Investigation of Localization Accuracy in Wireless Sensor Networks

Student authors: Vasil Dimitrov, Georgi Georgiev
Mentor: Rozalina Dimova

Abstract **– The paper presents results from some sensor network challenges investigations – accuracy of localization techniques. Critical demand in wireless sensor networks (WSN) introducing in industry is to be localized with high precision. Our investigations purpose is to present localization sensor nodes in one area with minimal error. The statistical results from MatLab 7.9 simulations show the dependence of mean square error and the limit of Cramer-Rao in unknown sensor nodes localization, using anchors.**

Keywords **– Localization, wireless sensor.**

I. Introduction

Wireless sensor networks are new technology, which atracts appreciable investigation interest through the last few years. The last developments in this area have made the sendor nodes small, inteligent and running many functions [1]. There are many interesting applications of the sensor networks like watching the environment, tracing orders and etc.These new applications require placing many sensor nodes in big geometric areas, as the benefit of them depends on the automatic and certain evaluation of their location. This would lead to correct indentifying of important places, when some event happens. Moreover, the certain location may be helpful for processing information, tasks, requests [2].

In the distributed localization is necessary to have a number of nodes,which coordinates are known. These nodes are ususally named anchors. The locations of the nodes is unknown, exept of the anchors. Due of the limits of the power, the communication between the nodes is restricted do local neighbours. The sensor nodes have the opportunity to measure the distance and the angle of the location to neighboring nodes [3]. Based on evaluation like that, they must define their spaced placement, by geometric techniques named: Trilateration and triangulation [4].

The method localization must be with good accuracy,and no matter of the space, the error should be reasonably slight.

Student autors:
[1]V.Dimitrov is with the Technical University of Varna, ul. Studentska 1, 9010 Varna, Bulgaria (phone: +359-52-383350; e-mail: v_1986@abv.bg).
[2]G.Georgiev, is now with the Technical University of Varna, ul. Studentska 1, 9010 Varna, Bulgaria; (phone: +359-52-383350, e-mail: georgi_ygeorgiev@yahoo.com)

Mentor:
Dr. R.Dimova is with the Technical University of Varna, ul. Studentska 1, 9010 Varna, Bulgaria; (phone: +359-52-383350, e-mail: rdim@abv.bg)

II. Measurement Accuracy Positioning

To be able to make a realistic assessment of the accuracy in localization, we test already approved in practice mathematical methods.

A. Mean square error - MSE

Mean square error (MSE) of estimator quantify the difference between estimator and true value of quantify being estimated [5]. MSE measure average of square of error. The error is the sum with which estimator differs from quantity to be estimated. The MSE of estimator $\hat{\theta}$ with respect to estimated parameter θ can be presented as:

$$MSE(\hat{\theta}) = E\left((\hat{\theta}-\theta)^2\right) \quad (1)$$

The MSE is equal to sum of variance and squared bias of estimator

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + \left(Bias(\hat{\theta},\theta)\right)^2 \quad (2)$$

In other words, the more precisely estimated location of unknown node is equal to less MSE [6].

B. Cramer-Rao lower bound (CRLB)

Cramer-Rao bound define the ultimate accuracy of any estimation procedure [7]. This lower bound is intimately related to the maximum likelihood estimator. If I_θ is the Fisher information matrix (FIM), for unknown parameter $p(z \mid \theta)$.

$$I_\theta = E\left\{\frac{\partial \log p(z\mid\theta)}{\partial\theta}\left(\frac{\partial \log p(z\mid\theta)}{\partial\theta}\right)^T\right\} \quad (3)$$

then CRLB can be presented as:

$$E\{(\hat{\theta}-\theta)(\hat{\theta}-\theta)^T\} \geq I_\theta^{-1} \quad (4)$$

where θ is unbiased estimate of θ and I_θ is the FIM.

