

A Practical Approach to Control of an Overhead Crane

Plamen Petrov¹ and Lubomir Dimitrov²

Abstract – This paper proposes a trajectory tracking controller for a two degree of freedom (2-DOF) overhead crane. First, a dynamic model of the crane suitable for feedback control is developed using robotic methodology. A desired trajectory for the trolley motion is generated using a reference differential equation. The proposed control law is based on collocated partial feedback linearization combined with trajectory tracking and linear feedback control which achieves local asymptotic stability of the closed-loop system. Simulation results illustrate the effectiveness of the proposed controller.

Keywords – Overhead crane, Dynamic modelling, Feedback control, Perturbed systems.

I. INTRODUCTION

In the last decades, the overhead cranes have been widely used for transportation in many industrial applications and become an interesting issue from automatic control point of view. The goal is to transport the payload quickly and in the same time to reduce the rope swing angle. Recently, different techniques been proposed for the design of Linear Quadratic [1, 2], adaptive [3], nonlinear coupling control [4] controllers for overhead cranes. The overhead cranes belong to the class of underactuated mechanical systems, which have fewer control inputs than degrees of freedom. One of the complexities of these systems is that they are not feedback linearizable. Due to the positive definiteness of the inertia matrix of this class of systems, the so-called collocated partial feedback linearization property [5] holds, which refers to the control that linearizes the equations associated with the actuated degrees of freedom of the system. Available control design methods mainly include approximate linearization [6] and saturation control [7].

In this paper, we propose a simplified control strategy based on collocated partial feedback linearization of the dynamic model combined with trajectory tracking and linear feedback control law, which achieves local asymptotic stability of the closed loop system. The organization of the paper is as follows: In Section II, a dynamic model of the crane suitable for feedback control applications is derived. The Problem formulation is given in Section III. In Section IV, a control law is designed. Section V contains simulation results. Conclusions are presented in Section VI.

II. DYNAMIC MODEL

A schematic view of an overhead crane is shown in Fig. 1. In order to derive a dynamic model suitable for control applications, we make the following assumptions: the payload mass is considered as a point-mass, and the mass and stiffness of the hoisting rope are neglected. The system has two degree-of-freedom and the associate generalized coordinates are

$$q = [d \ \theta]^T \in \mathbb{R}^2 \quad (1)$$

where d is the displacement of the trolley and θ is the swing angle of the load, (Fig. 1).

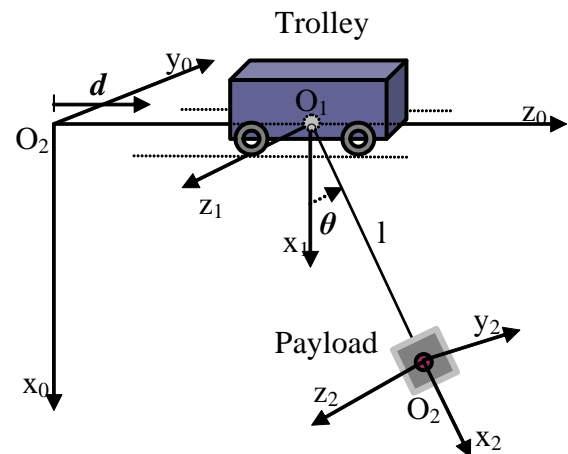


Fig. 1. Schematic of the overhead crane

We use the Denavit-Hartenberg convention [8] for the description of the crane kinematics. An inertial coordinate system $O_0x_0y_0z_0$ is assigned in the work space where the z_0 axis is in direction of the trolley displacement. The z_1 axis of a moving together with the trolley coordinate frame $O_1x_1y_1z_1$ is the axis of revolution of the rope. The z_2 axis of the coordinate frame $O_2x_2y_2z_2$ which is attached to the payload is parallel to z_1 . The link parameters are given in Table I where the four quantities a_i , d_i , α_i , and θ_i are parameters of link i and joint i , ($i = 1, 2$).

