The Analysis of Load Type Influence on Loss Allocation in Radial Distribution Networks

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Abstract – **The paper presents the results of power loss allocation in radial distribution networks for different load types. Three characteristic load types are considered: constant power, constant current and constant impedance load. Loss allocation is performed on the basis of current flows through network branches. The algorithm of the calculation regards all demands of fair allocation. Network topology is described by oriented graph. The folmulas for calculation and loss distribution in each branch of the network are given. It is shown that load type hardly influences on real power losses, and therefore it have to be regarded for loss allocation calculation.**

Keywords – **Loss allocation, Power losses, Distribution networks.**

I. INTRODUCTION

In conditions of power system deregulation, where electrical energy is treated as goods, it is necessary to determine the shares of all customers in total loss creation in distribution network. An intention is that every consumer could fulfill his obligation concerning paying of some part of loss expenses. To say by the other words, it is necessary to make loss allocation to the nodes or to the consumers supplied from these nodes. On that way, tariff system would take into account real influence of all consumers to expenses originating from energy losses. The problem of loss allocation is specially emphasized in transmission networks. In recent years, several papers have been appeared in literature offering special methods of loss allocation in distribution networks [1-5]. Main problems for loss allocation are: nonlinear connection between the losses and loads in the nodes and defining the way of common losses dividing.

Except aforementioned ones, additional problems appear due to unknown structure and load characteristic in some node. Namely, the calculations of active power losses always are carried out with load of constant power type, which is independent of voltage. In real conditions, load in some node depends on voltage and it reflects on power losses in distribution network [6]. This fact should be regarded if we want exact calculations of power and energy losses and after it allocation calculation on consumption nodes.

There are three characteristic load types: 1- constant power, 2-constant current and 3-constant impedance. In real situation no one of these three types suits to actual load which is changeable and unpredictable in time.

As authors of this paper know, there was no one attempt to solve this problem completely and to take into account load characteristics in every node of distribution network. Just therefore this work analyzes load type influence on calculation results of loss allocation. The calculations are carried out according the method proposed in [4]. Basic aim of this work was to research if, and in which extent, load type in some node of distribution network would impact on calculation results of loss allocation. In this sense, radial test network with 32 nodes is considered [7].

II. METHODOLOGY

Total active power losses, ΔP, in distribution network with n branches are obtained as a sum of losses in some branches ΔP_i,

$$\Delta P = \sum_{i=1}^{n} \Delta P_i \, . \tag{1}$$

Active power losses in *i*-th branch of network are:

$$\Delta P_i = 3 R_i J_i^2 \tag{2}$$

where:

R_i – active resistance of *i*-th branch,

J_i – effective value of current of *i*-th branch.

Current of *i*-th branch is determined as a sum of the currents of these consumers (Z) supplied via this branch, $\underline{I}_j$,

$$\underline{J}_i = \sum_{j \in Z} \underline{I}_j \, , \tag{3}$$

which is illustrated on Fig. 1.

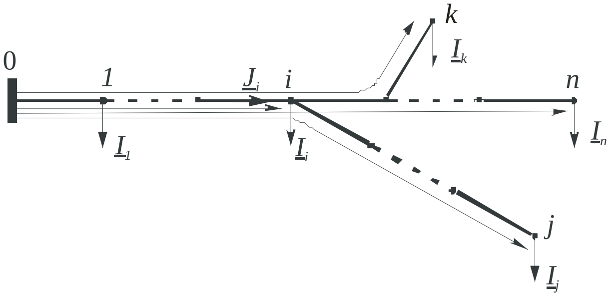

Fig. 1. Forming of radial network paths

When one express each load through their components $\underline{I}_j = I_{pj} + jI_{qj}$, it is easy to find the contributions of all loads/consumers to real and imaginary part of *i*-th branch current.

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$$\underline{J}_i = \sum_{j\in Z} \underline{I}_j = \sum_{j\in Z} I_{pj} + j\sum_{j\in Z} I_{qj} \,. \tag{4}$$

If Eq. (4) is put into Eq. (2), for active power losses in *i*-th branch we obtain:

$$\Delta P_i = 3R_i\left[\left(\sum_{j\in Z} I_{pj}\right)^2 + \left(\sum_{j\in Z} I_{qj}\right)^2\right]. \tag{5}$$

Developing Eq. (5) gives:

$$\Delta P_i = 3R_i\left[\sum_{j\in Z} I_{pj}^2 + 2\sum_{\substack{j\in Z\\ k\in Z\\ j\neq k}} I_{pj}I_{pk} + \sum_{j\in Z} I_{qj}^2 + 2\sum_{\substack{j\in Z\\ k\in Z\\ j\neq k}} I_{qj}I_{qk}\right], \tag{6}$$

$$= \sum_{j\in Z} \Delta P_{ij}$$

From Eq. (6) we can calculate impact of *j*-th load to *i*-th branch losses, designated here as *ΔP*$_{ij}$.

