The Effects of Multiple Reflection in Conducted RF Measurements

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Abstract – **During conducted measurements the reflection is one of the most varyingcomponents of the measurement uncertainty. For studying the cable reflections, in this paper we make a simple computer model and compare its output with measured results.**

Keywords– **Reflection, frequency dependency of velocity factor, measurement, measurement uncertainty.**

I. INTRODUCTION

Well known formula [1] for the solution of telegrapher's equations is

$$v(x,t) = |A| \cdot \cos(\omega \cdot t - \beta \cdot x) \cdot e^{-\alpha \cdot x} + |B| \cdot \cos(\omega \cdot t + \beta \cdot x) \cdot e^{\alpha \cdot x}, \quad (1)$$

Here,A and B are the complex amplitudes of the forward and reflected waves in the transmission line, andα, $\beta$are the real and imaginary parts of the propagation constant

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}. \quad (2)$$

In this case R, L, G and C are the per-unit-length parameters of the transmission line.

The velocity of the waves in the guide also depends on the cable parameters as

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{L \cdot C}} = \frac{1}{\sqrt{\varepsilon \cdot \mu}} = \frac{c}{\sqrt{\varepsilon_r \cdot \mu_r}}. \quad (3)$$

Well known is the effect of the reflection if the load impedance ($Z_L$in Fig.1) is not matched with waveguide impedance (Z_0 in Fig.1).The complex voltage reflection coefficient (Γ) is

$$\Gamma = \frac{B}{A} = \frac{Z_L - Z_0}{Z_L + Z_0}, \quad (4)$$

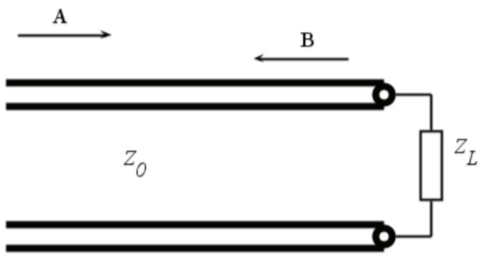

Fig. 1. Reflection at load

The characteristic impedance of the transmission line is expressed as [2]

$$Z_0 = \frac{V^+}{I^+} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}. \quad (5)$$

In case of conducted power measurements the reflection causes uncertainty. In this article we made a simple model to study the effect of multiple reflections.

The first step in developing a simulation model of the studied system is to know the precise properties of the measured transmission lines.One of the most important property of the reflection simulation is the β propagation constant of the used cables.

II. STUDY OF PROPAGATION VELOCITY IN COAXIAL CABLES

As Eq. (1) shows, the waves in the lossless transmission lines havea periodicity in time and space in case of sinusoidal excitation.

$$v(x,t) = V\left(0, t - \frac{x}{v}\right). \quad (6)$$

Measuring the propagation velocity a short circuited transmission line can be carried out as Fig. 2 shows.

The first attenuation (S_{21}) maximum belongs to the resonance frequency of the L long cable under test. The velocity factor in the cable under test is

$$Vf = \frac{\lambda_c}{\lambda_0} = \frac{2 \cdot L}{\lambda_0} = \frac{2 \cdot L \cdot f}{c}, \quad (6)$$

where f is the resonance frequency of the cable under test, and c is the speed of the waves in free space 299 792 458 m/sas Eq. (3) sows.

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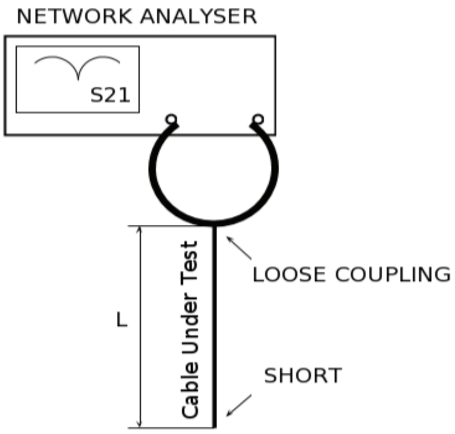

Fig. 2. Measurement setup for propagation velocity

Generally the observed frequency is determined by the length of the cable under test (*L*). Instead of applying different lengths of cables higher harmonics of the resonance can be used for determining the frequency dependence of the velocity factor and thus the dielectric constant.