TABLE I
PARAMETERS FOR A 2-DOFS OVERHEAD CRANE

| Parameters | Link 1 | Link 2 |
|------------------|---------|----------------|
| a_i [m] | 0 | l |
| d_i [m] | $d=var$ | 0 |
| α_i [rad] | $\pi/2$ | 0 |
| θ_i [rad] | 0 | $\theta = var$ |

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The corresponding transformation matrices which define the relative position and orientation between the adjacent coordinate systems are

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & l\cos\theta \\ \sin\theta & \cos\theta & 0 & l\sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

Using the transformation matrices (2), the coordinates of point O_2 with respect to $O_0x_0y_0z_0$ are

$$\begin{aligned} z_{O_2} &= d + l \sin \theta \\ x_{O_2} &= l \cos \theta \end{aligned} \quad (3)$$

The dynamic equations of motion of the crane are derived using Lagrange formalism

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i, \quad i = 1, 2 \quad (4)$$

where the Lagrangian L represents the difference between the kinetic and potential energy of the system, and Q_i are the generalized forces associated with the generalized coordinates.

The kinetic energy of the system is obtained as follows

$$T = \frac{1}{2}(M + m)\dot{d}^2 + \frac{1}{2}m(2l \cos \theta \dot{\theta} + l^2 \dot{\theta}^2) \quad (5)$$

where M and m are the mass of the trolley and the load, respectively and l is the length of the rope.

The potential energy of the system is given as

$$U = -mgl \cos \theta. \quad (6)$$

Using Eqs. (3), (4), (5) and (6), the dynamic equation of motion of the overhead crane are obtained in the form

$$D\ddot{q} + C\dot{q} + G = Q \quad (7)$$

where

$$\begin{aligned} D &= \begin{bmatrix} M+m & ml\cos\theta \\ ml\cos\theta & ml^2 \end{bmatrix}, & C &= \begin{bmatrix} 0 & -ml\sin\theta\dot{\theta} \\ 0 & 0 \end{bmatrix}, \\ G &= \begin{bmatrix} 0 \\ mgl\sin\theta \end{bmatrix}, & Q &= \begin{bmatrix} F \\ 0 \end{bmatrix} \end{aligned} \quad (8)$$

where F is the control force acting on the trolley.

Remark 1: It should be noted that the matrix D is positive definite and the matrix $1/2\dot{D} - C$ is skew-symmetric.

III. PROBLEM STATEMENT

In this paper, we consider the problem of position control of the overhead crane. The goal is to transport the payload quickly with high precision, and in the same time to reduce the swing angle which does not exceed 5° through the entire

trajectory of the trolley. The desired trajectory for the trolley motion is proposed in the integral form

$$z^d(t) = z_0^d \left[1 - (1 + \rho t) e^{-\rho t} \right] \quad (9)$$

where z_0^d is the desired distance which has to be traveled by the trolley, and ρ is a double root of a desired linear differential equation describing the trolley motion, (a larger ρ leads to a faster motion of the trolley and as a consequence, a bigger swing of the payload).

We make the following change of coordinate

$$z_e = d - z^d. \quad (10)$$

and input

$$w = u - \ddot{z}^d \quad (11)$$

where, due to the collocated partial feedback linearization property, u is obtained in the form

$$u = \frac{1}{M + m \sin^2 \theta} \left[F + m \sin \theta (l \dot{\theta}^2 + g \cos \theta) \right]. \quad (12)$$

Finally, using (10)-(12), after some work, the dynamic equations (7) for the crane can be written in error coordinate form as

$$\begin{aligned} \ddot{z}_e &= w \\ \ddot{\theta} &= -\frac{1}{l} \left[(w + \ddot{z}^d) \cos \theta + g \sin \theta \right] \end{aligned} \quad (13)$$

We assume that $e = [z_e \quad \dot{z}_e \quad \theta \quad \dot{\theta}]^T \in \mathfrak{R}^4$ are measurable. Given the crane dynamics in error coordinates described by (13), the control objective is to asymptotically regulate $z_e(t)$ to zero (transportation of the payload) and minimize the swing angle $\theta(t)$ of the payload.

