Comparison of Two Methods for Estimation of a Single-Phase Transformer's Magnetization Curve

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***Abstract* – This paper presents analysis of differences between shapes of a single-phase transformer's magnetization curves obtained through two different approaches. The first approach has been based on the standard no-load test performed at several different values of the applied voltage, using the laboratory instrumentation capable to register true RMS electrical quantities (voltage, current, active power and reactive power). In the second approach, analysis has been performed considering recorded waveforms of no-load currents and corresponding induced voltages. Using the MATLAB/Simulink software the way explained in the paper, for any pair of recorded induced voltage and no-load current waveforms it has been possible to obtain dynamic hysteresis loop. Further analysis has shown that both methods give similar final results, if magnetic core is not heavily saturated. However, if the applied voltage is higher than the rated value and the no-load current is highly distorted due to non-linearity of the magnetic core, a significant difference between estimated magnetization curves occurs.**

***Keywords* – Single-phase transformer, hysteresis loop, main magnetization curve.**

I. INTRODUCTION

There are numerous methods for measuring magnetization characteristics of electrical machines and magnetic materials in general. Most of them are based on direct current method and use specialized instrumentation that is not always available ([1]). However, there are situations in engineering and scientific practice when it is necessary to estimate nonlinear magnetization curve of an electrical machine, without use of specialized measuring equipment. In such cases, the only solution is to use an alternative approach and to exploit some of available AC measuring methods ([1-4]). Some of them are simple, and do not demand much equipment and time for analysis. On the other hand, some of AC measuring methods are more complex and have to be supported by sophisticated analytical process.

The aim of this paper is to perform comparison between two AC experimental methods in order to make conclusion, which one is more appropriate for use.

II. COMPARED EXPERIMENTAL METHODS

The simplest experimental procedure for quick estimation of magnetization curve is to perform no-load test at different values of applied voltage. According to this, single-phase transformer that was subject of experimental investigation in the paper has been powered from the secondary side, using an autotransformer as variable voltage source. In order to perform no-load test, terminals of the primary winding were opened, while the applied voltage U_0'', no load current I_0'' and consumed active power P_0'' were measured at terminals of the secondary winding. Since no-load current of the transformer always contains higher harmonics due to nonlinearity of the magnetic core, it is very important to use measuring equipment that retains rated accuracy even when non-sinusoidal electrical quantities are measured. For this purpose, a digital laboratory power analyzer, capable to register true RMS values has been used.

At any experimental point, the power factor can be estimated as

$$\cos\varphi_0'' = \frac{P_0''}{U_0'' I_0''}\,, \tag{1}$$

and that enables reactive part of no-load current to be calculated as

$$I_\mu'' = I_0'' \sin\varphi_0'' = I_0'' \sqrt{1-\cos\varphi_0''^2}\,. \tag{2}$$

Current I_μ'' is often called „magnetizing current“, since it establishes magnetic flux in a core of a transformer, but does not take into account active power dissipated in the core. Knowing the effective value (Eq. 2), maximum value of magnetizing current can be easily calculated, using the well-known relation between maximum and effective value

$$I_{\mu m}'' = \sqrt{2} I_\mu''\,. \tag{3}$$

However, in such approach mistake is consciously made, because no-load current is not sinusoidal and contains harmonics of higher order (especially third and fifth harmonic). In fact, a realistic, more or less distorted current waveform is being supplemented by fictitious sinusoidal current that produces equivalent sinusoidal magnetic flux in a core of a transformer.

If resistance and leakage reactance of secondary winding, R'' and X_γ'', are known, for any effective value of the applied voltage, corresponding effective value of induced voltage in secondary winding can be calculated using Eq. 4.

$$E'' = \Big[\big(U_0'' - R'' I_0'' \cos\varphi_0'' - X_\gamma'' I_0'' \sin\varphi_0''\big)^2 + \\ + \big(R'' I_0'' \sin\varphi_0'' - X_\gamma'' I_0'' \cos\varphi_0''\big)^2\Big]^{1/2}\,. \tag{4}$$

Finally, maximum value of magnetic flux is obtained from

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$$\Phi_m = \frac{E''}{4.44\, f N''}\,, \tag{5}$$

where N'' denotes number of turns in secondary winding and f denotes frequency.

