Comparison Between Steepest Ascent and Genetic Algorithm Optimization Methods in Series Based Software Direct Digital Synthesis of Signals

Mariana Shotova[1]

Abstract – **In this paper an optimization of the software method for Direct Digital Synthesis of signals, based on series approximation of sine wave is discussed. A 5th order polynomial is investigated and an optimization of the polynomial approximation is proposed and discussed. Two methods for spectral optimization, aimed at the reduction of the spurs' level, are compared – steepest ascent and genetic algorithm optimization. An increase of the dynamic range is achieved.**

Keywords – **DDS, series, polynomial, optimization, GAO, steepest ascent.**

I. Introduction

The Direct Digital Synthesis (DDS) is a technique for generating a high quality sine wave through a digitally defined frequency. The software implementation of DDS (SDDS) based on digital signal processor has two main versions – using ROM table of the sine wave, and series approximation of the sine wave. Measure of quality is the spurious free dynamic range D of the spectrum of the synthesized signal[1].

The advantages of the SDDS exploiting series approximation are: elimination of the ROM table, and better dynamic range. A drawback is the bigger number of the required mathematical operations, which results in lower sampling frequency.

There are several basic polynomial approximations which can be used in DDS [2,3,4].

The SDDS with 5^{th} order polynomial is investigated here. By taking advantage of the sinus' symmetry – using approximation in the range [-π/2, π/2], the even-order components are eliminated. The ***polyfit*** approximation, normally used, is based on minimization of the root-mean-square deviation, while in DDS case it is important to minimize the spectral spurs. Therefore two methods for spectral optimization, aimed at reduction of the spur' levels, are compared.

The first one is based on the steepest ascent method [5] – it searches for a global maximum over a 3 dimensional area by alternating two coefficient of a fifth order polynomial. The application of steepest ascent to SDDS results in several sets of coefficient of the suggested polynomial, which increase the dynamic range.

A comparison between the steepest ascent and the genetic algorithm optimization(GAO) methods[6,7,8] is made. While the steepest ascent method provides quick search over the parameter space, GAO deals with the individuals in a population over several generations, and thus the time for searching the fittest coefficients increases. GAO is expected to find the global maximum even in the case of several extremes, at the cost of the bigger number of iterations and calculations.

[1]Mariana Iv. Shotova is with the Faculty of Electronics, Technical University of Varna, 9010, Varna, Bulgaria, E-mail: marianashotova@gmail.com

II. Polynomial approximation of 5^{TH} order

*A. Polynomial approximation with **polyfit***

A polynomial approximation of 5^{th} order may be represented by the following equation:

$$\sin(\alpha) = a_1 - a_3\alpha^3 + a_5\alpha^5 , \qquad (1)$$

where a_i, i=1,3,5, are the coefficients of the sinus approximation. The range of the argument is [-π/2, π/2] and thus the even-order components of the polynomial are eliminated. The computation of the polynomial coefficients is implemented by MATLAB's *polyfit* function, which minimizes the root-mean-square error. Thus for the polynomial (1) the coefficients are : a_1= 0.99977007, a_3= = -0.16582379 and a_5= 0.00757279.

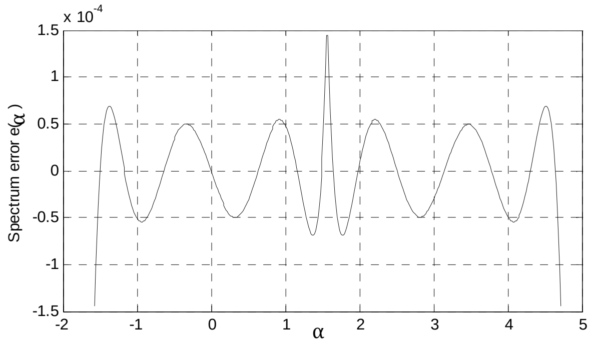

Fig.1 Sine wave error with 5^{th} order polynomial

A plot of calculation error $e_{(\alpha)}$ over the range [-π/2, 3·π/2] is shown in Fig.1.

The rearranged form of the series, closer to DSP is:

$$\sin(\alpha) = \alpha(\mathrm{a1} + \alpha^2(\mathrm{a3} + \alpha^2\mathrm{a5})) . \qquad (2)$$

The argument α^2 is calculated in advance. The number of mathematical operations here is 3 multiplications and 2 addition/subtractions.

In the case of DDS the error spectrum is of interest (Fig.2). An error signal, which contains 16 periods of error "wave" $e_{(\alpha)}$ is composed, and FFT is applied. Since the amplitude of the synthesized signal is A = 1, (0 dB), the SFDR is defined by the level of the fifth harmonic at k = 81, $D = -L_5$ = 88.6dB.

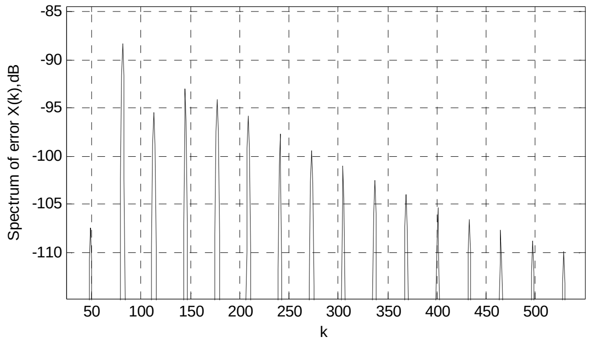

Fig.2. Error spectrum with 5th order polynomial.

