Composite Third Order Intermodulation Products in HFC/CATV Systems

Oleg Panagiev[1] and Valentin Hristov[2]

***Abstract* – This paper presents research on the influence of composite third order nonlinear products on the quality of the transmitted signals in HFC/CATV systems. The study was made of the impact between the transmitted signals. Created are algorithm and block diagram for determining the number of nonlinear composite products. The results are presented in graphical and table form.**

***Keywords* – third order composite nonlinear products, three-component beat, two-component beat, HFC/CATV, algorithm.**

I. Introduction

Third order intermodulation is the beating of one signal carrier with the second harmonic of another signal carrier ($2f_i \pm f_j$) or the beating of three signal carriers together ($f_i \pm f_j \pm f_k$) with or without modulation in a broadband multichannel system (HFC/CATV). A brief mathematical analysis of third order components will help to establish the relationship between the fundamental and spurious signals. When two, three or more sinusoidal voltages of different frequencies are applied to an amplifier or/and laser diode with distributed feedback (DFB), Mach-Zehnder modulator (MZM), etc. - several third order components are generated [1], [2].

II. Mathematical Analysis

In the general case the combination frequencies are determined by the formula:

$$f_{NP} = r_1 f_1 + r_2 f_2 + r_3 f_3 + ... = \sum_{i=1}^{N} r_i f_i \text{ , where} \tag{1}$$

r_i are arbitrary integers, possibly equal to zero. If the transmission characteristic of the system is of n-th order, then the coefficients $r_1, r_2, r_3 ...$ need to satisfy the inequality $|r_1| + |r_2| + |r_3| + ... \le n$. Since subject of this research are composite nonlinear products (NP) from third order, it is possible for Eq. (1) to be given as follows

$$f_{NP} = r_1 f_i + r_2 f_j + r_3 f_k . \tag{2}$$

Here $|r_1| + |r_2| + |r_3| = 3$ and f_i, f_j, f_k are the input signals' frequencies for the respective device. The number of the transmitted in the system signals is N, where $i=1 \div N$, $j=1 \div N$, $k=1 \div N$. If $i \neq j \neq k$ and $r_1=r_2=r_3=\pm 1$ is obtained a three-component beat ($f_i \pm f_j \pm f_k$). If $i \neq j$, $k=0$ or $i \neq k$, $j=0$ or $j \neq k$, $i=0$ and respectively $r_1=\pm 1/\pm 2$, $r_2=\pm 2/\pm 1$ or $r_1=\pm 1/\pm 2$, $r_3=\pm 2/\pm 1$ or $r_2=\pm 1/\pm 2$, $r_3=\pm 2/\pm 1$ a two-component beat ($2f_i \pm f_j$, $2f_i \pm f_k$, $2f_j \pm f_k$, etc.) is obtained.

In the following mathematical analysis is taken that $f_i < f_j < f_k$ and also a unmodulated signal is applied on the system input:

$$x(t) = \sum_{i=1}^{N} A_i \cos(2\pi f_i t + \theta_i) . \tag{3}$$

as for the D/K standard in range 111MHz÷862MHz Eq. (3) can be written in the following way:

$$x(t) = \sum_{i=1}^{N} A_i \cos[2\pi.((i-1).8 + f_1).t + \theta_i] . \tag{4}$$

Let the nonlinearity is described by the polynomial

$$y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) , \tag{5}$$

$$y(t) = y_1(t) + y_2(t) + y_3(t) . \tag{5a}$$

The nonlinear products from third order in Eqs. (5) and (5a) are generated from the third term, and the nonlinear products of second order are not a subject of this paper, the output signal can be shortened to:

$$y(t) = y_1(t) + y_3(t) = a_1 x(t) + a_3 x^3(t) . \tag{6}$$

After a substitution with Eq. (3) in Eq. (6) and getting done the respectively mathematical operations, it comes to the following expression:

a) In case of three-component beat

$$y_3^{(3)}(t) = A_{NP}^{(3)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \sum_{k=j+1}^{N} \cos[2\pi(f_i \pm f_j \pm f_k).t + \theta_i \pm \theta_j \pm \theta_k] , \tag{7}$$

$$y_3^{(3)}(t) = A_{NP}^{(3)} . \sum_{i=1}^{N} \sum_{j=i+1}^{N} \sum_{k=j+1}^{N} \cos[2\pi f_{NP}^{(3)}).t + \theta_{NP}^{(3)}] \text{ , where} \tag{7a}$$

$A_{NP}^{(3)} = G_{NP}^{(3)}.A_i.A_j.A_k = 3/2 a_3.A_i.A_j.A_k$ is amplitude, $G_{NP}^{(3)}$ is gain, $f_{NP}^{(3)}$ - frequency and $\theta_{NP}^{(3)}$ - phase of the nonlinear product at three-component beat.

In this case there are 7 different products generated. The strongest and most important third order composite products are the result of three frequencies. These can be expressed as:

- $f_{NP}^{(3)} = f_i - f_j + f_k$. $f_{NP}^{(3)}$ values are between f_i and f_k;
- $f_{NP}^{(3)} = f_j + f_k - f_i$. $f_{NP}^{(3)} > f_k$;
- $f_{NP}^{(3)} = f_i + f_j - f_k$. For close values of f_i, f_j and f_k the values of NP are under f_i. When $f_k >> f_j > f_i$ they are negative;

[1]Oleg Panagiev is with the Technical University of Sofia, Bulgaria, E-mail: olcomol@yahoo.com.
[2]Valentin Hristov is with the SWU"N. Rilski"-Blagoevgrad, Bulgaria, E-mail: v_hristov@aix.swu.bg .

- $f_{NP}^{(3)} = f_i + f_j + f_k$. $f_{NP}^{(3)} >> f_k$. If f_i, f_j and f_k are in the UHF band, the values of $f_{NP}^{(3)}$ go outside it.

The nonlinear products of f_i+f_j+f_k type, resulted out of the beating between analogue signals, are fallen onto the sound's carrier frequencies and on 3,75MHz over the digital signal's carrier frequencies (Fig.1). Only for frequencies from the upper part of the 110MHz÷470MHz range, NP fall out of the UHF band.

The nonlinear products of f_i+f_j-f_k type resulted out of the beating between digital signals, fall on 2,75MHz over the image's carrier frequencies.

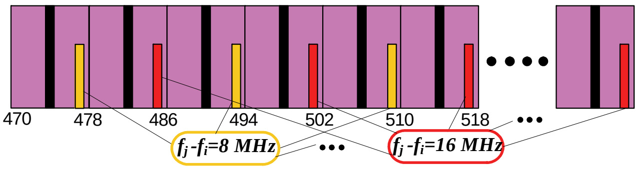

Fig.1. Influence of $f_i+f_j+f_k$

Nonlinear products from f_i-f_j+f_k and f_j+f_k-f_i type, influence substantially the signals of the whole working frequency range 110MHz÷862MHz (analog over analog, digital over digital, analog over digital, digital over analog: AM-VSB ↔ AM-VSB; M-QAM ↔ M-QAM; AM-VSB ↔ M-QAM), Table 1 and Fig.2.

b) In case of two-component beat

$$y_3^{(2)}(t) = A_{NP}^{(2)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \cos[2\pi(2f_i \pm f_j).t + 2\theta_i \pm \theta_j], \quad (8)$$

$$y_3^{(2)+}(t) = A_{NP}^{(2)}.\sum_{i=1}^{N} \sum_{j=i+1}^{N} \cos[2\pi f_{NP}^{(2)}).t + \theta_{NP}^{(2)}], \text{ where} \quad (8a)$$

