Performance Comparison of Chaotic and Classical Spreading Sequences

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Abstract – **In this paper theauto-correlation and BER performances of the chaotic sequencefor spectrum spreading are investigated by numerical simulations and compared with that of the pseudorandomM-sequence.**

Keywords– **spread spectrum communication, chaotic sequence, pseudorandom sequences, correlation properties.**

I. Introduction

In conventional spread spectrum (SS) communication systems pseudorandom(pseudo-noise(PN))signals are used for broadening thespectrum by modulating the phase (in direct-sequence (DS)), orthe frequency (in frequency hopping (FH)) of the carrier signal. The most popular pseudorandom spreading sequences are the maximal length sequences shift register (M- sequences) [1].

A different type of spreading sequence, that is used for broadening the spectrum, is the chaotic sequence [2]. Use of chaotic signals as spreading codes in a SS systems has been shown to be a promising way of applying chaos digital communication purposes [2, 3].

In this paper, the auto-correlation and BER performances of the chaotic sequence, belonging to the family of piecewise-affine Markov (PWAM) maps,are investigated by numerical simulations and compared with that of the well known pseudorandom spreading M- sequence.

II. Properties of Chaotic Sequence

Use of chaos in a SS communicationsystems consist in replacing the standard pseudo-noise generator by a chaotic dynamical system.

A chaotic dynamical system is an unpredictable, deterministic and uncorrelated system, that exhibits noise-like behaviour trough its sensitive dependence on its initial conditions.

A nonlineardynamical system model based on its *n*previousvalues can be described as [3]:

$$\boldsymbol{x}[n+1] = \boldsymbol{f}(\boldsymbol{x}[n],\lambda), \tag{1}$$

where

$$\mathbf{x}[n] = (\mathrm{x}_1[n], \mathrm{x}_2[n], \ldots\ldots,\mathrm{x}_\mathrm{m}[n]) \tag{2}$$

is the state,

$\boldsymbol{f} = (f_1, f_1, \ldots\ldots f_m)$ is the discrete functional transformation (maps) the state $\mathbf{x}[n]$ to the next state $\mathbf{x}[n+1]$,and $\lambda$is the bifurcating parameter.

Starting with an initial condition $\mathbf{x}[0]$, repeated applications of the map $\boldsymbol{f}$ give rise to the sequence of the points $\{\mathbf{x}[n]\}$.

A new sequence $\{\mathbf{y}[n]\}$ is obtained by mapping $\mathbf{x}[n]$ by a function $\boldsymbol{f_1}$, that is

$$\mathbf{y}[n] = \boldsymbol{f_1}(\boldsymbol{x}[n],\lambda). \tag{3}$$

So the chaotic sequences are non-converging and non-periodic sequences. A large number of these reproducible and random-like signals can be generated by changing initial value. Also the signals generated from chaotic dynamical system have broad power and flat spectrum in the frequency domain.

The correlation property of a spreading sequence plays an important role in the detection process

In this study, the family of (l, t)-tailed shifts with l even and $t < l/2$ [4] is employed to illustrate that the chaotic spreading sequence can achieve nearly optimal autocorrelation performance.The tailed shift map belongs to the family of piecewise-affine Markov (PWAM) maps [4].

(l, t)-tailed shifts areaffine in each of the intervals

$$x_j = \left[{(j-1)}/{l}, {j}/{l} \right]. \tag{4}$$

Theintervals from x_1 to x_{l-t} are mapped onto $x_{t+1} \ldots x_l$ while the last t intervals aremapped onto $x_1 \ldots x_t$.

Consider a function $\boldsymbol{f}$: [0,1] →[0,1] iterated starting from an initialcondition $\mathbf{x}[0]$ uniformly distributed in [0,1] to produce the sequence (1).Thissequence is quantized by the bipolar threshold function

$$\boldsymbol{f_1}: [0,1] \to [1,-1] \tag{5}$$

centered at $1/2$ and the spreading sequences are taken to be $\mathbf{y}[n] = \boldsymbol{f_1}(\boldsymbol{x}[n])$ for $n = 0, \ldots, N-1$.

The auto-correlation function of the tailed shift map can be written as [4]:

$$R_n = \frac{1}{l} \sum_{i=1}^{l} \sum_{j=1}^{l} f_1(x_i) f_1(x_j) \boldsymbol{K}^{\boldsymbol{n}}_{\boldsymbol{ij}}, \tag{6}$$

where $\boldsymbol{f_1}(\boldsymbol{x}_j)$ indicates the value of $\boldsymbol{f_1}$ for all thepoints in $\boldsymbol{x}_j$;

$$\boldsymbol{K}^{\boldsymbol{n}}_{\boldsymbol{ij}} = \frac{1}{l} \begin{bmatrix} 1-h^{n-1} & 1-h^{n-2} \\ 1-h^{n} & 1-h^{n-1} \end{bmatrix}; \tag{7}$$

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$$h = -\frac{t}{l-t}. \quad (8)$$

Figure 1shows the auto-correlation of a chaotic sequencefor *l*=4 , *t*=1 ((4,1)- tailed shift).