Pair of values ($I''_{\mu m}$, Φ_m) defines coordinates of the point that should belong to the main magnetization curve. If applied voltage is varied in reasonable small steps (e.g. one step could be about 10% of rated voltage), points obtained through previously described analysis should depict the nonlinear shape of the main magnetization curve. Results of such analysis performed on the real laboratory transformer are presented in Section 3.

The other method that can be used in order to identify nonlinear magnetization curve of a single-phase transformer is based on knowing of no-load current $i_0(t)$ (i.e. exciting current), and corresponding core flux $\Phi(t)$ waveforms, under different values of applied voltage. Basics of this method have been frequently described in the literature considering electrical machines and electromagnetism in general ([5-7]). The main idea is that, if several dynamic hysteresis loops have been successfully identified by plotting core flux $\Phi(t)$ versus exciting current $i_0(t)$, one can further obtain main magnetization curve by interpolating points whose coordinates are determined by peaks of hysteresis loops in the first quadrant. Sometimes, it is more appropriate to plot flux density $B(t)$ versus magnetic field $H(t)$, what can be easily done by scaling waveforms for $\Phi(t)$ and $i_0(t)$ with constant coefficients, depending on geometry of a magnetic core and construction parameters.

However, details of the experimental method are usually unexplained, which can be understood, because there is not only one and unique set of steps to perform in order to reach the final goal. The second author of the paper has already considered problem of identifying dynamic hysteresis loops of a single phase-transformer in his previous work ([1]), but the methodology used in that reference was significantly different.

The essential step in method used in this paper is to record accurate waveform of the no-load current in secondary winding $i''_0(t)$ and corresponding induced voltage in primary winding $e'(t)$. During experimental work, real current $i''_0(t)$ has been transformed to a voltage signal of appropriate amplitude, using LEM current module. Induced voltage $e'(t)$ has also been conditioned to an adequate voltage level, using linear isolating attenuator. These two signals, carrying all necessary information describing $i''_0(t)$ and $e'(t)$ waveforms, were recorded on the hard disc, using National Instruments PCI 6036E data acquisition card and LabView software. Both waveforms have been recorded with 400 samples per one cycle (sampling time was $5 \cdot 10^{-5}$ seconds).

After initial computations, performed in order to rescale recorded values to those equal with real electrical quantities, data can be used as input for a simple MATLAB/ Simulink model, based on use of Discrete Fourier blocks from SimPowerSystems Extra Library. This model was created in order to calculate magnitude (I_m or E_m) and phase angle (φ_i or φ_e) of odd harmonics in analyzed waveforms, up to 29-th.

Following this briefly described procedure, for any voltage applied to the secondary winding, no-load current in secondary and induced voltage in primary winding can be presented as:

$$i''_0(t) = \sum_{k=1}^{15} I_{m_{2k-1}} \sin\left((2k-1)\omega_1 t + \varphi_{i_{2k-1}}\right) \tag{6}$$

and

$$e'(t) = \sum_{k=1}^{15} E_{m_{2k-1}} \sin\left((2k-1)\omega_1 t + \varphi_{e_{2k-1}}\right), \tag{7}$$

where $2k-1$ is the order of the harmonic and ω_1 angular frequency of the main harmonic.

Regarding Eqs. 6 and 7, it is possible to create another simple Simulink model, proposed to generate smooth and accurate waveforms of current $i''_0(t)$ and induced voltage $e'(t)$ for each specific case. If the transformer was in steady state operation during the recording of mentioned waveforms (i.e. applied voltage had constant effective value), it is enough to generate only one cycle of no-load current and induced voltage.