$A_{NP}^{(2)} = G_{NP}^{(2)}.A_i.A_j = 3/4a_3.A_i.A_j$ is amplitude, $G_{NP}^{(2)}$ is gain, $f_{NP}^{(2)}$ - frequency and $\theta_{NP}^{(2)}$ - phase of the nonlinear product at two-component beat. $f_{NP}^{(2)}$ adopts and the following values (depending on the type of two-component nonlinear product): $2f_i \pm f_k$; $2f_j \pm f_k$; $f_i \pm 2f_j$; $f_i \pm 2f_k$; $f_j \pm 2f_k$. $\theta_{NP}^{(2)}$ adopts and the following values (depending on the type of two-component nonlinear product): $2\theta_i \pm \theta_k$; $2\theta_j \pm \theta_k$; $\theta_i \pm 2\theta_j$; $\theta_i \pm 2\theta_k$; $\theta_j \pm 2\theta_k$.

In this case there are 12 different products generated. The strongest and most important third order composite products are the result of one signal carrier with the second harmonic of another signal carrier. These can be expressed as:

- $f_{NP}^{(2)} = 2f_i - f_j$. $0 > f_{NP}^{(2)} < f_i < f_j$ and influences on channels from both (VHF and UHF) bands. This depends from values of f_i and f_j. Where f_i<< f_j, the values of NP is negative;
- $f_{NP}^{(2)} = 2f_j - f_i$. $f_i < f_j < f_{NP}^{(2)}$ and influences on channels from both (VHF and UHF) bands. This depends from values of f_i and f_j. Where f_i<< f_j, the values of $f_{NP}^{(2)}$ go outside UHF band.
- $f_{NP}^{(2)} = 2f_i + f_j$. $f_i < f_j << f_{NP}^{(2)}$.
- $f_{NP}^{(2)} = 2f_j + f_i$. $f_i < f_j << f_{NP}^{(2)}$.

<u>*Note:*</u> For last two kind nonlinear products:

- If f_i and f_j are in the VHF band, the values of $f_{NP}^{(2)}$ influence to M-QAM channels, distributed in UHF band.
- If $f_j > f_i$ >287,25MHz, the values of $f_{NP}^{(2)}$ go outside UHF band.

Nonlinear products from 2f_i-f_j and 2f_j-f_i type, influence mostly the frequency ranges, in which are f_i and f_j located.

Nonlinear products of 2f_i+f_j and 2f_j+f_i type, derived from the beating between the analog signals, influence into the working frequency range 340MHz÷862MHz and those of the beating between digital signals, are going out of the UHF range (Table 1 and Fig.3).

III. Determination of Number of the Non-linear Products from Composite Triple Beat

According to the complicity of the problem with studying of the influence of nonlinear products from a composite third beat, is necessary to be defined the number of nonlinear products, going onto or around the channel's carrying frequency. The methods presented in [2], [3] and [4] have some restrictions which do not allow a fully and comprehensive definition of the nonlinear products' number. We suggested here an algorithm (Fig.4) define the exact number of nonlinear products, and the results are to be given in a table or/and in a graphic type. Here f_r, f_b and f_h are respectively the carrying frequency, the lowest and the highest frequency of the studied signal. For example their values for a RVII channel are: f_r=183,25MHz, f_b=182MHz and f_h=190MHz. For the defining the number of nonlinear products, included in the studied channel ($f_{NP} \neq f_r$), is used a step by step changing of f_r to the left or to the right of her. The step is ±$k.f_o$, where f_o=0,25MHz and k=1÷16. In Table 2 are given the results for the number of nonlinear products for a transmitting of between 3 and 30 channels in a CATV system. The channels are spread according to the D/K standard. In Fig.5 are graphically presented the results for the same system, but now transmitting 35 channels.