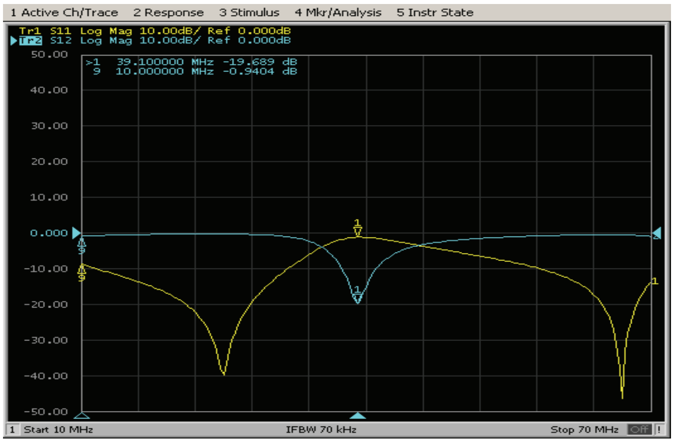

Fig. 3. The 1st resonance of the CUT (1) at 39.1000 MHz, with cable length 11.290 m. In the picture the minimum of the S_{12}can be seen at MARKER 1.

Unfortunately, for low frequency examination of the velocity factor this method needs long cable length*(L)*. The radiation of the open ended cablesintroduce more resonance disturbances, therefore it is worth to use two times longer short circuited cables for the precise measurements.

In this article the Cable Under Test (CUT) types are (1) Hirschmann KOKA 709 (75 Ω), and (2) H155 (50 Ω) low loss coaxial cables. The Fig. 3 shows the velocity factor'sfrequency dependency. The frequency dependency of ε can be calculated from Eq. (3), too.

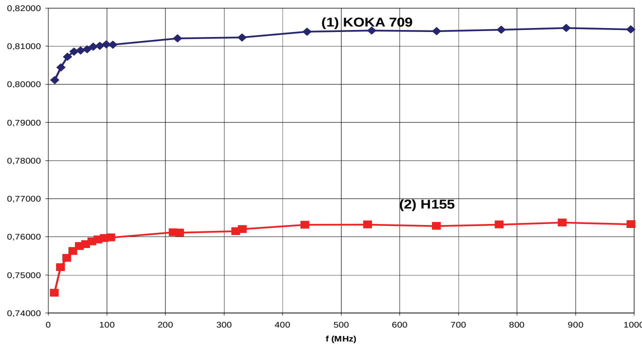

Fig. 4. Frequency dependency of velocity factor of (1) KOKA 709 and (2) H155 coaxial cables

wavelength variability have to be taken care of, at the same time. Therefore the used algorithm is

(1) seek the lowest resonance frequency (f_1),
(2) seek the next resonance frequency (f_2) near $2 \cdot f_1$,
(3) seek the n^{th} resonance frequency (f_n) near $n \cdot f_{n-1}/(n-1)$.

By using automated measurement control the algorithm can followed easily, but at manually measured values, if you want to measure at near discrete frequencies, it is difficult to identify the order number of resonances because of the frequency dependency of the cable parameters.

As Fig. 2 shows for the test we have to use loose coupling between the CUT and the measuring loop for decreasing the unwanted impedance transformation into the measured transmission line. In this case the minimumof S12 can be smaller as Fig. 3 shows.

As Fig 4 shows, from 100 MHz to 1000 MHz the variability of velocity factor is not dominant, therefore in the next model we use constant instead of it.

III. Model Elements for Multiple Reflecton of Transmission Line

A. Generator

The RF generator can be represented by itsoutput impedance (Z_g), and output voltage (V_g), at the nececcary frequency, of course. In practice V_g is calculable from the output power and Z_g, if it is matched. If the load impedance of the generator varies a lot, instead of the output power the *emf* (electromotive force) value should be used, witch is the output voltage of a generator without any load. This value is two times higher then the matched case.

