Modified Legendre Filters with Minimization of Summed Sensitivity

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Abstract – **The paper presents the design of active RC filter based on a new class of all-pole approximation. The approximating function is derived using Legendre orthogonal polynomial with the appropriate weights and usage of two Legendre multiplication factors at the origin and the passband edge frequency. Detailed analysis is done for the frequency response and tolerance analysis of active RC filters.**

Keywords – **Approximation, Tolerance analysis.**

I. INTRODUCTION

Continuous-time active RC filters are suitable for integration into the analog front end of mixed mode VLSI (Very-Large-Scale Integration) chips for communication systems. Programmable analog filters, and SC (Switched Capacitor) integrated filters, are replacing the classic analog filters (such as active opamp RC – operational amplifier + Resistor-Capacitor), but the design procedure is still based on the sensitivity and tolerance analysis [1, 2] in order to manufacture robust filters, to increase the production yield and to minimize the cost of mass production. The sensitivity and tolerance analysis allows the designer to predict variations of the filter performances and to predict the production yield (which is defined as a ratio of the number of manufactured filters satisfying the specification to the total number of manufactured filters) [1, 2].

It is well known that in the design of filters with real elements, the most influential factors on the filter performance may be the finite tolerances and temperature changes of their components. However, although the deviation between the designed and the measured attenuation characteristics, especially in the pass-band, is very important in the design of filter functions, which satisfies the specified characteristics when implemented with real components, it does not seem to be fully covered in the literature available.

It should be stressed that realizations using ideal components may disregard influence of changes due to the temperature variations. We are trying to overcome this problem using appropriate transfer function in such a way to minimize the summed sensitivity and thus to minimize temperature changes using component with the same temperature coefficient. In some papers, the problem was solved by designing filters with low Q factor of critical pair of poles [3–6].

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II. APPROXIMATING METHOD

The most general form of lowpass prototype filter all-pole transfer function is

$$H_n(s) = \frac{K\prod_{r=1}^{n}(-s_r)}{\prod_{r=1}^{n}(s-s_r)} \qquad (1)$$

The filter order is n, K is a constant to specify the attenuation at s=0 (for example 0 dB), and the poles of the transfer function are

$$s_r = \sigma_r + j\,\omega_r, \qquad r\,(1,\,2,\,3,\ldots n)$$

The squared magnitude response is

$$H_n(\omega)\;H_n(\omega) = \frac{1}{1+\varepsilon^2\;A_n\,(\omega^2)} = \frac{1}{1+\varepsilon^2\sum_{i}^{n} a_{2i}\,\omega^{2i}} \qquad (2)$$

The denominator of the squared magnitude response is a polynomial in ω, with rael contstants a_{2i}. The parameter $\varepsilon^2 = \rho^2/(1-\rho^2)$ determines the attenuation at the pass-band edge, while ρ is the pass-band reflection factor.

The characteristic function is normalized to1 at the pass-band edge frequency

$$A_n(\omega_p) = 1 \qquad (3)$$

The squared magnitude response of new class becomes

$$H_n(\omega)\;H_n(\omega) = \frac{1}{1+\varepsilon^2\left(\sum_{r=0}^{n} b_{2r}\,P_{2n}\;(\omega)\right)} \qquad (4)$$

$P_r(\omega)$ is the rth-order Legendre orthogonal polynomial.

The minimum of the ratio of the reflected power and the transmitted power is obtained by minimizing the following integral

$$I_{\min}(\omega) = \int_0^1 p(\omega)\,A_n\,(\omega^2)\,d(\omega) \qquad (5)$$

The weight $p(\omega)$ is 1. Firstly, we define a function using conditions of the proper prototype low-pass approximation for odd order

$$\phi(b_0, b_2, b_4, b_6, \ldots b_{2n}, \lambda_0, \lambda_1) = \int_0^1 [\sum_{r=0}^{r=n} b_{2r} P_{2r}(\omega)]^2 d(\omega) - \lambda_0[\sum_{r=0}^{r=n} b_{2r} P_{2r}(0)] - \lambda_1[\sum_{r=0}^{r=n} b_{2r} P_{2r}(1) - 1] \qquad (6)$$