TABLE 1

Input frequencies MHz			Nonlinear products MHz							
f_i	f_j	f_k	$f_i - f_j + f_k$	$f_j + f_k - f_i$	$f_i + f_j - f_k$	$f_i + f_j + f_k$	$2f_i - f_j$	$2f_j - f_i$	$2f_i + f_j$	$2f_j + f_i$
111,25	127,25	215,25	199,25	231,25	23,25	453,75	95,25	143,25	349,75	365,75
119,25	135,25	215,25	199,25	231,25	39,25	469,75	103,25	151,25	373,75	389,75
127,25	143,25	215,25	199,25	231,25	55,25	485,75	111,25	159,25	397,75	413,75
159,25	175,25	215,25	199,25	231,25	119,25	549,75	143,25	191,25	493,75	509,75
167,25	183,25	215,25	199,25	231,25	135,25	565,75	151,25	199,25	517,75	533,75
175,25	191,25	215,25	199,25	231,25	151,25	581,75	159,25	207,25	541,75	557,75
183,25	**199,25**	215,25	199,25	231,25	167,25	597,75	167,25	215,25	565,75	581,75
191,25	207,25	215,25	199,25	231,25	183,25	613,75	175,25	223,25	589,75	605,75
199,25	**215,25**	215,25	199,25	231,25	**199,25**	629,75	183,25	231,25	613,75	629,75
474	490	674	658	690	290	1638	458	506	1438	1454
482	498	674	658	690	306	1654	466	514	1462	1478
490	506	674	658	690	322	1670	474	522	1486	1502
530	546	674	658	690	402	1750	514	562	1606	1622
538	554	674	658	690	418	1766	522	570	1630	1646
546	562	674	658	690	434	1782	530	578	1654	1670
626	642	674	658	690	594	1942	610	658	1894	1910
642	**658**	674	658	690	626	1974	626	674	1942	1958
658	**674**	674	658	690	**658**	2006	642	690	1990	2006