Since induced voltage in primary winding and core flux are connected through

$$e'(t) = -N' \frac{d\Phi(t)}{dt} \tag{8}$$

waveform of the core flux $\Phi(t)$ can be further obtained as

$$\Phi(t) = -\frac{1}{N'} \int e'(t)\,dt + C\,, \tag{9}$$

where N' denotes number of turns in primary winding. Constant C is the initial value of core flux at the beginning of the analyzed cycle, and it is very important to be accurately determined, otherwise waveform $\Phi(t)$ will have an offset. Calculation of this constant can be easily done if simulation is once performed assuming that $C = 0$, since whole waveform will be shifted along flux axis for exactly $-C$ in that case.

Finally, plotting obtained waveform $\Phi(t)$ versus $i''_0(t)$ defines dynamic hysteresis loop that is valid for the applied voltage. If the value of the applied voltage is varied, a family of concentric hysteresis loops will be identified. In this case, points characterized by maximum value of flux and corresponding no-load current from any of obtained hysteresis loops, will be points that define shape of main magnetization curve.

III. EXPERIMENTAL RESULTS AND DISCUSSION

Both of methods explained in Section 2 were used in order to estimate main magnetization curve of a real single-phase transformer with rated values $S_n = 2kVA$, $U'_n = 380V$, $U''_n = 190V$, $I'_n = 5.3A$, $I''_n = 10.6A$, and number of turns per winding $N' = 360$ and $N'' = 179$.

During the investigation conducted according to the first described method, no-load test has been performed for 17 different values of the applied voltage U''_0, the highest being almost 1.5 times greater than rated voltage U''_n. Measured values are presented in first three columns of the Table I. Fourth and fifth column of the same table represent corresponding values of maximum magnetizing current $I''_{\mu m}$ and maximum core flux Φ_m. These values were calculated using Eqs. (1)-(5), regarding previously determined parameters of the secondary winding $R'' = 0.91\Omega$ and $X''_\gamma = 2.48\Omega$.

TABLE I
VALUES FROM STANDARD NO-LOAD TESTS

measured			calculated	
U''_0 [V]	I''_0 [A]	P''_0 [W]	$I''_{\mu m}$ [A]	Φ_m [Wb]
33.2	0.083	1.54	0.097	0.00083
55.8	0.111	4.19	0.115	0.00140
82.3	0.142	8.55	0.137	0.00206
106.7	0.174	13.57	0.168	0.00267
144.1	0.247	23.56	0.261	0.00361
167.8	0.337	31.5	0.395	0.00420
185.7	0.453	38.02	0.571	0.00464
193.7	0.526	41.06	0.681	0.00484
201.9	0.619	44.62	0.818	0.00504
213.9	0.788	50.06	1.064	0.00533
225	0.994	55.91	1.361	0.00560
235.6	1.229	62.54	1.698	0.00585
245.3	1.488	68.26	2.068	0.00608
253.8	1.735	74.86	2.419	0.00627
264.7	2.11	82.1	2.953	0.00652
276.1	2.581	94.07	3.621	0.00678
287.3	3.172	106.62	4.459	0.00702

Calculated values are graphically presented on Fig. 3. White circles represent points with coordinates ($I''_{\mu m}$, Φ_m), while the dashed line presents estimated basic magnetization curve, obtained by interpolation between experimental points. This curve has also been extrapolated above last experimental point.

Experimental investigation according to the second described method has been performed for 13 different values of the voltage applied to terminals of secondary winding. However, these voltages were not exactly measured, since their values were meaningless in this investigation. As it had been explained in Section 2, waveform of voltage induced in primary winding $e'(t)$ was recorded instead. It had only been important to get the range of flux core variation that could be compared to the range from the first experiment. In order to obtain uniform distribution of experimental points, RMS value of voltage induced in primary winding was measured as an orientation.

The lowest measured value was $E'_{\min} = 51V$, while the highest value was $E'_{\max} = 580V$. Knowing that transformer's turn ratio is $N'/N'' \approx 2$, it is obvious that the range of flux variation during the second experiment was somewhat wider, compared to the experiment whose results are presented in Table I. Following the procedure described in Section 2, 13 different dynamic hysteresis loops for investigated single-phase transformer have been plotted (Fig. 1).