One of easy way to compare different positioning algorithms is with their MSE. When MSE is lower bounded by the CRLB the result is minimum mean-square error (MMSE). The last one can be presented as:

$$MSE \triangleq E\left\{\left\|\hat{\theta}-\theta\right\|^2\right\} = trace\left[E\left\{(\hat{\theta}-\theta)(\hat{\theta}-\theta)^T\right\}\right] \geq \ldots$$

$$\ldots \geq trace[I_\theta^{-1}] \triangleq MMSE \quad (5)$$

MMSE for RSS-based positioning system [8] is:

$$MMSE_{RSS} = \left(\frac{\ln 10}{10n}\right)^2 \frac{\sum_{i=1}^{Nm} \sigma_{sh,i}^{-2} d_i^{-2}}{\sum_{i=1}^{N_m}\sum_{j=1}^{i-1} \sigma_{sh,i}^{-2}\sigma_{sh,j}^{-2} d_i^{-2} d_j^{-2} \sin^2(\psi_i - \psi_j)} \quad (6)$$

where n is the path lost exponent, $\sigma^2_{sh,i}$ is the variance of the log-normal shadowing for the ith measurement and

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad (7)$$

is the distance between unknown node and ith known node. The accuracy of RSS-based positioning depend on channel parameters and estimates at all nodes [9].

III. Numerical Results and Analysis of the Sensor Network

In order to evaluate the described approaches to sensor network localization, many numerical tests were performed. We performed a variety of simulation experiments to cover a wide range of network system configurations including the size of the network (number of nodes), the number of anchor nodes, anchor nodes deployment, the radio range, the distance measurement error and computation time. The key metric for evaluating all listed measurement was the accuracy of the location.

A. Description of the system

Let us consider L anchors, with coordinates (x_i , y_i) where i=1,...,L, are random placed in two-dimensional (2D) plane with size (x_{max} , y_{max}). The coordinates of the anchors are known. During the each of K simulations the positions of anchors are static. In this plane are random placed also LL unknown nodes, with coordinates (x_{0i} , y_{0i}) i=1,...,LL. From the propagation point of view the measurement are assumed to be made under line of sight condition. The true distance between one anchor and one unknow node is d_i and can be determine like:

$$d_i = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2} \quad (8)$$

The measured distance is:

$$r_{d_i} = d_i + \varepsilon_i \quad (9)$$

where ε_i ~N(0, σ_i^2) is the zero mean Gaussian noise with variance σ_i^2.

B. Examined parameters of the system

We examined the impact of the following parameters over localization in sensor network:

- Number of anchors - L
- Number of unknown nodes -LL
- Noise level - ε (represented as Gaussian random variable with normal distribution with mean 0 and standard deviation σ^2)
- Speed - v (v_x - speed in direction x, v_y - speed in direction y)
- Number of simulations - K
- Iterations to update estimates - S

C. Investigating the precision of localization of one and more unknown nodes

In one real network the number of unknown nodes usually is more than one. When one unknown node is with determined position based on fixed anchors, after that it can be use like fixed anchor for other unknown nodes. In this case the number of anchors increase, respectively MSE decrease, which mean more accuracy.

Whole algorithm consists of 4 steps:

- Step 1: determine positions of each unknown node based on the fixed anchors.
- Step 2: for all unknown positions determine distance between nodes

for node $i : \left(\hat{x}^0_{(i)}, \hat{y}^0_{(i)}\right)$,

for node $j : \left(\hat{x}^0_{(j)}, \hat{y}^0_{(j)}\right)$

$$\hat{d}^0_{i,j} = \sqrt{(\hat{x}^0_{(i)} - \hat{x}^0_{(j)})^2 + (\hat{y}^0_{(i)} - \hat{y}^0_{(j)})^2} + noise^0_{i,j} \quad (10)$$

- Step 3: determine distance to L and distances to LL

d – for fixed anchors

$d^{(0)}_{i,j}$ - for unknown nodes

- Step 4: repeat Step 2-3 S times - iterations to update estimate

$$\hat{d}^\alpha_{i,j} = \sqrt{(\hat{x}^\alpha_{(i)} - \hat{x}^\alpha_{(j)})^2 + (\hat{y}^\alpha_{(i)} - \hat{y}^\alpha_{(j)})^2} + noise^{(\alpha)}_{i,j} \quad (11)$$

In this case MSE for node i is

$$MSE_i = \frac{1}{K}\sum_{j=1}^{K}\left[\left(x_0 - \hat{x}_0^{(i,j)}\right)^2 + \left(y_0 - \hat{y}_0^{(i,j)}\right)^2\right] \quad (12)$$

