Calculation of the Attraction Force between Permanent Magnet and Infinite Linear Magnetic Plane Using Ampere's Currents

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Abstract – **The paper presents calculation of the attraction force between ring permanent magnet of trapezoidal cross section and infinite linear magnetic plane using distribution of Ampere's currents and discretization. The results obtained using the analytical method and discretization are compared with ones calculated numerically using FEMM software.**

Keywords – **Permanent magnet, Attraction force, Ampere's current.**

I. Introduction

Permanent magnets of various shapes are often utilized in magnetic actuators, sensors or releasable magnetic fasteners. Knowledge of the magnetic force, either levitation or attraction, is required to control devices reliably. The attraction force between ring permanent magnet and infinite linear magnetic plane of permeability μ_r, presented in Fig. 1, is derived using the analytical method based on surface Ampere's current distribution and discretization. Attraction force depends on the magnetic properties of permanent magnet (permanent magnetization *M*), permeability μ, on the distance between magnet and plane *h*, and on the geometrical design of the magnet [5].

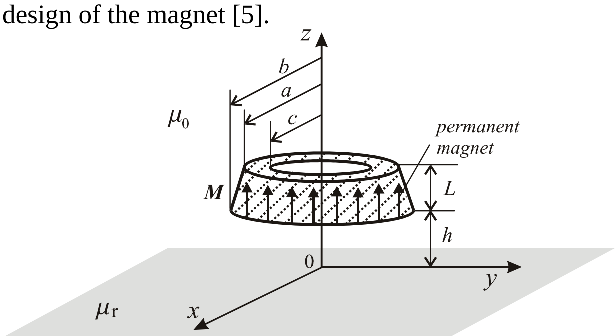

Fig. 1. Permanent magnet and infinite linear magnetic plane.

There are numerous techniques for analyzing permanent magnet devices and different approaches for determining attraction or levitation forces between magnets [1]-[4]. Many authors proposed simplified and robust formulations of the magnetic field components created by ring permanent magnets. Moreover, the evaluation of the magnetic field created by ring magnets is the step that can help calculating the force. Indeed, the force is the value of importance for the design and optimization of a bearing. In [2] permanent magnets are modeled as distributions of equivalent magnetic charge and the levitation forces are determined by computing the force between the two charge distributions. Attraction force between permanent magnets in [1] is established by magnetostatic interaction. In this paper the attraction force is derived using the analytical method based on surface Ampere's current distribution and discretization.

II. Procedure for Attraction Force Calculation

For solving this problem the image theorem in the plane mirror is used. The treated system can be replaced with system presented in the Fig 2. where

$$\alpha = \frac{\mu_r - 1}{\mu_r + 1}. \tag{1}$$

It is assumed that the magnetization is uniform throughout the magnet and it is

$$\boldsymbol{M}_1 = \boldsymbol{M} = M\hat{z}, \tag{2}$$

and for the image $\boldsymbol{M}_2 = \alpha\boldsymbol{M} = \alpha M\hat{z}$. (3)

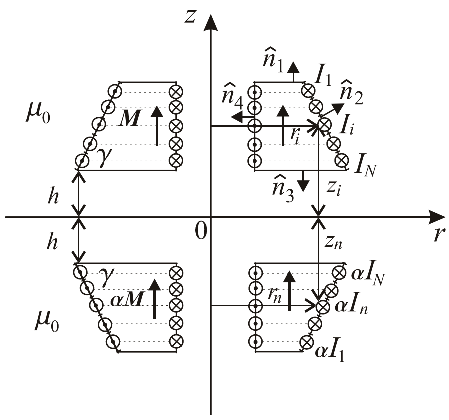

Fig. 2. Discretizing model.

Due to uniform axial magnetization of the magnet and image, the volume density of the Ampere's currents is

$$\boldsymbol{J}_a = \text{curl}\,\boldsymbol{M} = 0. \tag{4}$$

Only the surface Ampere's current, with density

$$\boldsymbol{J}_{sa} = \boldsymbol{M} \times \hat{n}, \tag{5}$$

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(where $\hat{n}$ is unite vector of outgoing normal), exists on the inner and outer torus covers, in angular direction and the permanent magnet and its image behave as a thin, single layer, uniformly winded solenoid coils.

