Attractive Ways Forward to Maximise Capabilities of the FD-BPM Technique

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Abstract – **Attractive ways forward to maximise capabilities of FD-BPM technique in photonics and nano-photonics design are presented and discussed. Novel improved FD schemes and formulas, the Alternating-Direction Implicit scheme, the explicit DuFort-Frankel scheme, the complex Jacobi iterative method, the efficient three-dimensional wide-angle beam propagation methods, the use of preconditioners allowing the solution of 3D problems of interest in reasonable computer runtimes – are just some of the few novel approaches highlighted.**

Keywords – **Beam propagation method, Finite-difference, Photonics, Optoelectronics, Numerical simulations.**

I. INTRODUCTION

Beam propagation methods (BPM) stand as a standard and computationally efficient design tools used in integrated photonics and optoelectronics in the last decades. BPM is an approach for numerical solving of the paraxial approximation of an exact vector Helmholtz's equation (also known as the Fresnel's equation). Although BPM can be formulated in time domain as well, frequency domain BPM techniques are still dominant in photonics analysis allowing suitable results with low run-time and memory computer costs.

The finite-difference beam propagation method (FD-BPM) is certainly the most popular BPM algorithm [1-3]. Since its first formulation in 1990 [4], the FD-BPM has undergone significant improvements, particularly during the last decade [5-7].

The main feature of the original FD-BPM, the paraxial approximation, at the same time presents the crucial limitation of the algorithm. A remedy was found in the early 1990s when the wide-angle (WA) BPM algorithm using Padé series expansion of the square root operator were introduced [8]. So far, several WA-BPM algorithms have been suggested allowing significant improvements of the computational efficiency of the standard paraxial BPM technique.

The FD-BPM is usually implemented in a rectangular co-ordinate system and accordingly the accuracy of the method is affected by inevitable so-called staircase approximation. Namely, if the structure under analysis contains oblique or curved interfaces or when the structure is changing in the direction of the propagation, the dielectric boundaries are modelled with error causing serious problems and certain restrictions of the method. A remedy was found by using improved FD formulas, or by using co-ordinate systems, which exactly describe the geometry of the photonic device studied. Several, usually the non-orthogonal, co-ordinate systems were recently proposed as well as novel very efficient forms of improved FD formulas.

The basic drawback of the three-dimensional (3D) implicit WA-BPM is its huge memory consumption and consequently lengthy computer runtimes. The way out has been sought during the last decade in the development of unconditionally stable Altering-Direction Implicit (ADI) schemes. ADI algorithms provide non-iterative solution and require less memory and computational time. One further attractive possibility is the use of the fast and unconditionally stable explicit BPM algorithms like the DuFort-Frankel (DFF) variant. The Spectral Collocation Method (SCM) is recently proposed to minimize memory storage and to offer highly accurate results [9]. The Fourier cosine BPM algorithm, based on simple and time efficient matrix calculations [10], deserves a particular attention.

Iterative FD-BPM schemes designed to achieve higher accuracy in numerical simulations (fine FD meshes for modelling complex geometries, the use of higher order Padé approximation in the WA-BPM) tend to be time very consuming and often instable. A novel complex Jacobi iterative algorithm [11] and construction of suitable preconditioners can substantially improve the convergence and minimize computer runtime involved in simulations.

Usage of the FD-BPM is not limited only for applications and devices in conventional integrated photonics; there are examples in the very recent literature about its possible application in the design of modern photonic devices, such as photonic crystals fibers, plasmonics, integrated optical memories, and other components in nano-photonics and bio-photonics.

This paper reviews the most recent advances in the field of FD-BPM implementation. Highlighted improvements are still attractive and active areas of research. Extensive bibliography follows the review presented in the paper.

Presented brief summary does not cover all advanced approaches proposed in the literature during the last decade. However, many of referred approaches have already significantly impacted the FD-BPM CAD manufacturing in photonics and optoelectronics.

II. WIDE-ANGLE FD-BPM PROPAGATION

The standard paraxial FD-BPM method limits the field simulations to paraxial beams along or close to the z axis. The WA-BPM schemes use Padé approximation of neglected second-order derivative with respect to the z in the wave equation, as it is assumed in the original BPM formulation. With the WA-BPM algorithms the field can be propagated through tilted and curved waveguide structures and circuits

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without loss of accuracy. The WA-BPM approach offers much more realistic results of the lightwave propagation. Unfortunately, the serious drawback of this approach is the increase of the matrix bandwidth, especially when higher order Padé operator is used, causing problems with available computational resources.