Next, we derive partial derivatives and set a system of equations that should be solved in terms of b_0, b_2, $b_4, b_6, \ldots, b_{2n}$

$$\frac{\partial}{\partial b_0} \phi(b_0, b_2, b_4, b_6, \ldots b_{2n}, \lambda_0, \lambda_1) = 2b_0 - \lambda_0 P_0(0) - \lambda_1 P_0(1) = 0 \qquad (7)$$

$$\frac{\partial}{\partial b_{2r}} \phi(b_0, b_2, b_4, b_6, \ldots b_{2n}, \lambda_0, \lambda_1) = \frac{2}{1+4r} b_{2r} - \lambda_0 P_{2r}(0) - \lambda_1 P_{2r}(1) = 0 \qquad (8)$$

$$r = 1, 2, 3, \ldots, n$$

$$\frac{\partial}{\partial \lambda_0} \phi(b_0, b_2, b_4, b_6, \ldots b_{2n}, \lambda_0, \lambda_1) = \sum_{r=0}^{r=n} b_{2r} P_{2r}(0) = 0 \qquad (9)$$

$$\frac{\partial}{\partial \lambda_1} \phi(b_0, b_2, b_4, b_6, \ldots b_{2n}, \lambda_0, \lambda_1) = \sum_{r=0}^{r=n} b_{2r} P_{2r}(1) - 1 = 0 \qquad (10)$$

The properties of the new class of functions will be demonstrated by example.

III. Example Design

Let us design a filter of the order $n = 10$ with $\rho = 0.25$. The poles of the transfer function are computed using equations (7)-(10). Magnitude response in dB is presented in Figure 1 for the 10th-order filter. It is important to notice that attenuation oscillates around 0dB. This type of approximation cannot be implemented using lossless LC filter because it has negative attenuation (gain) in the passband. The poles of the designed transfer function are

$$\begin{pmatrix} -0.3638978001828931 - 0.1566353531179695\,i \\ -0.3638978001828931 + 0.1566353531179695\,i \\ -0.3231936893533610 - 0.4592472353776506\,i \\ -0.3231936893533610 + 0.4592472353776506\,i \\ -0.2515205288535666 - 0.7206516018040339\,i \\ -0.2515205288535666 + 0.7206516018040339\,i \\ -0.1571211264738653 - 0.9145512196545400\,i \\ -0.1571211264738653 + 0.9145512196545400\,i \\ -0.0524125445027391 - 1.0204240278599806\,i \\ -0.0524125445027391 + 1.0204240278599806\,i \end{pmatrix}$$

The poles of the 11th-order transfer function are

$$\begin{pmatrix} -0.3423280721141051 \\ -0.3251529281079403 - 0.2819569693507116\,i \\ -0.3251529281079403 + 0.2819569693507116\,i \\ -0.2806469111451467 - 0.5454420792584142\,i \\ -0.2806469111451467 + 0.5454420792584142\,i \\ -0.2140524158334217 - 0.7671968743259037\,i \\ -0.2140524158334217 + 0.7671968743259037\,i \\ -0.1319460394353278 - 0.9293038103800189\,i \\ -0.1319460394353278 + 0.9293038103800189\,i \\ -0.0437164371089922 - 1.0171395361471176\,i \\ -0.0437164371089922 + 1.0171395361471176\,i \end{pmatrix}$$

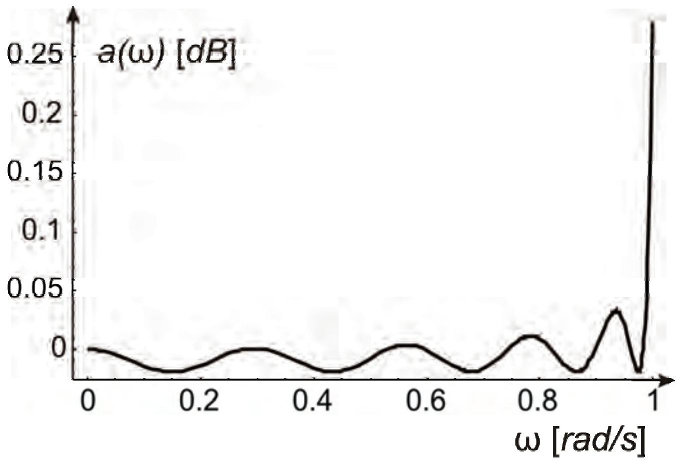

Fig. 1. Attenuation in dB of the 10th order filter.

