max(0,x) and λ_0 is determined such that the average interference received power is equal to Q.

By substituting (4), (6) and (10) in (7), we obtain channel capacity

$$\frac{C}{B} = K \int_{\beta=0}^{+\infty} \int_{\alpha=\beta/\gamma_0}^{+\infty} \left[\log_2\left(\frac{\gamma_0\alpha}{\beta}\right)\alpha^{4m_s-1} e^{-\alpha}\, d\alpha\right] \beta^{2m_p-1} e^{-\beta}\, d\beta, \quad (11)$$

where $1/K = (4m_s-1)!(2m_p-1)!$ and $\gamma_0 = 1/(\lambda_0 N_0 B)$.

After substituting $t = \alpha\gamma_0/\beta$, we obtain

$$\frac{C}{B} = \frac{K}{\ln 2} \int_{\beta=0}^{+\infty} \beta^{2m_p-1} e^{-\beta} \left(\frac{\beta}{\gamma_0}\right)^{4m_s} \left[\int_{t=1}^{+\infty} \ln(t)\, t^{4m_s-1} e^{-\frac{\beta t}{\gamma_0}}\, dt\right] d\beta. \quad (12)$$

It is shown in [6] that the integral in square brackets is

$$\int_{t=1}^{+\infty} \ln(t)\, t^{4m_s-1} e^{-\frac{\beta t}{\gamma_0}}\, dt = \frac{(4m_s-1)!}{(\beta/\gamma_0)^{4m_s}} \sum_{k=0}^{4m_s-1} \frac{\Gamma(k,\beta/\gamma_0)}{k!}, \quad (13)$$

where $\Gamma(k,x)$ denotes complementary incomplete gamma function defined in [9, Eq. (8.350-2)].

$$\frac{C}{B} = \frac{K(4m_s-1)!}{\ln 2} \sum_{k=0}^{4m_s-1} \int_{\beta=0}^{+\infty} \beta^{2m_p-1} e^{-\beta} \frac{\Gamma(k,\beta/\gamma_0)}{k!}\, d\beta. \quad (14)$$

The integral with respect to β is solved using [9, Eq .(6.455)], and the final expression is given with

$$\frac{C}{B} = \frac{1}{\ln 2 \cdot (2m_p)!} \sum_{k=0}^{4m_s-1} \frac{1}{k!} \frac{\Gamma(2m_p+k)\gamma_0^{2m_p}}{(\gamma_0+1)^{2m_p+k}} \times {}_2F_1\left(1; 2m_p+k; 2m_p+1; \frac{\gamma_0}{\gamma_0+1}\right), \quad (15)$$

where ${}_2F_1(a,b,c,z)$ denotes Gauss's hypergeometric function [10, 15.1].

The average interference power at the PU receiver is obtained by averaging over channel power gain realizations

$$\frac{Q}{N_0 B} = K \int_{\alpha=0}^{+\infty} \int_{\beta=0}^{\alpha\gamma_0} \left[\left(\gamma_0 - \frac{\beta}{\alpha}\right)\beta^{2m_p-1} e^{-\beta}\, d\beta\right] \alpha^{4m_s-1} e^{-\alpha}\, d\alpha = \\ = K \int_{\alpha=0}^{+\infty} \alpha^{4m_s-1} e^{-\alpha} \left[\gamma_0 \gamma(2m_p, \gamma_0\alpha) - \frac{\gamma(2m_p+1, \gamma_0\alpha)}{\alpha}\right] d\alpha. \quad (16)$$

and $\gamma(k,x)$ is incomplete gamma function [9, Eq. (8.350-1)]. Then, by using [9, Eq. (6.455)], we obtain the final expression

$$\frac{Q}{N_0 B} = \frac{\Gamma(4m_s+2m_p)}{(4m_s-1)!(2m_p-1)!} \frac{\gamma_0^{2m_p+1}}{(\gamma_0+1)^{4m_s+2m_p}} \times \\ \times \left[{}_2F_1\left(1; 4m_s+2m_p; 2m_p+1; \frac{\gamma_0}{\gamma_0+1}\right)\cdot \frac{1}{2m_p} \right. \\ \left. - {}_2F_1\left(1; 4m_s+2m_p; 2m_p+2; \frac{\gamma_0}{\gamma_0+1}\right) \frac{1}{2m_p+1}\right]. \quad (17).$$