MSE is:

$$MSE = \frac{1}{LL}\sum_{m=1}^{LL+1} MSE_j \quad (13)$$

The Cramer-Rao bound in this case will be:

$$CRB = \frac{4\sigma^2}{L + LL} \quad (14)$$

As can be seen in Figure 1. with increasing the number of anchors MSE decrease. Interesting of this picture is that, when assume unknown node to fixed anchors the minimum bound of MSE decrease. Respectively the accuracy on the system can increase. CRB and CRB(L+LL) are not parallel, and with increasing the number of anchors they stay closer and closer. Other interesting in this picture is that, when change *S*, the accuracy of the system does not increase. This mean the system work sufficiently good with only one estimation of the position of unknown nodes.

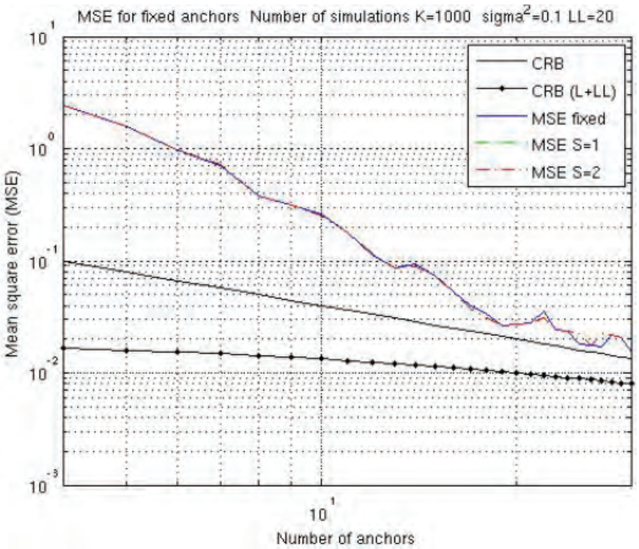

Fig. 1. Dependence between MSE and L for σ^2 =10^{-1}

Figure 2. shows the dependency between MSE and number of unknown nodes, which became anchors. The trend to decrease MSE with increasing the number of anchors holds true here.

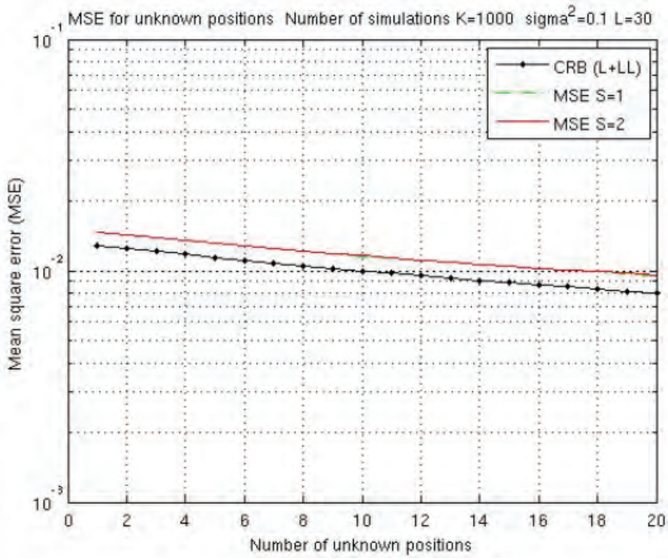

Fig. 2. Dependence between MSE and LL for σ^2 =10^{-1}

In Figure 3. shown the case, when the anchors are not sufficiently for good estimation of unknown nodes. As can be seen MSE increase with increasing number of unknown nodes, which are assumed to anchors.

Also with increasing the number of iteration to update estimates, MSE increase. This is called error propagation. On this example the number of unknown nodes does not matter for accuracy and MSE. They can not be used like anchors, because their positions can not be established sufficiently good.