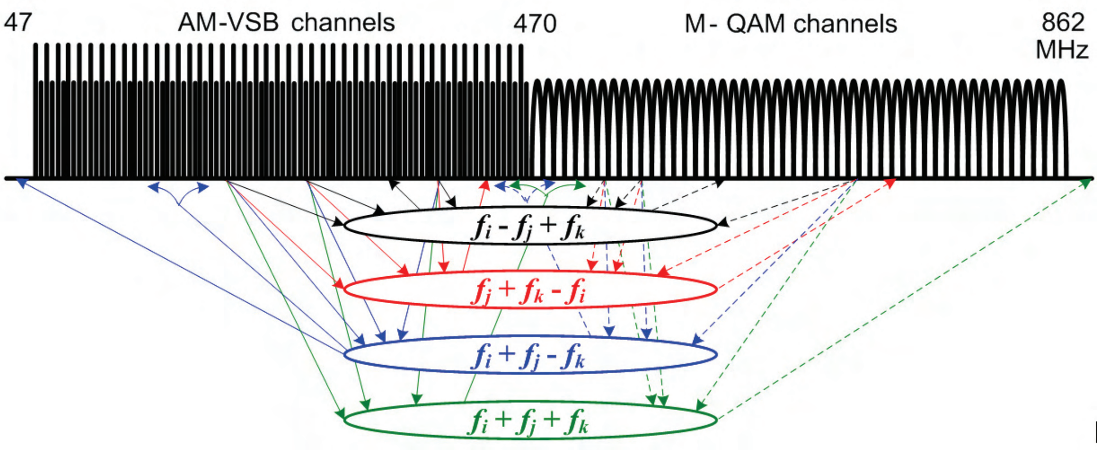

Fig.2. Influence of three-component beat

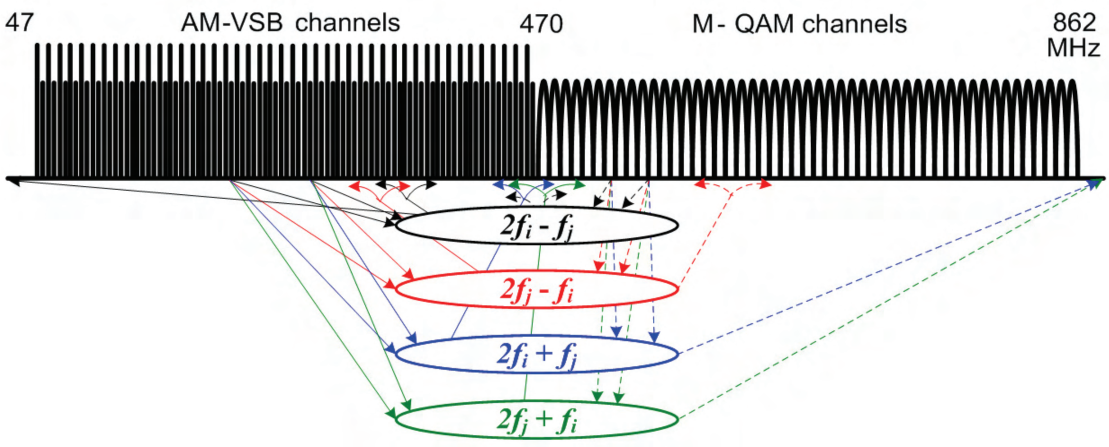

Fig.3. Influence of two-component beat

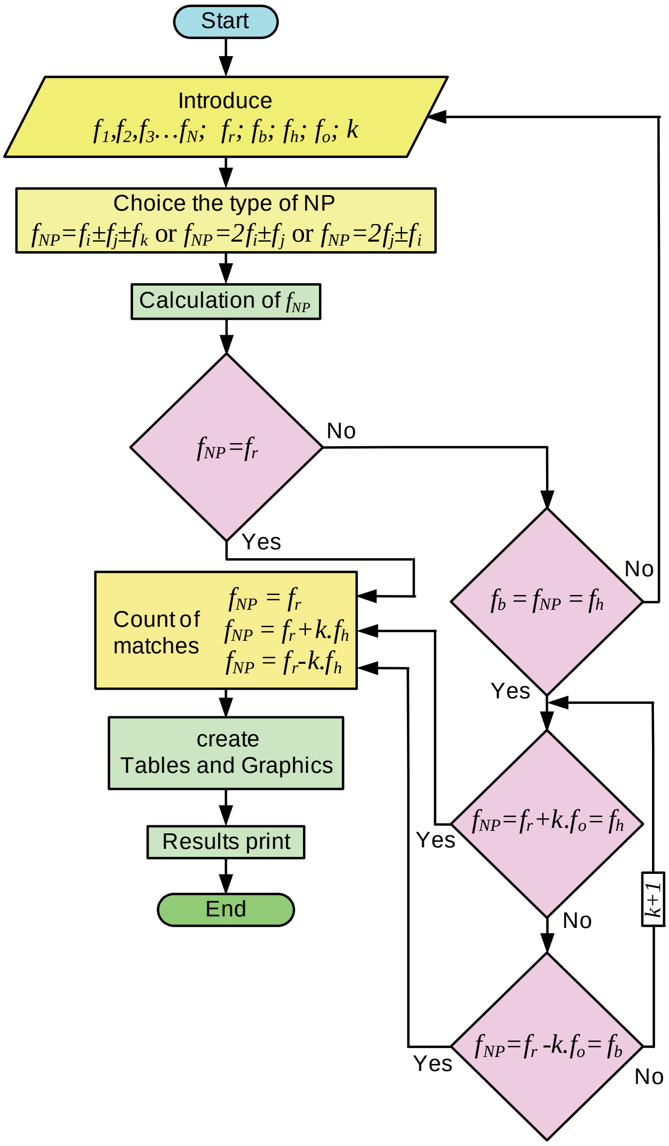

Fig.4. Block diagram of the algorithm

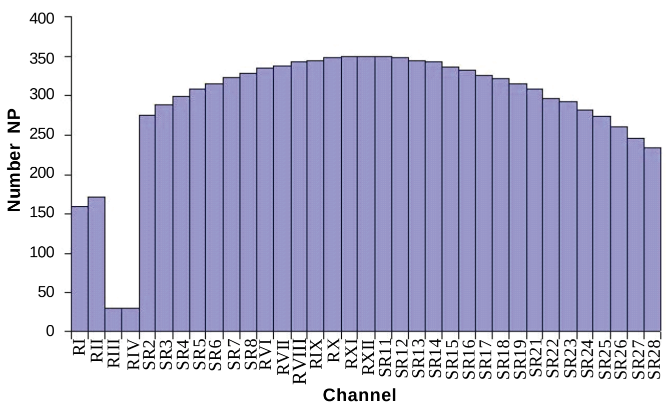

Fig.5. Distribution of third order beats for a 35 channel system

TABLE 2

No. of Channels	Channels	Central Channel	Max. No. on central channel $2f_i\pm f_j$	$f_i\pm f_j\pm f_k$
3	RX-RXII	RXI	0	1
4	RIX-RXII	RXI	1	2
5	RVIII- RXII	RX	2	4
6	RVII- RXII	RX	2	7
7	RVI- RXII	RIX	2	11
8	SR8- RXII	RIX	3	15
9	SR7- RXII	RVIII	4	20
10	SR6- RXII	RVIII	4	26
11	SR5- RXII	RVII	5	33
12	SR4- RXII	RVII	5	40
12 STD	RI- RXII	RIX	2	19
13	SR3- RXII	RVI	6	47
14	SR2- RXII	RVI	6	56
15	SR1- RXII	RVI	7	65
16	RV- RXII	SR8	7	77
17	RV- RXI	SR8	8	88
18	RV-SR12	SR8	8	100
19	RV-SR13	SR8	9	112
20	RV-SR14	RVI	9	125
21	RV-SR15	RVI	10	139
22	RV-SR16	RVII	10	157
23	RV-SR17	RVII	11	170
24	RV-SR18	RVIII	11	187
25	RV-SR19	RVIII	11	204
26	RV-SR21	RIX	12	204
27	RIV-SR21	RIX	12	206
28	RIII-SR21	RVIII	12	212
29	RII-SR21	RVIII	12	219
30	RI-SR21	RVII	13	226

IV. CONCLUSION