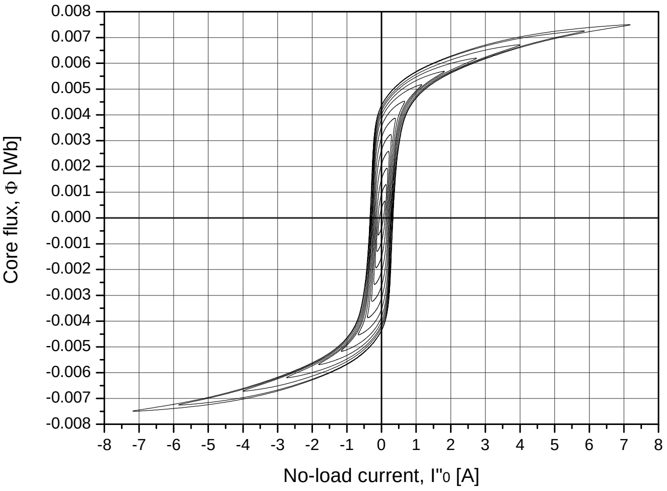

Fig. 1. Obtained hysteresis loops

In order to obtain basic magnetization curve through this approach, it was necessary to identify coordinates of points characterized by maximum value of flux, on each of 13 plotted hysteresis loops.

Fig. 2 shows only the part of hysteresis loops presented on Fig. 1, but for the desired analysis, this part is the most important. On any of presented loops, one can notice the point where flux reaches its maximum Φ_m, and consequently, magnetizing current also has maximum value $I''_{\mu m}$. These points are presented by black circles, and after they had been identified, it was possible to perform interpolation between them and to estimate main magnetization curve (thick line on Fig. 2).

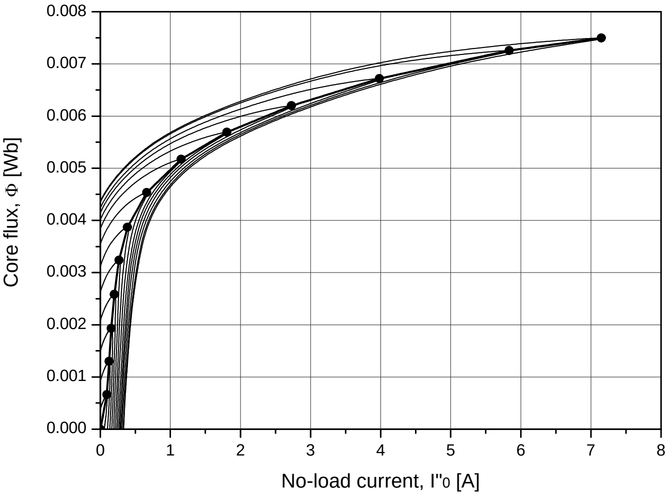

Fig. 2. Peaks of hysteresis loops and interpolation between them

Results obtained through two different experimental methods are finally presented on the same graph (Fig. 3), in order to enable easy comparison. From Fig. 3, it is obvious that experimental points obtained through two different approaches lay on two different nonlinear curves. It is not appropriate to compare results using point-by-point method, since their number is not equal, and they were not supposed to be comparable, as it has been already mentioned. However, nonlinear curves defined by those points should be the subject of consideration. It has to be mentioned that interpolations shown on Fig. 3 surely are not the best nonlinear approximations that could have been made, but even with points roughly connected by straight-line segments, significant difference can be noticed.

Complete main magnetization curve obtained through the first experimental method is placed above the curve obtained by the second method. If those curves are compared using constant flux as a criterion, it can be said that first curve underestimates maximum value of magnetizing current necessary to create desired magnetic flux in transformer's core.

Perhaps somebody might consider this situation in opposite direction, saying that second curve overestimates maximum value of the magnetizing current, however, it would not be correct. Previous statement is clear if one keeps on mind that first experimental method deals with fictitious, equivalent sinusoidal waveforms of no-load current and core flux, while the second one is established on analysis of real no-load current and core flux waveforms.