In our model the generator output voltage is

$$V_g(t) = V_0 \cdot \cos(\omega \cdot t)\,. \qquad (7)$$

Instead, by introducing$\boldsymbol{A_g}$ as forward complex peak amplitude, the generator voltage can be expressed as

$$V_g(t) = real(A_g \cdot e^{j\omega t})\,, \qquad (8)$$

B. Transmission line

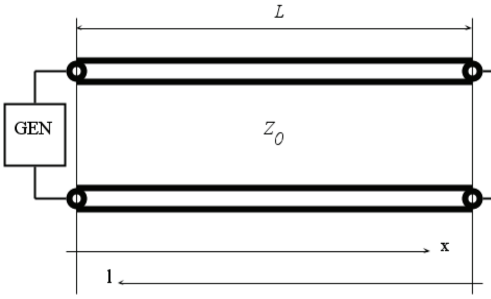

Fig. 5. Transmission line model

The $A_g$forward wavein the line at place $x$is

$$A(x) = A_g \cdot e^{-\alpha x} \cdot e^{-j\beta x}, \quad (9)$$

and

$$A(x,t) = A_g \cdot e^{-\alpha \cdot x} \cdot e^{-j \cdot \beta \cdot x} \cdot e^{j \cdot \omega \cdot t}, \quad (10)$$

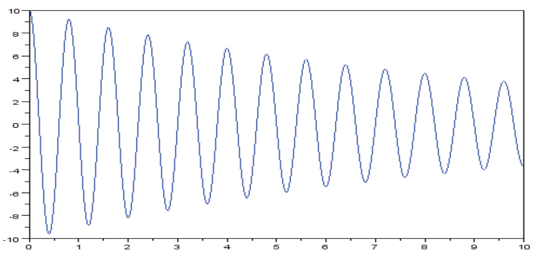

Fig. 6. The *real(A(x,t))* voltage in the transmission line (L=10m, A_g=10V, t=0, α=0.1, f=300MHz)

Observing the voltage of the transmission line in one period, its form can be seen in Fig. 7.

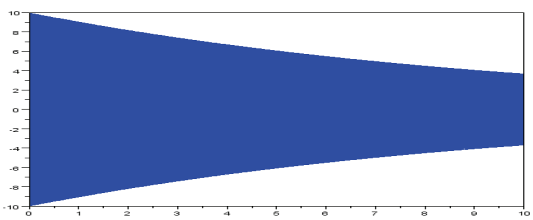

Fig. 7. Voltage in the transmission line (L=10m, A_g=10V, t=0…T, α=0.1, f=300MHz)

The B forward wave in the line at place $x$is

$$B(x,t) = B_0 \cdot e^{-\alpha l} \cdot e^{-j\beta l} \cdot e^{j \cdot \omega \cdot t} = \left[A_g \cdot e^{-\alpha \cdot L} \cdot e^{-j \cdot \beta \cdot L} \cdot \Gamma\right] \cdot e^{(x-L) \cdot \alpha} \cdot e^{j \cdot (x-L) \cdot \beta} \cdot e^{j \cdot \omega \cdot t} \quad (11)$$

The forward and reflected voltage at t=0 is shown in Fig. 8. forΓ= −1, and inFig. 9. forΓ= +1.

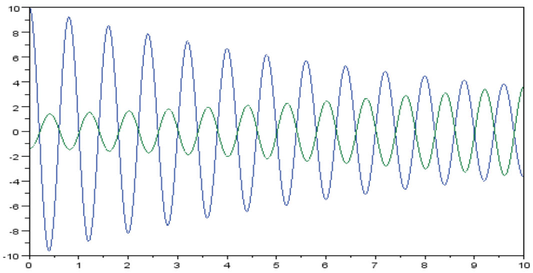

Fig. 8. Reflected voltage in the transmission line as a function of x(L=10m, Γ= −1)

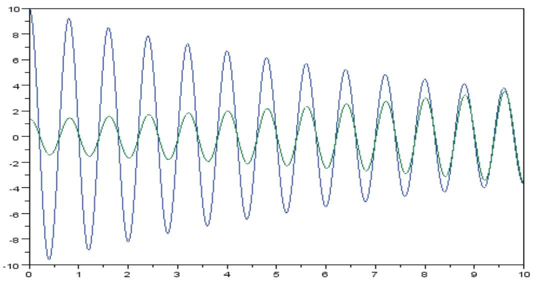

Fig. 9. Reflected voltage in the transmission line as a function of the distance x (L=10m, Γ= +1)

The voltage in the transmission line can be got by

$$V(x) = real(A(x) + B(x)). \quad (12)$$

The voltage shape in the transmission line for one period with Z_L=0 and with Z_L=∞can be seen in Figs. 10 and 11.