IV. FEEDBACK CONTROL DESIGN

The overhead crane is an underactuated single-input (w) two-output (z_e, θ) system. The control problem consists in finding a feedback control law for the system (13) such that

$$\lim_{t \rightarrow \infty} (z_e(t)) = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} (\theta(t)) = 0. \quad (14)$$

Consider the linear control

$$w = -k_1 z_e - k_2 \dot{z}_e + k_3 l \theta + k_4 l \dot{\theta} \quad (15)$$

where k_i , ($i = 1, 2, 3, 4$) are positive gains.

The resulting closed-loop system becomes

$$\begin{aligned} \ddot{z}_e &= -k_1 z_e - k_2 \dot{z}_e + k_3 l \theta + k_4 l \dot{\theta} \\ \ddot{\theta} &= -\frac{1}{l} \left((-k_1 z_e - k_2 \dot{z}_e + k_3 l \theta + k_4 l \dot{\theta}) + \ddot{z}^d \right) \cos \theta + g \sin \theta \end{aligned} \quad (16)$$

Denoting $\dot{z}_e = v_{ze}$ and $\dot{\theta} = \omega$, for small swing angles, the tangent linearization of Eqs. (16) about $\theta = 0$ can be written in state-space form as

$$\dot{x} = f(x) + g(t, x) \quad (17)$$

where

$$x = [z_e \quad v_{ze} \quad \theta \quad \omega]^T \quad (18)$$

$$f(x) = \begin{bmatrix} v_{ze} \\ -k_1 z_e - k_2 v_{ze} + k_3 l \theta + k_4 l \omega \\ \omega \\ \frac{k_1}{l} z_e + \frac{k_2}{l} v_{ze} - \left(k_3 + \frac{g}{l}\right) \theta - k_4 \omega \end{bmatrix}; \quad g(t, x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{l} \ddot{z}^d \end{bmatrix}. \quad (19)$$

The system (17) can be viewed as a perturbation of the nominal system $\dot{x} = f(x)$, which has exponentially stable equilibrium point at the origin $x = 0$, and $\ddot{z}^d(t)$ is a uniformly bounded disturbance that satisfies $|\ddot{z}^d| \leq \frac{z_0^d \rho^2}{l} = \delta$.

Furthermore, $\ddot{z}^d(t) \rightarrow 0$ as $t \rightarrow \infty$. Using Lemma 4.9, [9, p. 208], for the perturbed system (17), it can be shown that $x(t) \rightarrow 0$ as $t \rightarrow \infty$. Based on Lyapunov's linearization (indirect) method [9], one can be concluded that the corresponding nonlinear system is locally asymptotically stable.

V. SIMULATION RESULTS

Several simulations using MATLAB were carried out in order to illustrate the performance of the proposed controller. The desired trajectory of the trolley is given by (9) where the desired distance to travel is $z_0^d = 7m$ and $\rho = 0.5$. The overhead crane is tested with a mass of 200kg and 300kg for the trolley and the payload, respectively. The length of the rope was chosen to be $l = 5m$. In the first simulation, from Fig. 2, we can see the evolution in time of the swing angle θ during the displacement of the trolley. Fig. 3, presents the evolution in time of the movement of the trolley d according to desired trajectory $z^d(t)$. The results of the simulations confirm the validity of the proposed controller.

VI. CONCLUSION

In this paper, a trajectory tracking controller for a 2-DOF overhead crane has been proposed. A dynamic model of the crane was developed using robotic methodology. A desired trajectory for the trolley motion was generated using a reference differential equation. The proposed control law was based on collocated partial feedback linearization combined with trajectory tracking and linear feedback control and achieved local asymptotic stability of the closed-loop system. Simulation results were carried out and confirmed the effectiveness of the proposed controller.