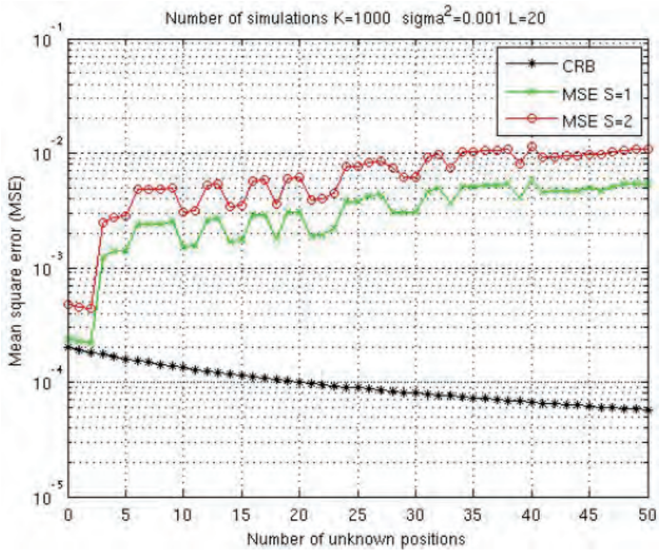

Fig. 3. Error propagation

D. Position tracking

In all up to now observed scenarios the unknown node does not change its position at the time of one simulation. In the real life the unknown node (most often human) change its position. The movement can be divide in three case: movement with constant speed, random walk and combination of both. The movement can be divided to ten steps (t=1,...,10). The first position is t=1. Then each new step can be presented as:

$$x(t+1) = x(t) + v_x T \text{ и } y(t+1) = y(t) + v_y T,$$

where $v_x T$ and $v_y T$ are the speed in direction x and the speed in direction y. According $v_x T$ and $v_y T$ movement is divided into three types:

• constant speed - $v_x T$ =const and $v_y T$ =const

• random walk - $v_x T$ =random and $v_y T$ =random

• constant speed and random walk - $v_x T$ =const+random and $v_y T$ = const + random

Therefore estimated position will be:

$$\hat{x}(t),\ \hat{x}(t+1),\ \ldots,\ \hat{x}(t+10),$$
$$\hat{y}(t),\ \hat{y}(t+1),\ \ldots,\ \hat{y}(t+10)$$

The next step can be predicted. In this case the speed for next step can be determine with these equals:

$$\hat{v}_x T(i) = \frac{1}{i}\sum_{i=1}^{i}(\hat{x}(i) - \hat{x}(i-1)) \qquad (15)$$

$$\hat{v}_y T(i) = \frac{1}{i}\sum_{i=1}^{i}(\hat{y}(i) - \hat{y}(i-1)) \qquad (16)$$

which use to predict next position of unknown node $\tilde{x}(i+1)$ and $\tilde{y}(i+1)$

$$\tilde{x}(i+1) = \tilde{x}(i) + \hat{v}_x T(i) \qquad (17)$$

$$\widetilde{y}(i+1)=\widetilde{y}(i)+\hat{v}_y T(i) \qquad (18)$$

So next step may be predict with determined accuracy. This algorithm can improve the performance of steepest descent algorithm and also accuracy of the system.

In the three cases the dependence between MSE and the speed is the same and it is show in Figure 4. As can be seen the MSE vary very little between 10^{-5} and 10^{-1}. After 10^{-1} MSE increased significantly.

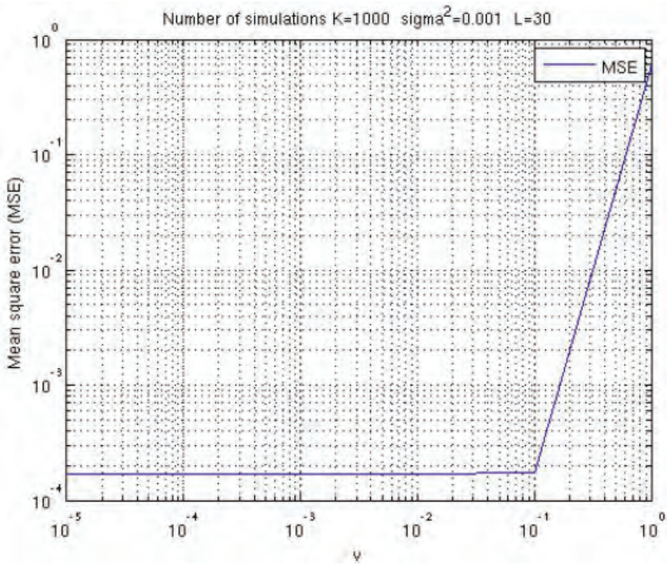

Fig. 4. Dependence between MSE and the speed

Dependence between MSE and σ^2, shown in Figure 5. When σ^2 increase, MSE increase linear.