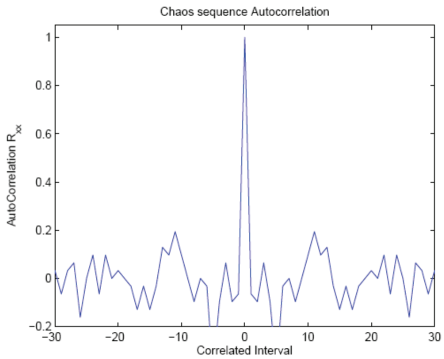

Fig.1. The auto-correlation of the chaotic sequence

III. PERFORMANCES COMPARISON OF CHAOTIC AND M- SEQUENCES

There are several properties of chaotic sequences that make them superior to conventional PN sequences.

Firstly PN sequences are generated by shift registers which are periodic in nature. Chaotic sequences on the other band are considered very secure because of its aperiodicity und unique property of high sensitivity to initial conditions.

Secondly, for a given M -stage linear feedback shift register, there is a limit to the maximum number of sequences that can be generated. So PN sequences are less in number and this limits the security. However, with the large number of chaotic maps available and the fact that changing the initial conditions, generates a completely new sequence, there are theoretically an infinite number of chaotic sequences that can be generated. Therefore the use of chaotic spreading sequence is more secure.

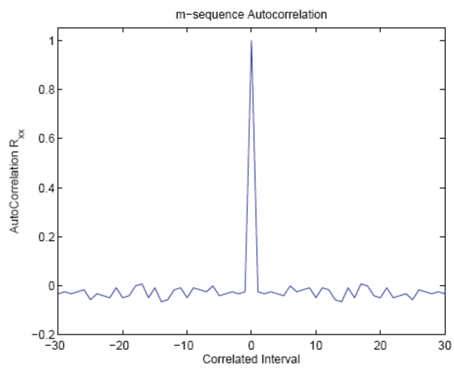

Fig.2. The auto-correlation of the M- sequence

We compare the auto-correlation properties of the chaotic sequence with that of the 31-bit M- sequence.Figure 2 shows the auto-correlation of a 31-bit M-sequence. The auto-correlation is given as N when the time lag is 0 and given as 1/N at all other times.In simulation it is found that the correlation bounds of the non-zero lags of the auto-correlation are about 0.21 and -0.24 for chaotic sequence. For M-sequence they are about 0 and (-0.24).

The bit error rate (BER) is one of the best performance measures in comparingdifferent communication systems. Following is a comparison of the BERs of conventional31-bitM-sequence to chaotic spreading sequence.Figure 3shows the performance of these sequences in AWGN Channel.

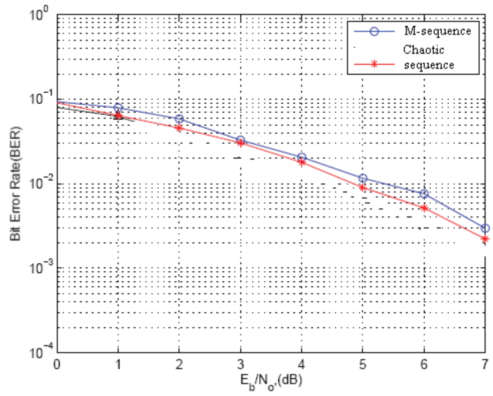

Fig.3. The BER results of using different spreading sequences

The chaotic sequenceseems toperform the best, followed by M-sequence.There seems tobe only about 0.5dB difference between these sequences.

IV. CONCLUSION

According to the analysis of the auto-correlation performance, it is found that the chaotic spreading sequences generated by PWAM mapshave a better auto-correlation performance than classical spreading M-sequences.

Besides this, the use of chaotic spreading sequences has some otheradvantages in the application of SS communicationsystems to classical sequences.One advantage is the availability of an enormous numberof different sequences of a given length as compared to the maximal length sequences. Generation and regeneration of chaotic sequences is very simple and involvesthe storage of only a few parameters and functions even for very long sequences. The inherent aperiodic and sensitive initial conditions features in chaotic sequences are definitelyproperties that can be used to make a system more secure.

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