Investigation into Filter with Hausdorff's Weighted Window Function Designed for Wideband Channels

Borislav Naydenov[1], Ginka Marinova[2] and Valentina Markova[3]

Abstract – **In this paper the results received from investigation into non-recursive digital filter are presented. Hausdorff's weighted window is used as well known Kaiser window. For the weighted window is applied expression representing the delta function approximation to algebraic polynomial with Hausdorff's values of the coefficients. Correlated interferences observed in UWB cannels are used.**

Keywords – **non-recursive digital filter, WB channel, Hausdorff's window**

I. Introduction

Digital filters with the finite impulse response (FIR) are discrete systems with one input and one output and constant in time parameters. They are characterized by strictly linear phase characteristic. This determines their wider use in practical implementations compared to filters with infinite impulse responce (IIR). Transfer function of the physical realizable FIR filter has the following form[2]:

$$H(z) = \sum_{n=0}^{N-1} h(n).z^{-n} \ , \qquad (1)$$

where $H(z)$ is a polynomial of z^{-1} of degree $N-1$. Thus, $H(z)$ has $N-1$ zeros, which can be located arbitrarily in the final z-plane. Poles are $N-1$ and are located in the central point z = 0. The frequency response is trigonometric polynomial

$$H(e^{iw}) = \sum_{n=0}^{N-1} h(n)e^{-iwn} \ . \qquad (2)$$

The design of FIR filters can be accomplished either by finding the coefficients of the pulse characteristics, or the determination of N samples of the frequency response. The basic approach in defining filters FIR is by crossing his infinite length pulse characteristic and thus obtain the impulse characteristic with the final length. Assuming that $H_d(e^{iw})$ is the ideal frequency response characteristic, then the corresponding sequences of the samples of the pulse characteristic has the form:

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{i\omega})e^{i\omega n}d\omega . \qquad (3)$$

In most cases, the ideal frequency response characteristic $H_d(e^{iw})$ of frequency selective filter is constant for different parts of the passband and stopband, with a break at the endpoints between them. The interruption determines the sequence of pulse characteristics, which have infinite lengths $h_d(n)$ and requires it to be truncated to obtain physically realizable filters. Periodic frequency characteristic in (3) can be viewed as presented by means of Fourier series, where pulse sequence $h_d(n)$ acts as Fourier coefficients. Intersection of the ideal impulse characteristic is equivalent to to study convergence of Fourier sequence. This convergence in the theory is presented as Gibbs phenomenon.

The errors within the passband and stopband are specified as δp and δs. The frequency response is allowed to fluctuate both positively and negatively within these error limits. We can translate these specifications into the decibel gain by using (4) and (5), [3].

$$a_{pass} = 20\log(1-\delta_p) \qquad (4)$$

$$a_{stop} = 20\log(\delta_s) \, , \qquad (5)$$

where δ_s and δ_p are the error coefficients in stopband and passband.

II. Using Windows to Obtain FIR Filters

Physically realizable FIR filter is obtained by restricting the ideal pulse characterization [2]. From a mathematical point of view, limiting the ideal pulse characteristic is equivalent to multiplication with a weight function:

$$w(n) = \begin{cases} 1, & 0 \le n \le N-1 \\ 0 \end{cases} . \qquad (4).$$

The actual pulse characteristics are:

[1]Borislav Naydenov is with the Faculty of Electronic, Technical University - "Studentska" 1, 9010 Varna, Bulgaria, E-mail: borna@abv.bg.
[2]Ginka Marinova is with the Faculty of Computing and Automation, Technical University - "Studentska" 1, Varna, Bulgaria, E-mail: gin_kaleva@abv.bg.,
[3]Valentina Markova is with the Faculty of Electronics, Technical University – "Studentska" 1, 9010 Varna, Bulgaria, E-mail: valliq@abv.bg.

$$h(n) = h_d(n).w(n)\,. \tag{5}$$

Transfer function of the actual filter is obtained by convolution between the transfer function of the ideal filter and frequency response of the window.

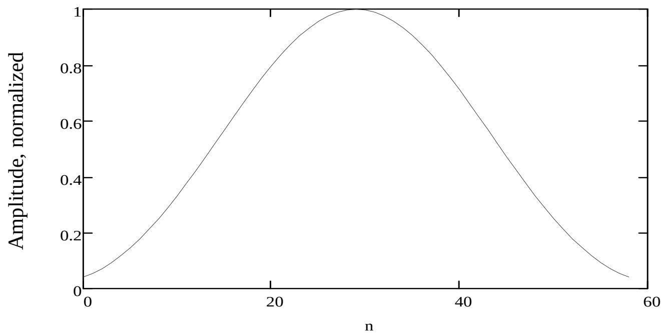

Fig. 1. Kaiser window

The frequency response of the window should be as narrow as to be able to reproduce precisely specified frequency response. With increasing the number of elements N of the transfer function width of the main leaf and the side leaf of the frequency responses of the window is decreased. By this reason, their amplitudes increased, as the area under these main and side leaves remains the same. This leads to larger fluctuations in the frequency response of the synthesis filter. In the theory of Fourier series is known Gibbs phenomenon, which defines this uneven convergence. It can be reduced by using a smooth intersection of Fourier series. With the gradual shrinking of the window to zero on each side, can reduce the height of the side leaf, which obtain by increasing the width of the main leaf and hence a wider zone at the point of sharp transition.