The presented mathematical analysis and algorithm for determining the number of nonlinear products of composite triple beat, make it possible to explore these nonlinear products not only by the D/K standard. Furthermore, the distribution of channels might be different than presented above. Analog and digital channels can be carried through the whole range of 47MHz to 862MHz in the desired order and number. This flexibility allows a frequency planning of HFC/CATV system with a minimum number of intermodulation products and provides a quality and reliable transmitting of the signals.

REFERENCES

[1] Y.-I. Kim, J. H. Kim, S. Lee, D. H. Woo, S. H. Kim, and T.-H. Yoon, “Broad-band all-optical flip-flop based on optical bistability in an integrated SOA/DFB-SOA”, IEEE Photon. Technol. Lett., vol. 16, no. 2, pp. 398–400, Feb. 2004.

[2] L.T. Jordanova, V. I. Topchiev, “Reduction of the Signal Nonlinear Distortion in CATV Systems applying Dual Mach-Zehnder Modulators’, J. Opt. Commun. 30, pp.74-79, 2009.

[3] “Some Notes on Composite Second and Third Order Intermodulation Distortions” (2005) http://www.matrixtest.com/literat/MTN108.pdf

[4] O. B. Panagiev, ”Determinating the amplitudes of intermodulation products of higher order by means Volterra kernels”, ICEST, Proc. of Papers, vol.1, Bitola, pp. 215-216, 16-19 June 2004.