IV. Numerical Results

A waveform sequence with $L=10^7$ samples is generated for every diversity branch by using an improved Jakes fading simulator [11]. For each realizations of the channel gain random variables α and β, the optimal transmit power is calculated according to Eq. (10). Channel capacity and average interference power are evaluated by averaging over random channel realizations. These results are compared with the analytical results obtained using (15) and (17). The obtained results are presented in Figs. 4-5.

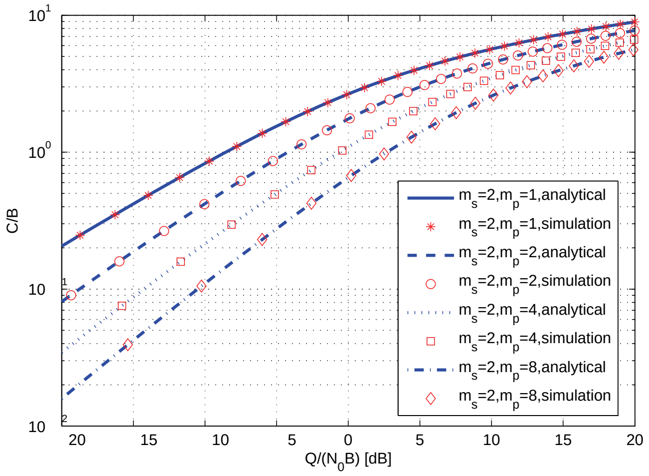

Fig. 4. Channel capacity vs. average interference power for fading parameters m_s=2 and various m_p values.

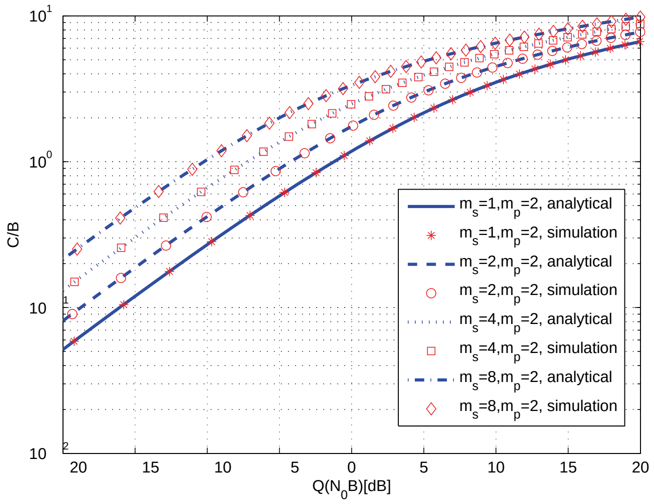

Fig. 5. Channel capacity vs. average interference power for fading parameter m_p=2 and various m_s values.

Channel capacity vs. average received interference power is presented in Fig. 4, for the case when Nakagami fading parameters in all branches of SU link are m_s=2, and values of fading parameter m_p=1, 2, 4, 8. Channel capacity is decreasing with the raise of parameter m_p, as the propagation characteristics on the links of the interference signals are improved.

In Fig. 5, channel capacity vs. average received interference power is presented, for the various secondary link fading parameter m_s=1, 2, 4, 8, and fading parameters of links between SU transmitter and PU receiver m_p=2. It can be noticed that for the fixed average interference power value, channel capacity is increasing with the improvement of propagation conditions in secondary link channel, that correspond to the raise of parameter m_s.

Conclusion

In this paper the closed-form expressions for the secondary link channel capacity is given, for the spectrum-sharing system with the limited average power at PU receiver.

We analyzed the case where OSTBC with Alamouti scheme is employed at the SUs, and fading on all links is Nakagami-*m* distributed. The analytical results are confirmed using an independent simulation method. This analysis can be further extended to the case of higher order diversity with OSTBC and correlation between branches [12].

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