In practice, a variety of weight functions, which in most cases have names of their discoverers, is using. Kaiser offers a family of weight functions using modified Bessel function of first kind and zero order [2]

$$\omega(n) = \frac{I_0\left[\omega_a\sqrt{\left(\frac{N-1}{2}\right)^2 - \left[n - \left(\frac{N-1}{2}\right)\right]^2}\right]}{I_0\left[\omega_a\left(\frac{N-1}{2}\right)\right]}\,. \tag{6}$$

They obtained the greatest power in the main leaf when is set the amplitude of side leaves. This property makes Kaiser window (KW) nearly optimal. The Kaiser Window function is shown on Fig. 1 - $\omega_a = 5.605$.

III. Application of Hausdorff's Weight Function

In scientific publication [5] is proposed a weight function with Hausdorff's metrics. Weight function is obtained by approximation of a delta function with algebraic polynomial (7) [6], that performs the best approximation in Hausdorff's metrics in the interval [-1, 1].

$$P_m(x) = \varepsilon T_m\left(\frac{2x^2 - 1 - \alpha^2\varepsilon^2}{1 - \alpha^2\varepsilon^2}\right), \tag{7}$$

where ε means Hausdorff's distance; T_m is polynomial of Chebishev of first order and degree m; α - parameter, $\alpha\varepsilon$ - value, which determins the width of the function in the main maximum. Dependencies between the parameters of the polynomial are determined by expression (8):

$$\alpha\varepsilon = \sqrt{\frac{ch\left[\frac{1}{m}Ach\left(\frac{1}{\varepsilon}\right)\right] - 1}{ch\left[\frac{1}{m}Ach\left(\frac{1}{\varepsilon}\right)\right] + 1}}\,. \tag{8}$$

Weight function is obtained by Hausdorff's polynomial translation in a positive direction with a value one, definitional field is reduced to field of main maximum $\left[1 - \alpha\varepsilon, 1 + \alpha\varepsilon\right]$ and raise of degree 1.27 [5]:

$$w_m(x) = \left\{\varepsilon\cos\left[m\arccos\left|\frac{2(\alpha\varepsilon x - \alpha\varepsilon)^2 - 1 - \alpha^2\varepsilon^2}{1 - \alpha^2\varepsilon^2}\right|\right]\right\}^{1,27}. \tag{9}$$

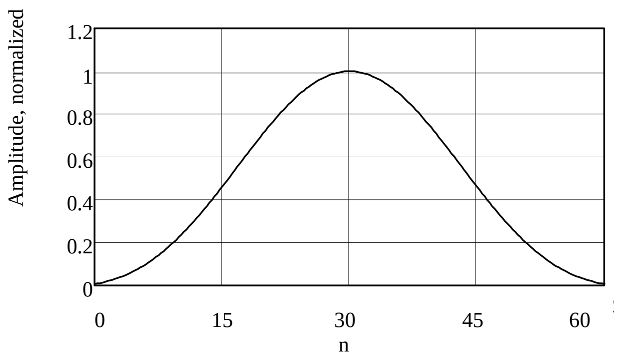

Fig.2. Hausdorff's weight function, attenuation a = 60 dB, Hausdorff's distance ε = 0.021 and parameter α = 1.693

With a mark attenuation in stopband expressed in decibels. Hausdorff's distance is determined by the dependencies:

When *a<24 dB*

$$\varepsilon = 1\,. \tag{10}$$

When *24 dB≤a<50 dB*

$$\varepsilon = \frac{0.66}{\left(2.7e - 5a^2 - 8e - 4a + 1.073\right)^{a-25}}\,. \tag{11}$$

When *50 dB<a≤130 dB* (Fig. 2)

$$\varepsilon = \frac{0.66}{1.1035^{a-25}}\,. \tag{12}$$

When *a>130 dB*

$$\varepsilon = \frac{0.66}{(0.0001a+1.09)^{a-25}} . \qquad (13)$$

It is appropriate the degree of Hausdorff's polynomial m to be equivalented to N.

IV. Results from the Investigation into Digital Filter with FIR using Weighted Window Described by Hausdorff's Function and Kaiser Window

For the investigation into digital filter with FIR is used models of correlated interference, obtained by the avtoregresion rows. The coefficients of avtoregersion sequence are selected so as to dominate the low frequency component of interference (Fig. 3).