It means that *j*-th load has impact to *i*-th branch losses through items I^2_{pj} and I^2_{qj}, corresponding exclusively to *j*-th consumer and via items $2\sum\limits_{\substack{j\in Z\\ k\in Z\\ j\neq k}} I_{pj}I_{pk}$ and $2\sum\limits_{\substack{j\in Z\\ k\in Z\\ j\neq k}} I_{qj}I_{qk}$ showing simultaneous impact of *j*-th and *k*-th loads to the losses in *i*-th branch.

If *j*-th currents does not pass through *i*-th branch ($j \notin Z$) then would be $\Delta P_{ij}=0$.

Question arises how to divide power losses in some branch to individual consumers (ΔP_{ij}). This problem is not unique due to nonlinearity of products appearing in [4]. Two easy feasible principles are possible: linear and square. Here the first principle is used. The losses should be divided proportionally to loads current components. So it is obtained:

$$\Delta P_{ij} = 3R_i[I_{pj}^2 + 2\sum_{\substack{j\in Z\\ k\in Z\\ j\neq k}} I_{pj}I_{pk}\,\frac{I_{pj}}{I_{pj}+I_{pk}}$$

$$+ I_{qj}^2 + 2\sum_{\substack{j\in Z\\ k\in Z\\ j\neq k}} I_{qj}I_{qk}\,\frac{I_{qj}}{I_{qj}+I_{qk}}]. \tag{7}$$

Beginning from adopted distribution principle (Eq. (7)), network losses belonging to *j*-th load/node are obtained as a sum of the losses in network branches on the way from considered node to source one (Fig. 1)

$$\Delta P_j = \sum_{i=1}^{n} \Delta P_{ij} \,, \tag{9}$$

where n shows number of branches.

III. NUMERICAL EXAMPLE AND CALCULATION ANALYSIS

Test network of rated voltage U_n=12.66kV with 32 nodes, shown in Fig. 2 is considered [7]. Loads and network elements are balanced. Loads are presented according their static characteristics:

$$P_i = P_{ni}\left(\frac{U_i}{U_n}\right)^{k_{pi}}$$

$$Q_i = Q_{ni}\left(\frac{U_i}{U_n}\right)^{k_{qi}} \tag{10}$$

Different load types can be simulated by choosing the values of coefficients k_{pi} and k_{qi}. Load of constant power *(*$k_{pi} = k_{qi}=0$*)*, constant current *(*$k_{pi} = k_{qi}=1$*)* and constant impedance *(*$k_{pi} = k_{qi}=2$ are considered here.

Voltage and load flow calculation is done by power summation method [8]. After iterative procedure finishing, load and current flows are obtained and current components in network branches so. On the basis of these currents, power losses in each branch and total network losses are calculated.

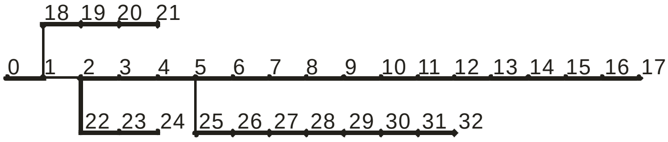

Fig. 2. Test network

Active power losses in the network of given configuration depend on individual loads, their locations, type of load and voltage of source node [6]. As the theme of this work is load type influence on loss allocation, furthermore the losses will be considered only when source node voltage is U_0=12.66kV.

For source node voltage $U_0 = 12.66\,\text{kV}$ and basic loads, total active power losses in considered network are: $\Delta P = 202{,}677\,\text{kW}$, if all loads are of constant power type, $\Delta P = 176{,}628\,\text{kW}$, if all loads are of constant current type and $\Delta P = 156{,}872\,\text{kW}$, if all loads are of constant impedance type. Calculation results of loss allocation according exposed method, for linear principle of common loss distribution (Eq. 7) in some branch, are shown in Table 1. Percentage shares of active power of individual consumers in total network load (columns 4, 7 and 10) are given in the same table. Graphical presentation of the results from table 1 is given in two figures due to clearness. Fig. 3 shows absolute values of allocated losses in the nodes. Fig. 4 presents percentage values of allocated losses in relation to total losses.