Figure 2 shows the summed sensitivity in the passband expressed in dB. Very small sensitivity in the passband implies that the variation of the magnitude response due to temperature changes will be small for the case of the same temperature coefficient of all elements of the same type.

More details about summed sensitivity can be found in [1] and [2].

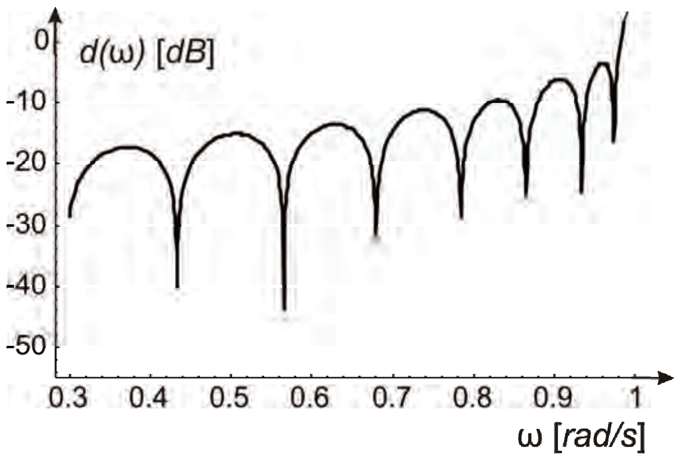

Fig. 2. Summed sensitivity of the 10th order filter.

Figure 3 shows the summed sensitivity in the passband expressed in dB for 11th-order filter. Again, the summed sensitivity is very low in the passband.

The passband variation in dB is illustrated in Figure 4. This type of approximation provides very low passband variation in the most of the passband, and attenuation increases at frequencies close to the edge frequency. Since the larges sensitivity and larges deviation of the attenuation due to element values changes are at the passband edge frequency, the filter can be more robust after moving the edge frequency into the transition region, as it is done in [7].

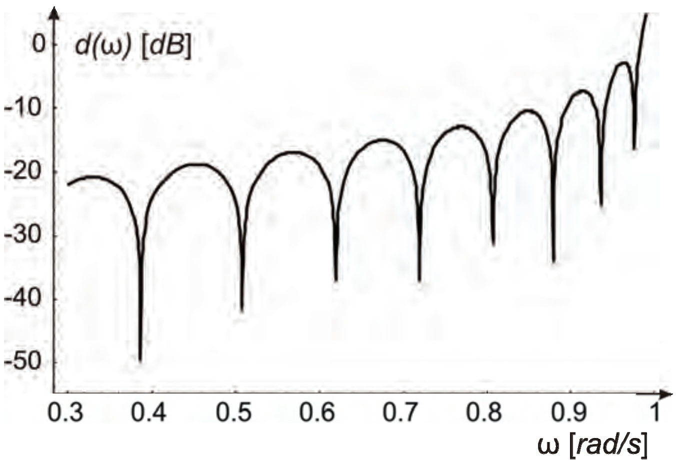

Fig. 3. Summed sensitivity of the 11th order filter.

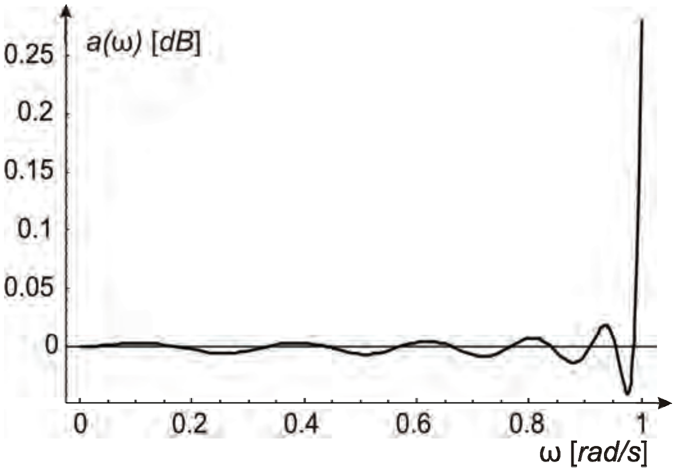

Fig. 4. Attenuation in dB of the 11th order filter.