The goal of this approach is to determine analytically the magnetic flux density generated by the image in any point and then to calculate the force on the magnet. In order to determine the magnetic flux density produced by the lower permanent magnet, the circular current loop C is considered (Fig.3).

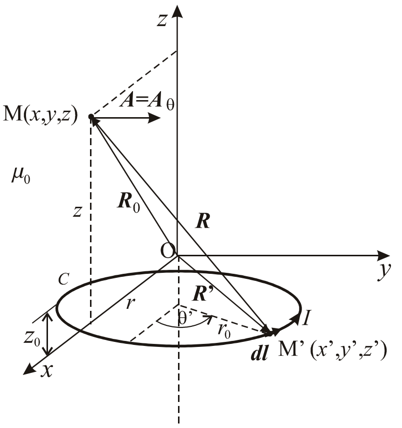

Fig. 3. The circular current loop.

In [3] the final form of the magnetic flux density components generated by circular current loop are obtained as:

$$B_r = \frac{\mu I}{2\pi} \frac{z - z_0}{r\sqrt{(r + r_0)^2 + (z - z_0)^2}} \left(-K\left(\frac{\pi}{2}, k\right) + \frac{r^2 + r_0{}^2 + (z - z_0)^2}{(r - r_0)^2 + (z - z_0)^2} E\left(\frac{\pi}{2}, k\right) \right), \tag{6}$$

$$B_z = \frac{\mu I}{2\pi} \frac{1}{\sqrt{(r + r_0)^2 + (z - z_0)^2}} \left(K\left(\frac{\pi}{2}, k\right) + \frac{r_0{}^2 - r^2 - (z - z_0)^2}{(r - r_0)^2 + (z - z_0)^2} E\left(\frac{\pi}{2}, k\right) \right). \tag{7}$$

Using the results that are obtained for the circular loop the magnetic flux density generated by the image could be determined in any point. The idea is to discretize each magnet cover into system of segments (circular loops) where N is the number of segments. Considering Fig. 2 it is obvious that the following formulas are satisfied:

$$\boldsymbol{M} \times \hat{n}_1 = 0 \,,\; \boldsymbol{M} \times \hat{n}_2 = \boldsymbol{J}_{\mathrm{sa}} \,,\; \boldsymbol{M} \times \hat{n}_3 = 0 \,,\; \boldsymbol{M} \times \hat{n}_4 = \boldsymbol{J}_{\mathrm{sa}} \,. \tag{8}$$

Since only the surface Ampere's currents, with density $J_{\mathrm{sa}} = M$, exists on the inner and outer magnet covers, the total magnetic flux density produced by the lower magnet can be determined by summing the contribution of both magnet covers.

By taking into account the magnet geometry, from Fig. 2, the following parameters can be defined

$$z_n = h + \frac{2n - 1}{2N} L \,; \tag{9}$$

$$r_n = b + \frac{a - b}{L}(z_n - h) \,; \tag{10}$$

$$\sin(\gamma) = \frac{L}{\sqrt{(b - a)^2 + L^2}} \,; \tag{11}$$

$$\Delta l = \frac{\sqrt{(b - a)^2 + L^2}}{N} \,; \tag{12}$$

and the current of each circular loop of radius r_n is

$$I_n = \alpha M \Delta l \sin(\gamma), \qquad n = 1, \ldots, N \,. \tag{13}$$

The magnetic force that acts on the one segment of upper magnet, that is circular loop of radius r_i or radius c, due to the magnetic field generated by lower magnet, can be expressed as

$$\mathrm{d}\boldsymbol{F} = I_i \,\mathrm{d}\boldsymbol{l} \times \boldsymbol{B} \,, \tag{14}$$

where current of the circular loop is

$$I_i = M \Delta l \sin(\gamma) \tag{15}$$

and

$$r_i = r_n = b + \frac{a - b}{L}(z_n - h) \,. \tag{16}$$

Since $\mathrm{d}\boldsymbol{l} = -r_i \,\mathrm{d}\theta\, \hat{\theta}$, using following relations

