Structuring the Nominal Mathematical Model of the Electric Power System for the Aims of Robust Analysis

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Abstract – **A structured representation of the mathematical description of the electric power system (EPS) is proposed. It allows flexible introduction of uncertainties in the separate constituent elements of EPS. An algorithm is discussed for reduction of the order of the analyzed model by frequency equivalentation in respect to a certain generator bus.**

Keywords – **structuring the nominal mathematical model, robust analysis.**

I. Introduction

The most widely used tool for damping electro-mechanical oscillations in the modern electric power systems (EPS) is the device power system stabilizer (PSS). The PSS task is to create additional damping momentum which is in phase with the rotor mechanical speed deviation. Only then the device can successfully damp the mechanical oscillations. A common practice is to calculate the PSS settings from a determinate mathematical model of EPS, linearized around a certain operating point [1-3]. It is obvious that change of the EPS operating point (i.e. the regime) can result in inappropriate PSS settings. The authors of this article have the purpose to create an algorithm for calculation of robust PSS settings. This means that the EPS mathematical model has to include uncertainties which represent the constantly changing operating point and the inaccuracies in the EPS scheme parameters. In order to be able to model them easily, the paper proposes an appropriate structuring of the EPS mathematical description of the mechanical oscillations under small disturbances.

II. Mathematical Model

The structural scheme from Fig. 1 represents the proposed EPS electro-mechanical oscillations mathematical model which will be used for calculation of robust PSS settings of a certain generator.

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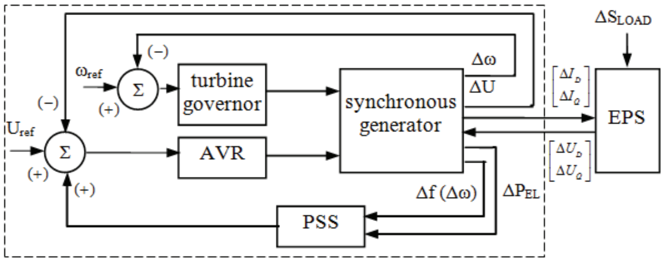

Fig. 1. Structure of the proposed EPS mathematical model

The generator linearized mathematical description, written in per unit system for nominal base conditions in *dq*-coordinates is:

$$\left|\begin{aligned}
&\Delta U_d = \Delta E''_d - x''_q \cdot \Delta I_q - r \cdot \Delta I_d;\\
&\Delta U_q = \Delta E''_q + x''_d \cdot \Delta I_d - r \cdot \Delta I_q;\\
&\frac{d\Delta E''_d}{dt} = \frac{-1}{T''_{q0}} \cdot \Delta E''_d + \frac{-(x_q - x''_q)}{T''_{q0}} \cdot \Delta I_q;\\
&\frac{d\Delta E''_q}{dt} = \frac{k_1}{T''_{d0}} \cdot \Delta E_{fD} + \frac{k_3 - k_1}{T''_{d0}} \cdot \Delta E'_q + \frac{-k_3}{T''_{d0}} \cdot \Delta E''_q +\\
&\qquad \frac{(x_d - x'_d)\cdot k_1 + (x'_d - x''_d)\cdot k_3}{T''_{d0}} \cdot \Delta I_d;\\
&\frac{d\Delta E'_q}{dt} = \frac{1}{T_{d0}} \cdot \Delta E_{fD} + \frac{-(1+k_2)}{T_{d0}} \cdot \Delta E'_q +\\
&\qquad \frac{x_d - x'_d - (x'_d - x''_d)\cdot k_2}{T_{d0}} \cdot \Delta I_d + \frac{k_2}{T_{d0}} \cdot \Delta E''_q;\\
&\frac{d\Delta\omega}{dt} = \frac{-P_0}{T_J} \cdot \Delta\omega + \frac{1}{T_J} \cdot \Delta P_{мех} - \frac{\left(E''_{q0} - I_{d0} \cdot \left(x''_q - x''_d\right)\right)}{T_J} \cdot \Delta I_q -\\
&\qquad \frac{\left(E''_{d0} - I_{q0} \cdot \left(x''_q - x''_d\right)\right)}{T_J} \cdot \Delta I_d - \frac{I_{d0}}{T_J} \cdot \Delta E''_d - \frac{I_{q0}}{T_J} \cdot \Delta E''_q;\\
&\frac{d\Delta\theta}{dt} = 2\pi f_{nom} \cdot \Delta\omega + \left(-2\pi f_{nom}\right) \cdot \Delta\omega_R
\end{aligned}\right. \tag{1}$$