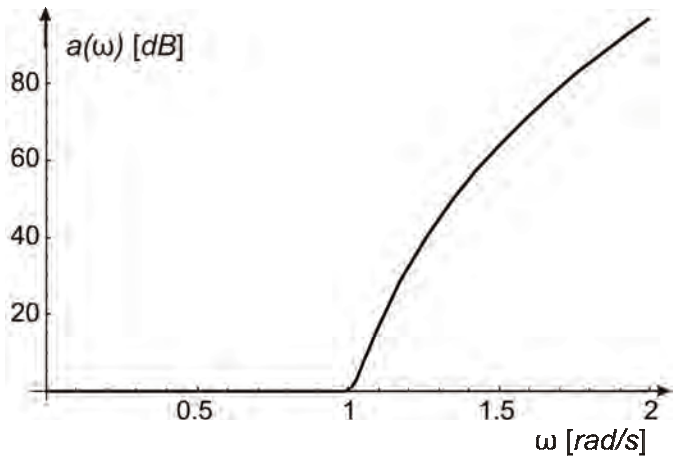

Fig. 5. Attenuation in dB of the 11th order filter.

Figure 4 shows that the maximum deviation can be controlled in a similar way as in the case of reducing the effect of imperfection by reducing Q factor of second order sections.

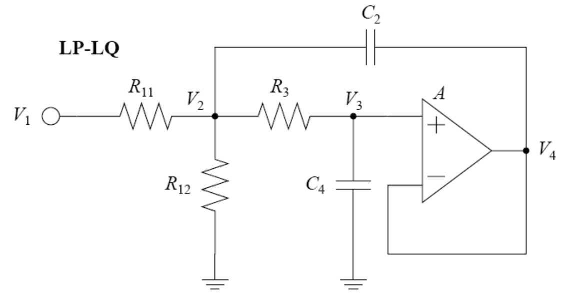

Fig. 6. Low-pass low-Q factor biquad.

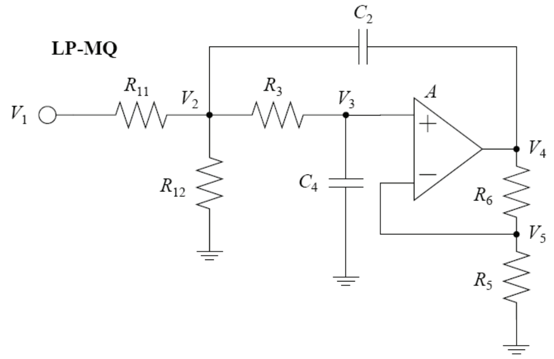

Fig. 7. Low-pass medium-Q factor biquad.

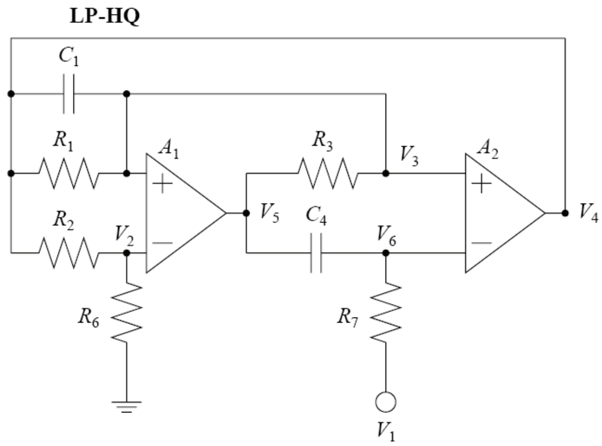

Fig. 8. Low-pass high-Q factor biquad.

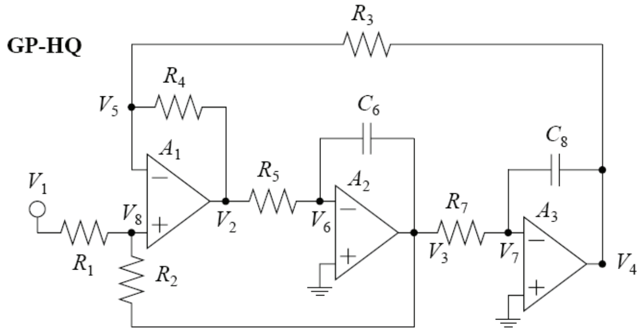

Fig. 9. Low-pass high-Q factor filter - general purpose biquad.