$$\hat{\theta} = -\sin(\theta)\,\hat{x} + \cos(\theta)\,\hat{y} \,,$$
$$\hat{r} = \cos(\theta)\,\hat{x} + \sin(\theta)\,\hat{y} \text{ and}$$
$$\hat{z} = \hat{z} \,, \tag{17}$$

from Eq. (14) the magnetic force is finally obtained as

$$\mathrm{d}\boldsymbol{F} = I_i\, r_i\, B_z \cos(\theta)\,\mathrm{d}\theta\,(-\hat{x}) + I_i\, r_i\, B_z \sin(\theta)\,\mathrm{d}\theta\,(-\hat{y}) + I_i\, r_i\, B_r\, \mathrm{d}\theta\, \hat{z}. \tag{18}$$

The z component of this force represents the attraction force that acts on the one magnet's segment

$$\mathrm{d}F_z = I_i r_i B_r \,\mathrm{d}\theta \,. \tag{19}$$

The x and y components of the force are equal to zero. Therefore, the attraction force of the one magnet's segment, can be obtained using Eqs. (19) and (15) as

$$F_{z\,i} = 2\pi r_i B_r M \Delta l \sin(\gamma) \,. \tag{20}$$

Contributions of inner and outer cover of both magnets must be included:

$$F_{z1} = \frac{\mu_0 \alpha M^2 L^2}{N^2} \sum_{i=1}^{N} \sum_{n=1}^{N} \frac{z_i + z_n}{\sqrt{(r_i + r_n)^2 + (z_i + z_n)^2}} \times \left(-K\left(\frac{\pi}{2}, k_1\right) + \frac{r_i{}^2 + r_n{}^2 + (z_i + z_n)^2}{(r_i - r_n)^2 + (z_i + z_n)^2} E\left(\frac{\pi}{2}, k_1\right) \right), \tag{21}$$

$$k_1^2 = \frac{4 r_i r_n}{(r_i + r_n)^2 + (z_i + z_n)^2} \,, \tag{22}$$

$$F_{z2} = \frac{\mu_0 \alpha M^2 L^2}{N^2} \sum_{i=1}^{N} \sum_{n=1}^{N} \frac{z_i + z_n}{\sqrt{(r_i + c)^2 + (z_i + z_n)^2}} \times \left(-K\left(\frac{\pi}{2}, k_2\right) + \frac{r_i^2 + c^2 + (z_i + z_n)^2}{(r_i - c)^2 + (z_i + z_n)^2} E\left(\frac{\pi}{2}, k_2\right) \right), \tag{23}$$

$$k_2^2 = \frac{4 r_i c}{(r_i + c)^2 + (z_i + z_n)^2}, \tag{24}$$

$$F_{z3} = \frac{\mu_0 \alpha M^2 L^2}{N^2} \sum_{i=1}^{N} \sum_{n=1}^{N} \frac{z_i + z_n}{\sqrt{(c + r_n)^2 + (z_i + z_n)^2}} \times \left(-K\left(\frac{\pi}{2}, k_3\right) + \frac{c^2 + r_n^2 + (z_i + z_n)^2}{(c - r_n)^2 + (z_i + z_n)^2} E\left(\frac{\pi}{2}, k_3\right) \right), \tag{25}$$

$$k_3^2 = \frac{4 c r_n}{(c + r_n)^2 + (z_i + z_n)^2}, \tag{26}$$

$$F_{z4} = \frac{\mu_0 \alpha M^2 L^2}{N^2} \sum_{i=1}^{N} \sum_{n=1}^{N} \frac{z_i + z_n}{\sqrt{4c^2 + (z_i + z_n)^2}} \times \left(-K\left(\frac{\pi}{2}, k_4\right) + \frac{2c^2 + (z_i + z_n)^2}{(z_i + z_n)^2} E\left(\frac{\pi}{2}, k_4\right) \right), \tag{27}$$

$$k_4^2 = \frac{4c^2}{4c^2 + (z_i + z_n)^2}. \tag{28}$$

Finally the total attraction force between ring permanent magnet and plane can be calculated easily and it is valid for any distance between magnet and plane

$$F_z = F_{z1} - F_{z2} - F_{z3} + F_{z4}. \tag{29}$$

III. Numerical Results

The results of the presented approach are given in the graphical form and they are compared with ones calculated numerically using FEMM software.