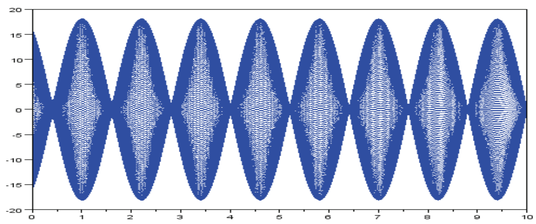

Fig. 10. Wave in the loss transmission line as a function of x (L=10m, Γ= −1, f=100 MHz)

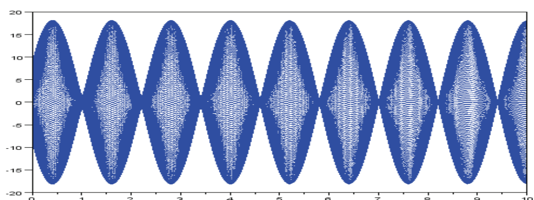

Fig. 11. Wavew in the loss transmission line as a function of x (L=10m, Γ= +1, f=100 MHz)

The difference depends on the load, as it can be seen at the above plots at the cable ends (x=10 m).

C. Transmission line steps

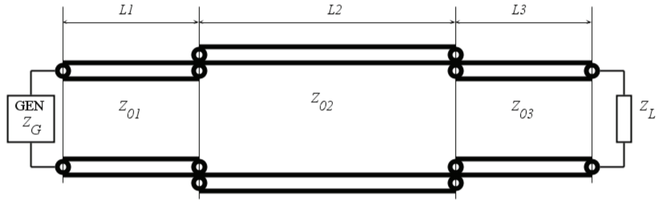

Fig. 12. Cascade coupled transmission lines

The input impedance of the 3rd transmission line in general case is [3]

$$Z_{in} = Z_0 \cdot \frac{Z_L + Z_0 \cdot th(\gamma L)}{Z_0 + Z_L \cdot th(\gamma L)}. \quad (13)$$

In case of lossless transmission lines, where α=0

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot tg(\beta L)}{Z_L + j \cdot Z_0 \cdot tg(\beta L)}. \quad (14)$$

The reflection coefficient at step $Z_{01} \rightarrow Z_{02}$ is Γ_{12} from Eqs. (4) and(13). In this simple model the Z_g=Z_{01}, Z_{03}=Z_L, therefore there are no reflections at the generator and at the load.

The input voltage of the 2nd line must be equal to the voltage of the 1st line at the end. (It depends on the Γ_{12}.) At the end of 1st line the voltage is V_{2IN}

$$V_2(x_2 = 0) = real(A(x_1 = L_1) + B(x_1 = L_1)). \quad (15)$$

$$B = \Gamma \cdot A. \quad (16)$$

Therefore

$$A_2(0) = A_1(L_1) \cdot [1 + \Gamma_{12}]. \quad (17)$$

At step$Z_{02} \rightarrow Z_{03}$ the reflection is Γ_{23}, therefore the reflected value is

$$B_2(L_2) = A_2(L_2) \cdot \Gamma_{23}. \quad (18)$$

This reflected wave is reflecting at step$Z_{02} \rightarrow Z_{01}$

$$A_{2[1]}(0) = B_2(0) \cdot \Gamma_{21}. \quad (19)$$

$$A_{2*}(0) = A_2(0) + \sum_{i=1}^{\infty} A_{2[i]}. \quad (20)$$

IV. Conclusion

Generally the higher order reflections are not dominant, i.e.,

$$A_{2*}(0) \sim A_2(0) + \sum_{i=1}^{\infty} (\Gamma_{23} \cdot \Gamma_{21})^i. \quad (21)$$

The phases of the 1st, 2nd etc. reflected waves will be the same. The simulation result by SciLab [4] can be seen in Fig. 13.

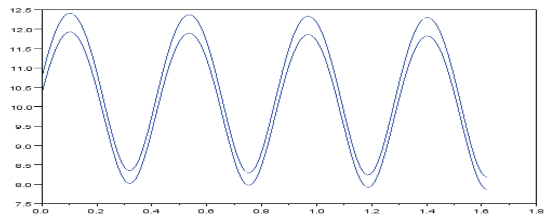

Fig. 13. A_{20}+B_{20}(lower curve), and A_{2*}+B_{2*}(upper curve) in the transmission line(L=1.62m, α=0.01, Z_G=Z_{01}=Z_{03}=50Ω, Z_{02}=75Ω)

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