where

$$\begin{aligned}
&k_1 = \frac{T''_{d0}}{T_{d0}} \cdot \frac{x_{ad} - (x_d - x''_d)}{x_{ad} - (x_d - x'_d)};\\
&k_2 = \frac{x_d - x'_d}{x_{ad} - x_d + x'_d};\\
&k_3 = 1 - k_1 \cdot k_2;
\end{aligned} \tag{2}$$

U_d, U_q, I_d, I_q are the stator winding voltage and current, respectively for *d* and *q* axis, in p.u.;

E''_d, E''_q – subtransient voltages, respectively for d and q axis, in p.u.;

E_{fD} – the rotor voltage, proportional to the excitation voltage in p.u.;

x_{ad}, x_d, x'_d, x''_d – mutual, synchronous, transient and subtransient generator reactances in d-axis, in. p.u.;

x_q, x''_q – synchronous and subtransient reactances in q-axis, in p.u.;

r – stator winding active resistance, in p.u.;

T_{d0}, T''_{d0} – the catalogue time constants in d-axis for open stator winding, in s;

T''_{q0} – the catalogue time constant in q-axis for open stator circuit, in s;

T_J – mechanical time constant of the synchronous generator rotor, in s;

t – time, in s;

ω – electrical angular velocity of the generator's rotor, in p.u.;

ω_R – electrical angular velocity of the EPS referent coordinate system, in p.u.;

θ – the mutual angle between generator's q-axis and Q-axis of the referent EPS coordinate system, in rad;

P_{mech} – turbine mechanical power, in p.u.

The mathematical descriptions of PSS, AVR and the turbine governor most often are represented by structural schemes of directional blocks. For example, Fig. 2 shows the structural scheme of the widely used in the EPS of Bulgaria PSS-2A.

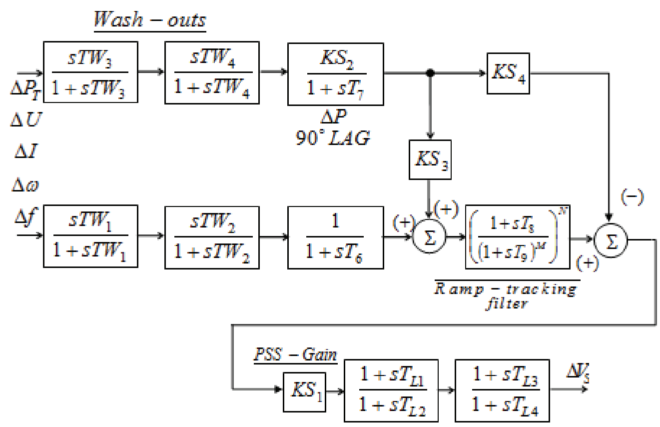

Fig. 2. PSS-2A block diagram

The synchronous generator model, whose PSS is being tuned, is represented by the mathematical descriptions of its constituent elements: synchronous generator, turbine and turbine governor, excitation system and automatic voltage regulator (AVR), power system stabilizer (PSS). The connection between the model of the synchronous unit and the model of the EPS is made by the variations of the current and voltage, defined in Cartesian coordinate system (DQ) which rotates with the referent generator speed ω_R.

These are transformed into transfer matrix connecting the input and output quantities, as shown on Fig. 1. In same details is described and each other generator in the EPS. Their descriptions are joined by the network description into a single transfer matrix connecting the variations of current in the generator with the PSS to be tuned (ΔI_D, ΔI_Q) with variations of the voltage in the same generator (ΔU_D, ΔU_Q) and the referent generator speed ($\Delta\omega_R$).