In order to improve the efficiency of the WA-BPM, the algorithm is combined with various multistep methods [12,13] and ADI methods [14-16]. The implementation of various improvements of the WA-BPM, including different meshes, non-standard co-ordinate systems [17] and multistep methods is perhaps the most attractive area in FD-BPM research.

III. STRUCTURE-RELATED FD-BPM PROPAGATION

The co-ordinate transformation approaches reformulate the BPM in non-orthogonal, so-called structure related (SR) co-ordinate systems [18]. Successful approach to eliminate non-physical scattering due to the staircasing effect in FD discretization of oblique dielectric interfaces in rectangular co-ordinate system is the use of the co-ordinate transformation methods, such as SR FD-BPM [18]. SR co-ordinate systems, such as tapered, oblique, bi-oblique co-ordinate systems, naturally follow the local geometry of the structure under analysis. The BPM Helmholtz's equation can be rewritten and numerically solved in any orthogonal or non-orthogonal transverse co-ordinate system.

SR FD-BPM algorithm allows simulations with noticeably reduced numerical noise and shortened simulation time. The non-orthogonal co-ordinate FD-BPM has been applied to the analysis of structures with oblique, bi-oblique, tapered, and tapered-oblique cross-sections in the transverse plane. Recently, the oblique FD-BPM has been proposed based on the fast explicit DFF algorithm [19].

IV. IMPROVED FD FORMULAS

Implementations of the FD-BPM for structures in a rectangular co-ordinate system are characterized by low-order truncation errors, e.g. standard difference equations in two dimensions in homogeneous regions are second-order accurate, $n = 2$, or $O(h^2)$, where h is the FD mesh size. Near the step-index dielectric interfaces, accuracy usually drops to $n - 1$, and near dielectric corner points difference equations are $(n - 2)$th-order accurate, resulting with $(n - 1)$th-order of accuracy of the modal index and modal electromagnetic field.

The starting point in improving FD discretization procedure was Stern's work [20] where the concept of a semi-vectorial mode has been introduced, resulting in $O(h^0)$ truncation error. Vassallo [21] proposed an improved three-point FD formulation for the semi-vectorial case providing $O(h)$ accuracy. Yamauchi et al. [22] improved Vassallo's approach to give $O(h^2)$ accuracy regardless of interface position. Chiang et al. [23] generalized Vassallo's and Yamauchi's approach to full-vectorial case to give $O(h^2)$ accuracy for oblique, even curved step-index boundaries. Hadley [24,25] derived highly accurate FD formulas, assuming TE polarization, with truncation error in the uniform region $O(h^4)$ to $O(h^6)$ depending on the type of grid employed, and up to the $O(h^5)$ near dielectric interfaces under certain grid-interface conditions. In [24,25] Hadley utilized 2D solutions of the Helmholtz's equation in cylindrical co-ordinates. This approach resulted in the tremendous increase in accuracy, however with increase in algebraic and numerical efforts in formulas derivation and implementation. Three distinct cases of uniform regions, dielectric interfaces and dielectric corners are handled separately and these derivations are finally incorporated into a TE mode waveguide modelling tool. Although Hadley's FD formulas are rather tedious to derive and implement, they have been incorporated in the improved accuracy eigenmode solvers for benchmark purposes in some waveguide simulations.

A novel technique for obtaining the truncation error with $O(h^{2N})$ accuracy (where N is number of sampled FD points) is recently proposed [26] for the 2D case. The extension to 3D cases is expected, promising benefits with tremendously less computation time and memory.

V. IMPROVED FD DISCRETISATION

FD-BPM modelling of optical and photonic crystal fibers with non-cylindrical cross-sections and complex geometries, including nonlinearity effects involved, have been successfully solved by using FD dicretisation with triangular-mesh [27]. Nonlinear contributions to the index of refraction, due to high-power regimes of optical systems, are successfully treated within the algorithm. Curved dielectric boundaries of any shape can be accurately approximated with irregular deformable triangular FD-grid, although the derivation of accomplished FD formulas is more sophisticated and therefore more complicated.

The main disadvantage of the standard FD discretisation approach is a need to define a line-structured grid of points. Non-standard FD approaches, such as Generalized Finite-Difference Method (GFDM) [28], relax the grid requirements. By using the radial or polynomial basis functions and a moving least-squares scheme, the FD interpolation formulas can be constructed on localized sets of points to enable dealing with complex geometries.