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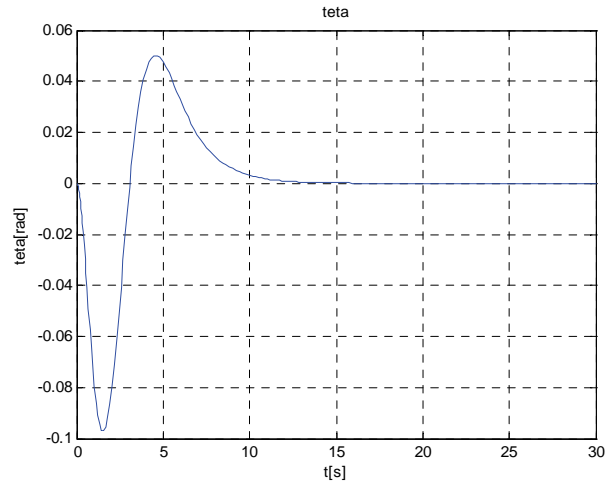


Fig. 2. Time history of the swing angle of the payload

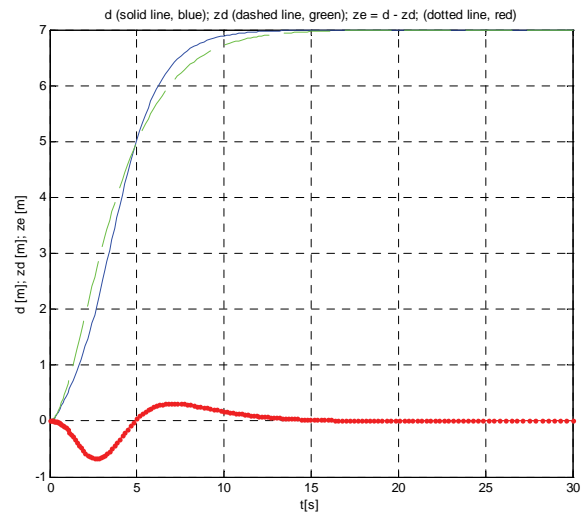


Fig. 3. Time history of the trolley displacement (solid line), desired trajectory (dashed line), and tracking error (dotted line)

REFERENCES

- [1] Z. Wang, B. Surgenor, A problem with the LQ control of overhead cranes, *Trans. ASME J. Dyn. Syst. Meas. Control*, vol. 128, 2006, pp. 436-440.
- [2] K. Prommaneevat, P. Roengruen, V. Kongratana, Anti-sway Control for Overhead Crane, in *Proc. Int. Conf. Contr. Autom. Systems*, 2007, pp. 1954-1957.

- [3] B. Ma, Y. Fang, X. Zhang, Adaptive tracking control for an overhead crane system, in *Proc. 17th IFAC World Congress*, 2008, pp. 12194-12199.
- [4] Y. Fang, E. Zergeroglu, W. Dixon, D. Dawson, Nonlinear coupling control laws for an overhead crane system, in *Proc. IEEE Int. Conf. Contr. Applications*, 2001, pp. 639-644.
- [5] M. Spong, Underactuated mechanical systems, Lecture Notes in Control and Information Sciences, Springer, vol. 230, 1998, pp. 135-150.
- [6] N. Bedrossian, Approximate feedback linearization: the cart-pole example, in *Proc. Rob. Automation*, 1992, pp. 1987-1992.
- [7] A. Teel, Using saturation to stabilize a class of single-input partially linear composite systems, in *IFAC Symp. Nonlinear Control Design*, 1992, pp. 224-229.
- [8] M. Spong, S. Hutchinson, M. Vidyasagar, Robot modeling and control, John Wiley and Sons, Inc., 2005.
- [9] H. Khalil, Nonlinear systems, Macmillan Publishing Company, 1992.