The realizations of these transfer matrixes in the state space have too big dimensions. Due to the big number of generators in the detailed model of a real EPS these dimensions can reach order of a few thousand. This is a significant problem for the optimization procedure for calculation of optimal PSS settings. To overcome this difficulty it is necessary to reduce the order of the system in respect to the bus of the generator whose PSS is going to be tuned.

III. MODEL ORDER REDUCTION

For this purpose the calculated frequency responses of the whole transfer matrix

$$G_{EPS}(j\omega) = \begin{bmatrix} \dfrac{\Delta\omega_R}{\Delta I_D} & \dfrac{\Delta\omega_R}{\Delta I_Q} \\ \dfrac{\Delta U_D}{\Delta I_D} & \dfrac{\Delta U_D}{\Delta I_Q} \\ \dfrac{\Delta U_Q}{\Delta I_D} & \dfrac{\Delta U_Q}{\Delta I_Q} \end{bmatrix} \tag{3}$$

for the frequency range significant for the EPS electromechanical oscillations ($\omega = 0 \div 30$ rad/s) are approximated and this way a reduction of the order of the mathematical description is achieved without loss of the real EPS influence over the PSS which is going to be tuned.

The approximation gives the following transfer matrix (in Laplace space):

$$\mathbf{G}(s) = \begin{bmatrix} Y_{RD}(s) & Y_{RQ}(s) \\ Y_{DD}(s) & Y_{DQ}(s) \\ Y_{QD}(s) & Y_{QQ}(s) \end{bmatrix} \tag{4}$$

The elements of the transfer matrix $\mathbf{G}(s)$ are transformed into state space and have the following realizations

$$Y_i(s) = \left[\begin{array}{c|c} \mathbf{A}_i & \mathbf{B}_i \\ \hline \mathbf{C}_i & \mathbf{D}_i \end{array}\right] \tag{5}$$

where $i = RD, RQ, DD, DQ, QD, QQ$. Then the realization in state space of the whole transfer matrix $\mathbf{G}(s)$ is [4]:

$$\mathbf{G}(s)=\left[\begin{array}{cccccc|cc}
\mathbf{A}_{RD} & 0 & 0 & 0 & 0 & 0 & \mathbf{B}_{RD} & 0 \\
0 & \mathbf{A}_{RQ} & 0 & 0 & 0 & 0 & 0 & \mathbf{B}_{DQ} \\
0 & 0 & \mathbf{A}_{DD} & 0 & 0 & 0 & \mathbf{B}_{DD} & 0 \\
0 & 0 & 0 & \mathbf{A}_{DQ} & 0 & 0 & 0 & \mathbf{B}_{DQ} \\
0 & 0 & 0 & 0 & \mathbf{A}_{QD} & 0 & \mathbf{B}_{QD} & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{A}_{QQ} & 0 & \mathbf{B}_{QQ} \\
\hline
\mathbf{C}_{RD} & \mathbf{C}_{DQ} & 0 & 0 & 0 & 0 & \mathbf{D}_{RD} & \mathbf{D}_{DQ} \\
0 & 0 & \mathbf{C}_{DD} & \mathbf{C}_{DQ} & 0 & 0 & \mathbf{D}_{DD} & \mathbf{D}_{DQ} \\
0 & 0 & 0 & 0 & \mathbf{C}_{QD} & \mathbf{C}_{QQ} & \mathbf{D}_{QD} & \mathbf{D}_{QQ}
\end{array}\right] \quad (6)$$

It is worth noticing that the obtained state space realization $\mathbf{G}(s)$ is not always the minimal realization. To get the minimal realization, decomposition in respect to controllability and observability has to be made, which will eliminate the uncontrollable and/or unobservable states. This can be done with the MATLAB function `minreal`. Then additional model order reduction can be done by means of the the hankel norm.