Table I presents the attraction force calculated when dimensions of the magnet are: $a = 3$mm, $b = 4$mm, $c = 2$mm, $L = 1$mm, magnetization $M = 900$ kA/m and relative permeability of the plane $\mu_r = 3$ when the number of segments of each magnet's cover is $N = 100$. Results of FEMM 4.2 software are obtained for 1.7 million elements. Difference between presented approach and FEMM is greater as the distance is smaller. That difference lowers when the separation distance, *h*, increases. Fig. 4 shows normalized attraction force dependency versus outer radius of the circular bottom of the magnet for various separation distance, when $\mu_r = 3$.

TABLE I
ATTRACTION FORCE FOR DIFFERENT SEPARATION DISTANCE.

h [mm]	F_z^{femm} [N]	F_z [N]
0.5	-0.550230	-0.549755
0.6	-0.438152	-0.437758
0.7	-0.353875	-0.353520
0.8	-0.289550	-0.289242
0.9	-0.239870	-0.239567
1.0	-0.201039	-0.200721
1.1	-0.170331	-0.170000
1.2	-0.145767	-0.145441
1.3	-0.125960	-0.125599
1.4	-0.109783	-0.109402
1.5	-0.096491	-0.096049

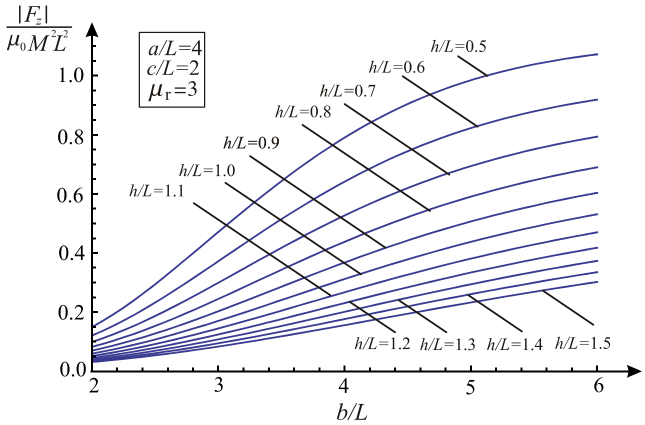

Fig. 4. Attraction force versus outer radius of a circular bottom of the magnet.

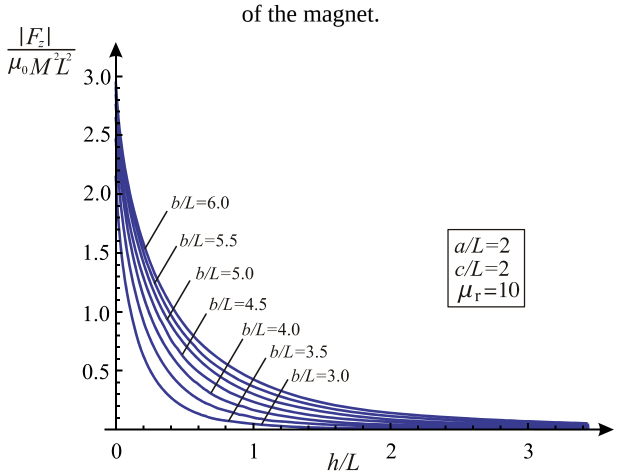

Fig.5. Attraction force versus separation distance.

In Fig.5 normalized attraction force versus separation distance is presented when relative permeability of plane is $\mu_r = 10$. Figs. 6 and 7 present distribution of magnetic flux density and distribution of magnetic field, respectively. They are obtained using semi-analytical method when the radii of circular top and circular bottom are equal $a = b = 3$mm, $c = 1$mm, $L = 1$mm and separation distance $h = 1$mm.

When the inner radius of magnet is equal to zero, $c = 0$, the system is composed of truncated cone shaped permanent magnet and infinity plane. Fig.9 presents attraction force versus separation distance when $\mu_r = 50$ while Fig.8 shows attraction force dependency versus relative permeability.