TABLE I

LOSS ALLOCATION FOR DIFERENT LOAD TYPES

Node	P	Q	Constant power			Constant current			Constant impedance		
	[kW]	[kVAr]	$\frac{P_i}{\sum P_i}\cdot 100$ [%]	ΔP_i [kW]	$\frac{\Delta P_i}{\Delta P}\cdot 100$ [%]	$\frac{P_i}{\sum P_i}\cdot 100$ [%]	ΔP_i [kW]	$\frac{\Delta P_i}{\Delta P}\cdot 100$ [%]	$\frac{P_i}{\sum P_i}\cdot 100$ [%]	ΔP_i [kW]	$\frac{\Delta P_i}{\Delta P}\cdot 100$ [%]
1	100	60	2.692	0.244	0.120	2.814	0.236	0.134	2.925	0.230	0.147
2	90	40	2.423	1.066	0.526	2.499	1.010	0.572	2.566	0.962	0.613
3	120	80	3.230	2.854	1.408	3.309	2.659	1.505	3.377	2.497	1.592
4	60	30	1.615	1.177	0.581	1.643	1.089	0.616	1.666	1.016	0.648
5	60	20	1.615	1.661	0.819	1.614	1.487	0.842	1.612	1.350	0.861
6	200	100	5.384	11.604	5.725	5.361	10.292	5.827	5.339	9.268	5.908
7	200	100	5.384	13.176	6.501	5.336	11.594	6.564	5.292	10.372	6.611
8	60	20	1.615	2.332	1.151	1.591	2.034	1.152	1.570	1.806	1.151
9	60	20	1.615	2.643	1.304	1.582	2.281	1.292	1.553	2.007	1.279
10	45	30	1.211	2.077	1.025	1.185	1.790	1.014	1.163	1.573	1.003
11	60	35	1.615	3.229	1.593	1.578	2.775	1.571	1.546	2.433	1.551
12	60	35	1.615	3.589	1.771	1.569	3.051	1.727	1.529	2.650	1.689
13	120	80	3.230	10.955	5.405	3.130	9.289	5.259	3.046	8.053	5.133
14	60	10	1.615	3.035	1.497	1.563	2.562	1.451	1.519	2.213	1.411
15	60	20	1.615	3.361	1.658	1.561	2.831	1.603	1.515	2.440	1.555
16	60	20	1.615	3.458	1.706	1.558	2.902	1.643	1.510	2.494	1.590
17	90	40	2.423	6.914	3.411	2.335	5.804	3.286	2.262	4.988	3.180
18	90	40	2.423	0.229	0.113	2.532	0.223	0.126	2.630	0.218	0.139
19	90	40	2.423	0.507	0.250	2.523	0.496	0.281	2.611	0.486	0.310
20	90	40	2.423	0.558	0.275	2.521	0.545	0.309	2.608	0.534	0.340
21	90	40	2.423	0.602	0.297	2.519	0.588	0.333	2.604	0.576	0.367
22	90	50	2.423	1.281	0.632	2.490	1.208	0.684	2.549	1.147	0.731
23	420	200	11.306	13.842	6.830	11.545	12.876	7.290	11.742	12.070	7.694
24	420	200	11.306	15.193	7.496	11.506	14.086	7.975	11.666	13.161	8.390
25	60	25	1.615	1.809	0.892	1.611	1.614	0.914	1.606	1.461	0.932
26	60	25	1.615	1.892	0.934	1.607	1.681	0.952	1.599	1.517	0.967
27	60	20	1.615	2.081	1.027	1.589	1.812	1.026	1.566	1.607	1.024
28	120	70	3.230	7.828	3.862	3.152	6.722	3.806	3.085	5.890	3.754
29	200	600	5.384	50.447	24.890	5.235	43.117	24.411	5.109	37.634	23.990
30	150	70	4.038	11.193	5.522	3.910	9.491	5.374	3.803	8.228	5.245
31	210	100	5.653	18.347	9.052	5.470	15.547	8.802	5.315	13.471	8.587
32	60	40	1.615	3.160	1.559	1.562	2.668	1.511	1.518	2.305	1.469
	3715	2300	100.00%	202.677	100.00%	100.00%	176.362	100.00%	100.00%	156.655	100.00%

Exposed allocation method is taking into account location and power factor by summing belonging parts of losses in all network branches through which some consumer is supplied. It is the best to see comparing of calculation results of loss allocation concerning the nodes 5, 8, 9, and 16 and the nodes 17, 18, 19, 20 and 21 in which loads of the same powers exist. So, for example, loads in the nodes 17 and 18 are of the same powers, but due to unequal distances from source node significantly bigger losses are allocated to the consumer in node 17. In dependence of load type the losses in node 17 are 3.4112%, 3.2858% and 3.1799% for load of constant power type, constant current type and constant impedance type, respectively. For the same power and load type, the losses of 0.1127%, 0.1260% and 0.1386% are allocated to node 18.