It is well known [5] that Hankel norm of a system with transfer matrix **G** is equal to:

$$\|\mathbf{G}\|_H=\sqrt{\rho\cdot(\mathbf{P}\cdot\mathbf{Q})} \quad (7)$$

where ρ is the spectral radius of the product of **P**.**Q**; **P** – controllability Gramian of **G**; **Q** – observability Gramian of **G**. Because ρ is the maximal eigenvalue of the **P**.**Q**, they are often called Hankel singular values, i.e.

$$\sigma_{Hi}=\sqrt{\lambda_i(\mathbf{P}\cdot\mathbf{Q})} \quad (8)$$

From all this follows that the Hankel norm of a **G** is equal to the maximal Hankel singular value, i.e.

$$\|\mathbf{G}\|_H=\max\sigma_{\mathrm{H},i}=\sigma_{\mathrm{H},1} \quad (9)$$

The ratio of the Hankel singular values to the Hankel norm

$$O_{\mathrm{H},i}=\frac{\sigma_{\mathrm{H},i}}{\|\mathbf{G}\|_H}=\frac{\sigma_{\mathrm{H},i}}{\sigma_{\mathrm{H},1}},\ i=\overline{2,n} \quad (10)$$

is indicator for the possibilities for **G** order reduction.

IV. Results

In the study, for test model was used the detailed mathematical model of real EPS consisted of: 787 generators, 124 PSS, 5749 power lines, 1495 transformers, 5176 buses and 3255 loads and shunts.

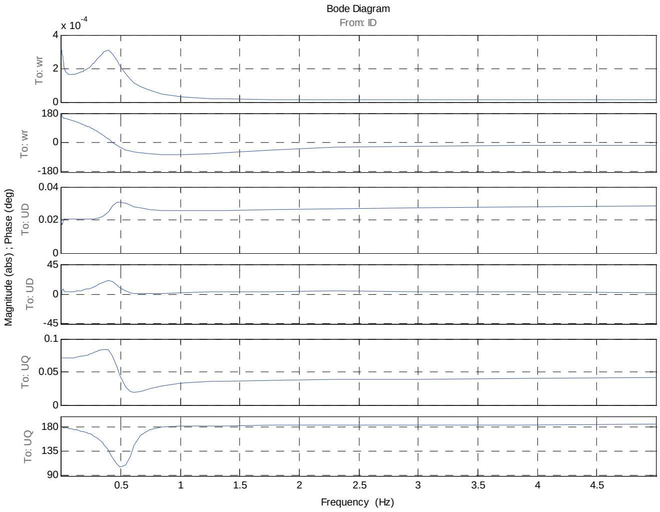

Fig. 3. Frequency response of the analyzed EPS in respect to the bus of a certain 300MW generator in the system for input I_D

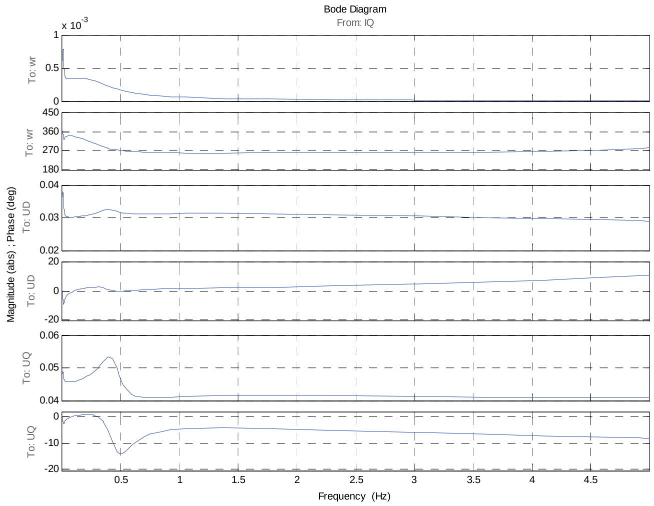

Fig. 4. Frequency response of the analyzed EPS in respect to the bus of a certain 300MW generator in the system for input I_Q