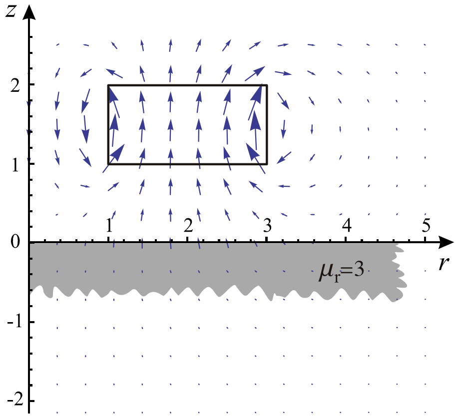

Fig. 6. Distribution of magnetic flux density.

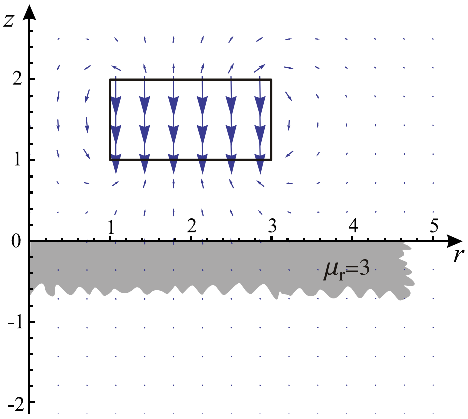

Fig. 7. Distribution of magnetic field.

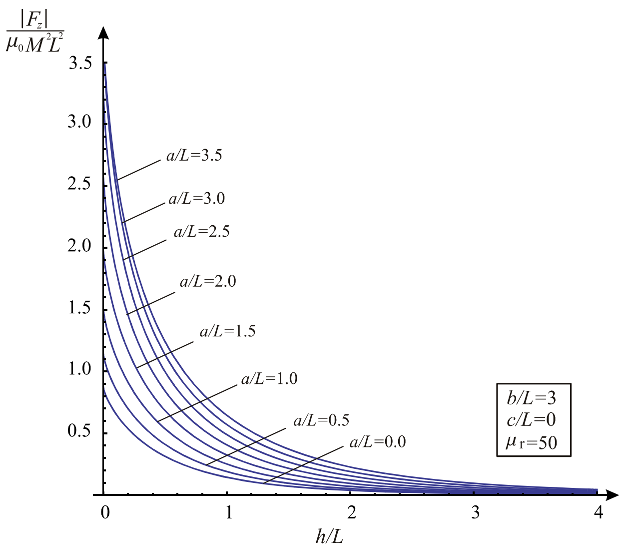

Fig.8. Attraction force versus separation distance.

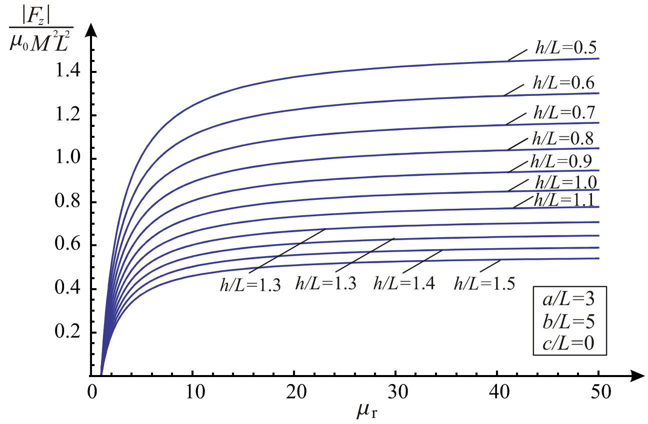

Fig.9. Attraction force versus relative permeability.

IV. CONCLUSION

The derived algorithm for attraction force calculation between ring permanent magnet and infinite magnetic plane is easily implemented in any standard computer environment and it enables rapid parametric studies of the force. The resulting expression is given in terms of elementary functions that are available in all programming languages. The results of the presented approach are successfully confirmed using FEMM 4.2 software. Attraction force calculation using presented approach for mentioned parameters and $N = 100$ is performed with Intel Core 2 Duo CPU at 2.4GHz and 2GB RAM memory and it took ten seconds of run time. The force is also determined on the same computer using FEMM 4.2 software and the computation time was 14 minutes. Therefore the advantage of presented approach is its simplicity and time efficiency.

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