Figure 6 illustrates that the attenuation increases monotonically in the stopband, because this type of approximation belongs to all-pole approximations.

The design procedure is based on the second order filter sections described in [2]. Some of typical biquads are swown in Figures 6, 7, 8, and 9.

The general purpose second-order section can be implemented using programmable analog integrated circuits, such as biquad shown in Figure 10.

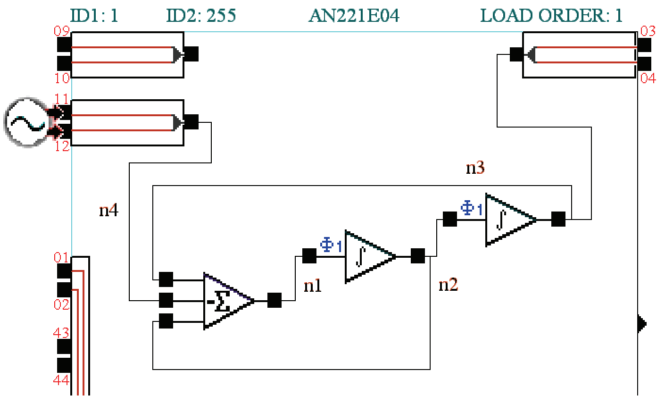

Fig. 10. General purpose opamp biquad implemented using programmable analog integrated circuits.

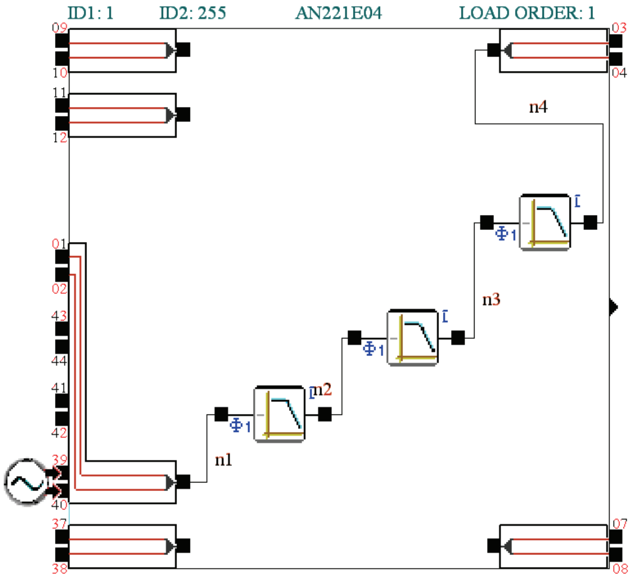

Fig. 11. Sixth-order filter implemented using programmable analog integrated circuits and 3 biguads.

The designed second-order sections as a building block can be used for implementing highest order filters, see Figure 11. Q factor can be extremely high with respect to classic opamp implementations, see Figure 12.

The design procedure is programmed in Mathematica [8]. The whole analysis is evaluated for arbitrary precision, and automated using equations (7) – (10).

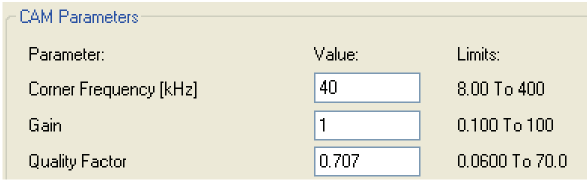

Fig. 12. Programmable parameters of biguads.

IV. Conclusion

Detailed analysis of the frequency response and summed sensitivity in the passband for active RC filters are presented for a class of modified Legendre filters. The filter parameters can be computed for optimization of some of the filter property, such as to minimize the magnitude response deviation in the passband.

In practice, it is more comfortable to have an application as executive programme, using C++ or Java, or web application for computing element values, filter parameters and filter characteristics. The future work is to develop software that will be available for users on different platforms.

Acknowledgement

This work was partially supported by the Ministry of Science of Serbia under Grant TR-32023.

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