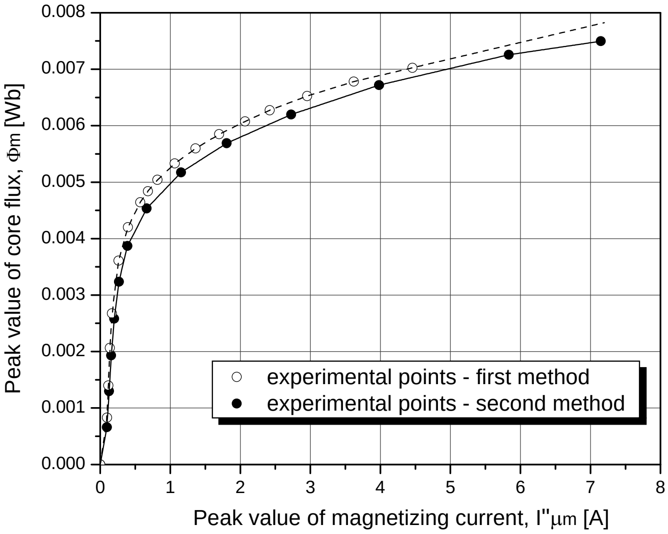

Fig. 3. Comparison of obtained magnetization curves

It can be noticed that small difference exists even between those segments of estimated magnetizing curves that are usually treated as „linear part of magnetization curve“ in the technical literature ([4], [6]). In fact, there is no part of magnetization curve that can be considered as absolutely linear, however, an usual engineering approach is to treat the first segment of the curve in that way. As the value of the applied voltage increases, transformer's magnetic core becomes more and more saturated, and in this segment greater difference between estimated magnetization curves can be observed. This can be explained by the fact that when magnetic circuit is heavily saturated, no-load current becomes highly distorted, due to harmonics of higher order emerging in it's reactive component (i.e. in magnetizing current).

IV. CONCLUSIONS

Based on previous analysis, it can be said that considered experimental methods for estimation of a single-phase transformer's magnetization curve give similar, but not identical results. If magnetic core is not saturated and operating point remains on linear segment of magnetization curve, difference between obtained curves can be neglected. However, even at rated operating point, which is usually on the knee of the curve, slight difference is notable. Finally, if transformer's operating point enters the saturated region of magnetization curve for any reason, basic magnetization curve obtained through classical no-load test will underestimate maximum values of magnetizing current.

The final conclusion is that before any experimental activity, one should have clear idea what is the real purpose of identification of magnetizing curve. If estimated curve will be used for analysis of non-saturated or slightly saturated regimes, it is convenient to use the first method, which is less complex and demands less equipment and time. Otherwise, the advice is to use more complex, but also more accurate experimental method, based on analysis of no-load current and core flux waveforms.

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REFERENCES

[1] Z. P. Stajić, S. Aleksandrov, Đ. R. Vukić and N. Rajaković, „Značaj poznavanja karakteristika magnećenja magnetskih kola pri remontu transformatora” (in Serbian), Zbornik referata III Simpozijuma Industrijska Elektronika INDEL 2000, pp. 60-65, Banjaluka, 2000.

[2] Z. Wlodarski, “Extraction of Hysteresis Loops From Main Magnetization Curves”, Journal of Magnetism and Magnetic Materials No 308, pp. 15-19, 2006.

[3] S. Bogarra, A. Font, I. Candela and J. Pedra, “Parameter Estimation of a Transformer With Saturation Using Inrush Measurements”, Electric Power System Research No 79, pp, 417-425, 2009.

[4] R. Wolf, *Ispitivanje električnih strojeva* (in Croatian), Zagreb, Sveučilište u Zagrebu, 1964.

[5] J. Hindmarsh, *Electrical Machines and Their Applications*, Oxford, Pergamon Press, 1977.

[6] G. Slemon, A. Straughen, *Electric Machines*, Reading, Massachusetts, Adison-Wesley Publishing Company, 1982.

[7] A. Fitzgerald, C. Kingsley, S. Umans, *Electric Machinery*, Tokyo, McGraw-Hill International Book Company, 1983.