Loss allocation method cares about power factor and reactive load flows. It is important because of the fact that reactive powers impact on amount of active power losses in great extent. So, the consumer in node 31 has greater active power and it is situated on bigger distance, but it shares in the losses less than consumer in node 29, which have distinctly bad power factor.

This example shows there is no proportion between the shares of active powers of individual consumptions in total power toward the shares of individual consumptions in network losses. In concrete example, the biggest losses in accounts of 24.8902%, 22.853% or 21.0217% depending on load type, are allocated to node 29, although it shares with 5.383%, 5.236% and 5.109% in network active power. The least losses are allocated to node 18.

The results from Table 1 show that load type impact in great extent as on total losses value as on allocated losses in the nodes (Fig. 3). However, comparing allocated losses in the nodes, for different load types, expressed in percentage of total losses, shows then the difference is much less (Fig. 4). With load type changes it comes to insignificant reallocation of percentage losses. The consumers located close to source node accept proportionally bigger allocated losses if they are of constant impedance than if they are of constant power. For distant ones it is opposite, what can be seen on Fig. 4.

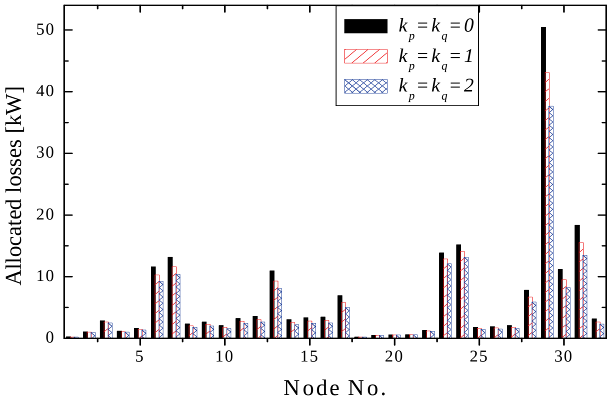

Fig. 3. Allocated losses in the nodes for different load types

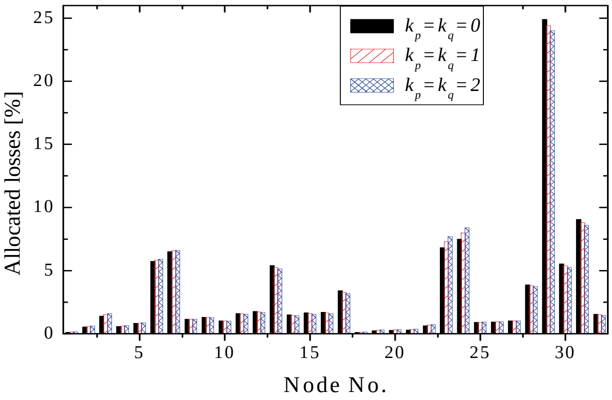

Fig. 4. Percentage values of allocated losses for different load types

TABLE II

ALLOCATED LOSSES TO NODE 29 FOR INDIVIDUAL LOAD TYPES IN DISTRIBUTION NETWORK

Load type	Allocated losses to node 29					
	$k_p = k_q = 0$		$k_p = k_q = 1$		$k_p = k_q = 2$	
	[kW]	[%]	[kW]	[%]	[kW]	[%]
Constant ppower	50.44	24.89	44.91	22.85	40.20	21.02
Constant current	48.11	26.45	43.12	24.41	38.83	22.56
Constant imped.	46.16	27.88	41.58	25.84	37.63	23.99

In some distribution network all loads are not of the same type. As a rule, they are different with unequal self-regulation coefficients for active and reactive power. With load type changing the total losses are changing also and certainly allocated losses in the nodes. From many possible combinations, three hypothetic cases are considered here. Load type is changed only in one node and all others are of the same type: a) constant power, b) constant current, c) constant impedance. A node number 29 is chosen for load type variations as the node with greatest allocated losses. Calculation results are shown in Table 2. In nine analyzed combinations, the losses allocated to node 29 are varying in the range from 37,643kW to 50,477kW in dependence on load type or in percentage from 21,022% to 27,884%. This example shows it is necessary to regard load type in every node to make fair loss allocation.

Due to the fact that load level in the nodes varies during the day, allocated losses of active power will be varying also. Therefore it is necessary to repeat the procedure of power loss allocation for every hour or another time interval. On that way allocated losses per nodes can be obtained. By implementation of complete procedure of loss allocation for longer period, for example a season or a year, loss factor could be established for distribution network and for the part of network supplied from some node.

IV. CONCLUSION

This work gives presentation and comparative analysis of calculation results of loss allocation in distribution networks for different load types. It is shown that load type in great extent impact on the level of allocated losses. Calculation results could be used as the basis for criterion choice for power and energy account and for forming one more impartial tariff system.

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