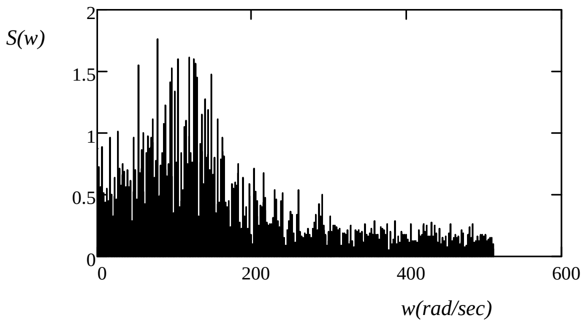

Fig.3. Spectrum of correlated interference

There are several approaches in the synthesis of filters. In investigation in this publication is set ideal impulse characteristic of the type $h_d(n) = \frac{\sin[\omega_c(n-\kappa)]}{\pi(n-\kappa)}$ [7]: where ω_c - cut-off frequency, $\kappa = \frac{N-1}{2}$ and N - order of the filter. Real pulse characteristic is obtained by expression (5). Transfer function of the filter is obtained by convolution between the transfer function of the ideal filter and frequency response of the window. When is used the rectangular window is received non satisfactory results in approximation of frequency response characteristic in the transitional area. In the stopband is observed large ripple of response. When is applied the filter processing with a rectangular window of the investigated correlated interference we received ripples expressed by (5) - a_s=7 *dB* and attenuation about 17 dB.

In these studies used lowpass filter with Kaiser window and highpass filter with Hausdorff's window (HW). For lowpass filter ω_c = 90 Hz. For the highpass filter ω_c = = 220 Hz.

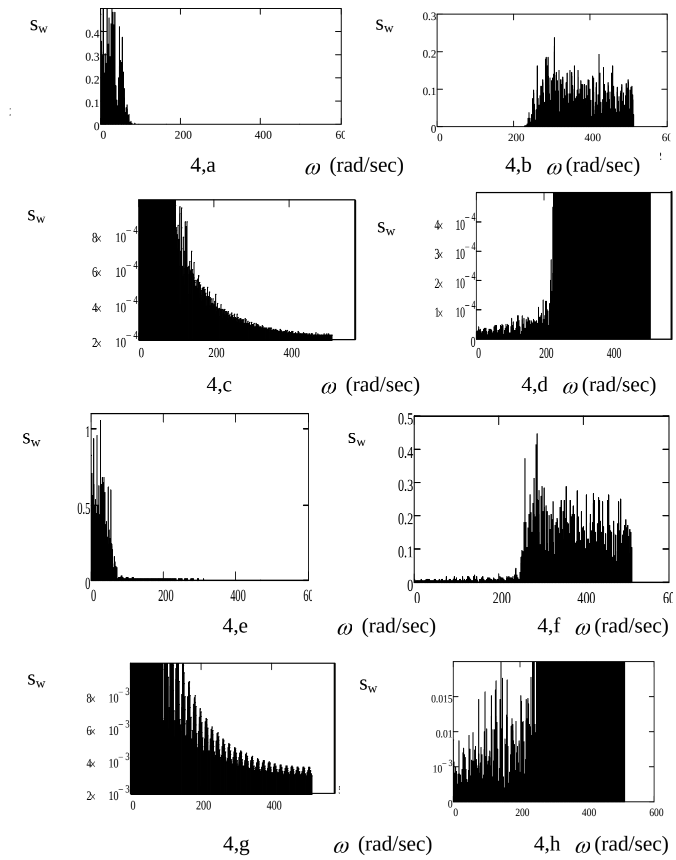

Fig. 4. Output signals of filter with KW (4a, 4c, 4e, 4g) and HW (4b, 4d, 4f, 4h). The input signals are processed with KW (4a, 4c), HW (4b, 4d) and rect. window (4e, 4f, 4g, 4h)

V. Conclusion

When used KW (Fig.4c) in stopband occurs gradually increasing of the attenuation, as the maximum attenuation is 80 dB. When using the HW (Fig. 4d) there is less change in the stopband. Attenuation is about 95 dB, but there are more ripples. In HW processing transitional area is steeper (Fig.4).

The results confirm the conclusions made in the article [5].

References

[1] Z. Nikolova, V. Poulkov, G. Iliev, K. Egiazarian. New adaptive complex IIR filters and their application in OFDM systems. J. S., Image and Video Proc. Springer. V. 4, I. 2, 2010, pp. 197-207.

[2] A. Oppenheim, R. Schafer, Digital signal processing. Prentice hall, inc., Englewood cliffs, New Jersey, 1979.

[3] Les Thede, Practical Analog and Digital Filter Design, Artech House, Inc., 2004.

[4] Р. Иванов, Цифрова обработка на едномерни сигнали, Almamater international, Габрово, 1999 г.

[5] Peter Apostolov, Application of Hausdorff's Window Function by FIR Filters Synthesis, ICEST 2008, Serbia, Nis, 25-27 June.

[6] Бл. Сендов, Хаусдорфовые приближения. С., БАН, 1979.

[7] R. Bogdanov, B. Bedzhev, Z. Tasheva, B. Stoyanov, An Algorithm for Lossless Signal Processing, Proc. of the Third IEEE W-p on IDAACS 2005, IX, 5-7, 2005, Sofia, Bg, pp.498.