Promising approach has been reported in [29], where the generalised two-dimensional full-vectorial FD approach has been used for the electromagnetic field discretisation near dielectric interfaces within rectangular grid featuring in $O(h^4)$, and higher, truncation error.

The irregular generalized FD schemes are still to come in the use in the whole scope of the FD-BPM applications.

VI. IMPROVED ITERATIVE FD-BPM ALGORITHMS

The implicit FD-BPM methods (like CN – Crank-Nicolson method) are known as unconditionally stable FD-BPM algorithms. For fine FD meshes those algorithms require often time lengthy usually iterative procedures for matrix inversions. This is particularly problem when the propagation matrix is not sparse enough as in the original FD-BPM applications. The WA-BPM and the reflective FD-BPM algorithms are the typical examples. In those cases every step-forward in cutting the computer runtimes is welcomed. A new complex Jacobi iterative (CJI) method, proposed by Hadley [8], is one of the most promising and competitive approach. The CJI method has been successfully applied to the 3D WA-BPM simulations [12,13,30] enabling a development of higher order 3D Padé approximant–based algorithms within modest runtimes and memory requirements.

Another way forward to improve the iterative FD-BPM algorithms is to speed-up the convergence rate by constructing and applying a suitable preconditioner. Usually, this is the typical linear algebra problem, where the CJI method [11] can serves as a preferable approach for obtaining precondition parameters for optimum algorithm convergence. An efficient idea is to use a preconditioner based on paraxial approximation [31].

VII. Non-iterative FD-BPM algorithms

The standard FD-BPM technique employs the CN scheme which is unconditionally stable, however, as an implicit procedure, has a drawback because it uses iterative matrix solver in every propagation step and thus requires huge computational resources. ADI-FD-BPM makes use of the highly efficient non-iterative Thomas algorithm (the direct tridiagonal matrix solver) by splitting the FD operator in two one-dimensional terms – i.e. two FD equations.

ADI schemes have been successfully applied to the WA-BPM algorithm to enhance the efficiency of the WA schemes [14,15]. Further, the Hoekstra's scheme has been utilized with the ADI and WA algorithm [32]. Non-iterative Local One-Dimensional (LOD) schemes were recently introduced for 3D FD-BPM [33].

The attractive alternative to implicit (iterative or non-iterative) schemes is sought within the use of the three-level explicit DuFort-Frankel (DFF) algorithm [19,34]. The DFF algorithm is the rare example of the explicit BPM procedure being unconditionally stable. The DFF approach does not need matrix solver; the associate computer code can be parallelized easily and very efficiently onto distributed memory parallel computers. Besides of these highly competitive benefits, the DFF scheme has some serious inherent disadvantages (or weakness). First of all, this is the appearance of the spurious mode solutions (the FDD algorithm is empirically constructed). To avoid and suppress spurious (fake) modes, the propagation step has to be reduced, or obtained solution has to be filtered. Furthermore, the FDD-BPM approach is still limited to paraxial and semi-vectorial cases. Therefore, the trade between unprecedented simplicity and efficiency of the DFF-BPM method and serious drawbacks on the other hand will certainly continue to deserve attention of researches in the future.

VIII. Modern implementation of the FD-BPM

Photonic devices are being constantly improved and developed in the last decade. These newly-designed structures have placed high demands on the numerical modellers. Although the finite element method (FEM) and finite-difference time domain method (FD-TD) are traditionally used today to solve the propagation and modal properties of these newly-designed photonic devices, the FD-BPM is highly applicable in this direction as well.

The employment of the FD-BPM in the design of novel optical fibers with complex geometries and modal properties operating in nonlinear and high power regimes [27,35,36] is already highlighted in Section 5. Numerical simulations of the wave propagation in plasmonics (metallic waveguides supporting surface plasmons having the enormous bandwidth of a light pulse) have been recently accomplished by the use of the FD-BPM [37]. Photonic crystal fibers [38] and other newly-designed modern photonic structures [39], which appear as the result of the advances in the modern nano-fabrication and characterization techniques, can be successfully modelled with the FD-BPM approaches.

IX. Conclusion

The FD-BPM remains one of the most widely used techniques for numerical field simulations in integrated photonics and optoelectronics. Some of very promising recently proposed FD-BPM procedures have been addressed and discussed. The overall conclusion is that the improvement of the capabilities of the FD-BPM is still a very attractive area of research in numerical photonics.

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