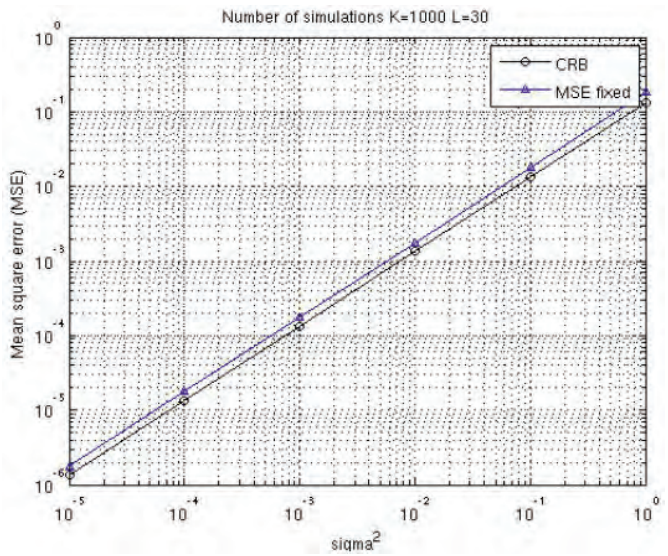

Fig. 5. Dependence between MSE and σ^2

IV. Conclusion

Finding the location of the sensor nodes is from substantially matter for building sensor networks.

To evaluate the accuracy in localization of the sensor nodes in the space,we use mean square error and the limit of Cramer-Rao.

From research done can be seen that with geometric techniques for determining the position we can accomplish relatively good lokalization. The results of the sumitaion are these:

- MSE increase linear with increasing σ^2.
- the number of anchors for sufficiently accuracy is more than theoretical number.
- increase the number of iterations to update estimates can not improve accuracy.
- unknown node can be use like anchors only if anchors are more than few times theoretical number.
- the algorithm of predicting can be applied to improve the localization of the unknown nodes.

Based on the wrought researches in future may be improved the communication between the rest of the sensor nodes and to be relegated the cost price of the system.

Acknowledgement

This paper was developed in the frames of the research project № DVU01/0109 "Research on Cross Layer Optimization of Telecommunication Resource Allocation" in the frames of FNI, Ministry of Education, Bulgaria .

References

[1] D. Estrin, L. Girod, G. Pottie, M. Srivastava, "Instrumenting the world with wireless sensor networks," *In Proceedings of the International Conference on Acoustics, Speech and Signal Processing (ICASSP 2001),* Salt Lake City, Utah, May 2001.

[2] C.-Y. Chong and S. P. Kumar, "Sensor networks: evolution,opportunities, and challenges," Proceedings of the IEEE, vol. 91, pp.1247-1256, 2003.

[3] D. Braginsky and D. Estrin, "Rumor Routing Algorithm For Sensor Networks," Under submission to International Conference on Distributed Computing Systems, Novemb. 2001.

[4] H. Bulusu, D. Estrin, L. Girod and J. Heidemann, "Scalable Coordination for wireless sensor networks: Self-Configuring Localization Systems," *In Proceedings of the Sixth International Symposium on Communication Theory and Applications*, Ambleside, Lake District,UK, July 2001.

[5] Z H. Cramer, *Mathematical Methods of Statistics.* Princeton, NJ: Princeton Univ. Press, 1946.

[6] Y. C. Eldar, A. Ben-Tal, and A. Nemirovski, "Robust mean-squared error estimation in the presence of model uncertainties," IEEE Trans. Signal Process., vol. 53, pp. 168–181, Jan. 2005.

[7] C. R. Rao, Linear Statistical Inference and Its Applications, second ed. New York: Wiley, 1973.

[8] Hongchi Shi, Xiaoli Li, Yi Shang, Dianfu Ma "Cramer-Rao Bound Analysis of Quantized RSSI Based Localization in Wireless Sensor Networks" Proceedings of the IEEE, vol. 2, pp. 32-36, 1521-9097, July 2005

[9] Y. C. Eldar, "Minimum variance in biased estimation: Bounds and asymptotically optimal estimators," IEEE Trans. Signal Process., vol. 52, pp. 1915–1930, Jul. 2004.