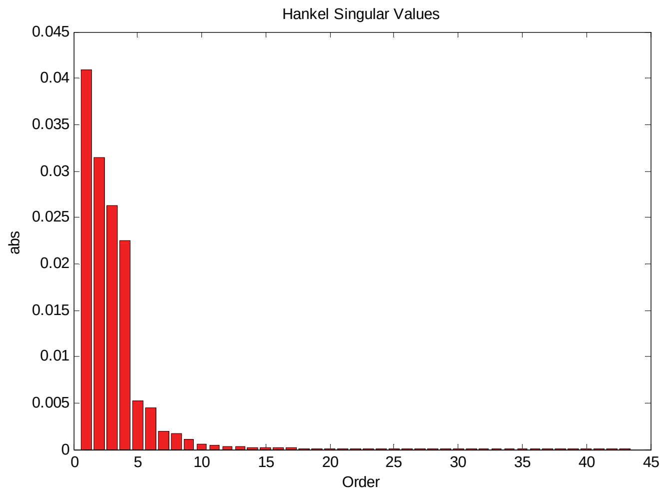

Fig. 5. Hankel singular values of the approximated EPS

Figs. 3 and 4 shows the calculated frequency responses of the EPS model in respect to the bus of a certain 300MW generator in the system.

The full EPS model matrix in state space is of order 7139x7139. After the approximated the system is reduced to order 43x43. The Hankel singular values of the approximated model are shown in Fig. 5. They are calculated with the MATLAB function hankelmr.

It is clearly seen that the order of the obtained equivalent system description can further be reduced to 17x17. A confirmation of this is seen from the closeness of the step responses of the two systems (Figs. 6 and 7).

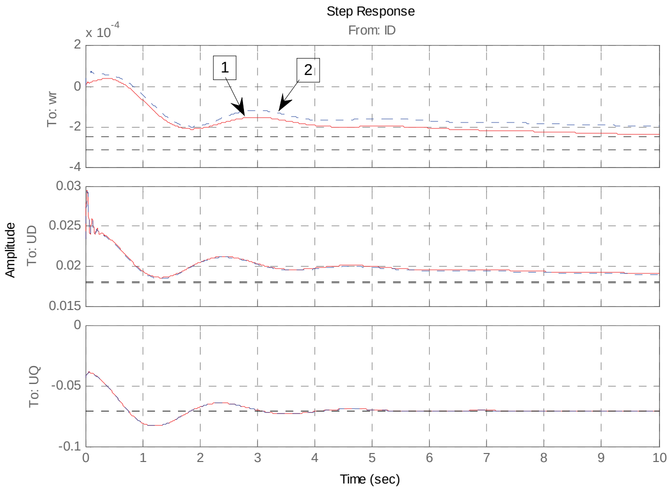

Fig. 6. Comparison of step responses for input I_D of:
1 – approximated to 43x43 EPS model;
2 – reduced to 17x17 approximated EPS model

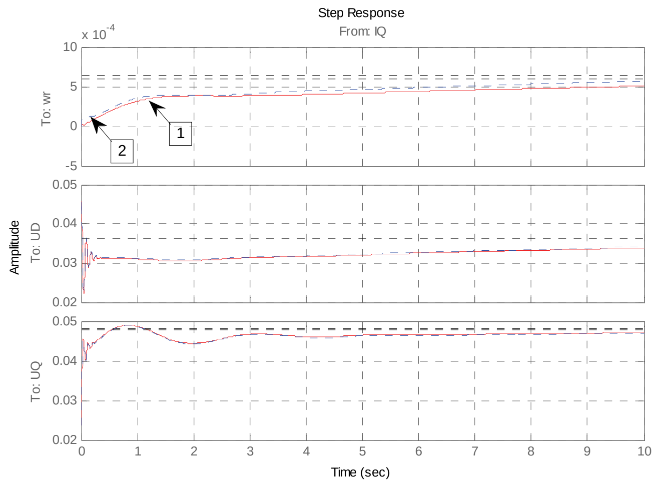

Fig. 7. Comparison of step responses for input I_Q of:
1 – approximated to 43x43 EPS model;
2 – reduced to 17x17 approximated EPS model

V. Conclusion

The proposed structuring of the mathematical model of EPS and the method for reduction of the order of the system allow easy introduction of variations of the EPS constituent elements due to the variations of the operating point parameters and inaccuracies in the knowledge about the scheme parameters. As a result a family of transfer matrixes of the corresponding elements can be calculated and thus easily appropriate weight functions for unstructured representation of the uncertainties in the EPS elements can be constructed.

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