B. Reducing the number of mathematical equations

As the first coefficient a_1 from (1) is very close to 1, it could be rounded. The other two coefficients a_3 and a_5 remain the same. Thus the number of mathematical operations is reduced to 2 multiplications and 2 addition/subtractions.

$$\sin(\alpha) = 1 - a_3\alpha^3 + a_5\alpha^5 \ . \tag{3}$$

The calculation error $e_{(\alpha)}$ for polynomial (3) over the range [-π/2, 3·π/2] is shown in Fig.3.

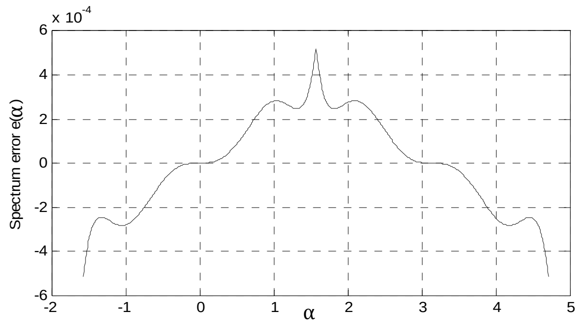

Fig.3 Sine wave error with 5th order polynomial

The spectrum of the obtained error $e_{(\alpha)}$ is depicted in Fig.4. The SFDR is defined by the level of the third harmonic at k = =49, $D = -L_3$ = 88.7dB.

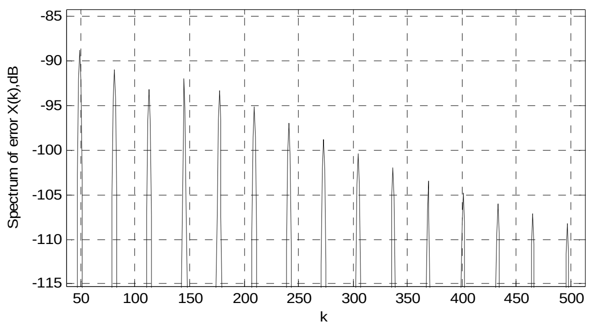

Fig.4. Error spectrum with 5th order polynomial.

Rounding the first coefficient a_1 doesn't affect the SFDR of the synthesized signal. Furthermore the number of mathematical operations is reduced without increasing the amplitude of the spectral spurs.

III. AN OPTIMIZATION ALGORITHM BASED ON STEEPEST ASCENT METHOD

A drawback of the ***polyfit*** function is that it is based on minimization of the root-mean-square error. In DDS it is important to minimize the spectral spurs. Therefore an algorithm aiming at the minimization of the spectral spurs based on steepest ascent/descent method is presented.

It searches for a global maximum over a two dimensional area by alternating two coefficient of a fifth order polynomial (3) with a step size alpha adjusted so that the function value is maximized along the direction by line search technique.

The goal is to find the best coefficients a3 and a5 at which the highest SFDR is achieved. The search area is shown in Fig.5. It is obtained by combining the two coefficients a3 and a5 and the resulting SFDR is given. The area is a ridge (Fig.6) with one global maximum which is the goal. To find the coefficient at which the maximum SFDR is achieved, an algorithm based on steepest ascent/descent method is presented.

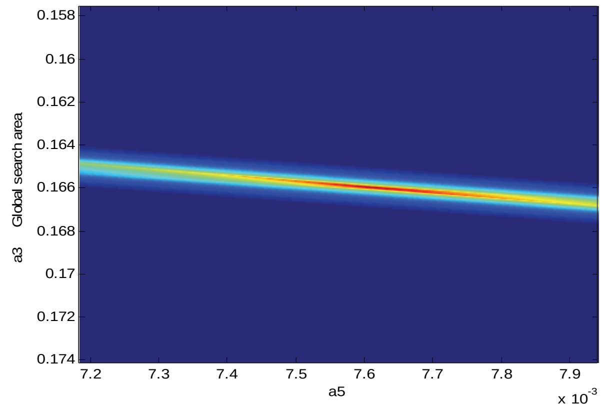

Fig.5. Search area of the steepest ascent method

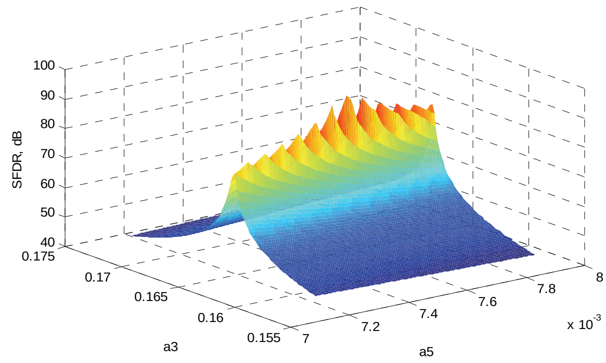

Fig.6. The search area of the steepest ascent method in 3D.

The algorithm is presented in Fig.7, where at the start two random coefficient a3, a5 and step sizes *alpha3* and *alpha5* respectively are defined and then the SFDR is calculated. For finding the ascending direction the two coefficient a3 and a5 are changed with the defined step sizes *alhpa3* and *alpha5* (addition). First the SFDR of the old coefficient a_3 and the new coefficient a_5 is calculated, then the SFDR of the new coefficient a_3 and the old coefficient a_5. After that the 3 obtained SFDRs are compared and the difference d3 and d5 is calculated. The direction of ascending is defined by comparing the SFDR of the current cycle and the obtained SFDR of the cycle before – if the difference between the newly calculated SFRD and the old one (calculated in the cycle before) DD, is lower than the defined threshold, the direction for searching isn’t changed. Else the two step sizes are changed and the algorithm returns to the beginning. The algorithms ends when the difference DD is bigger than the defined value of the threshold. Then it is assumed that the maximum SFRD is achieved.

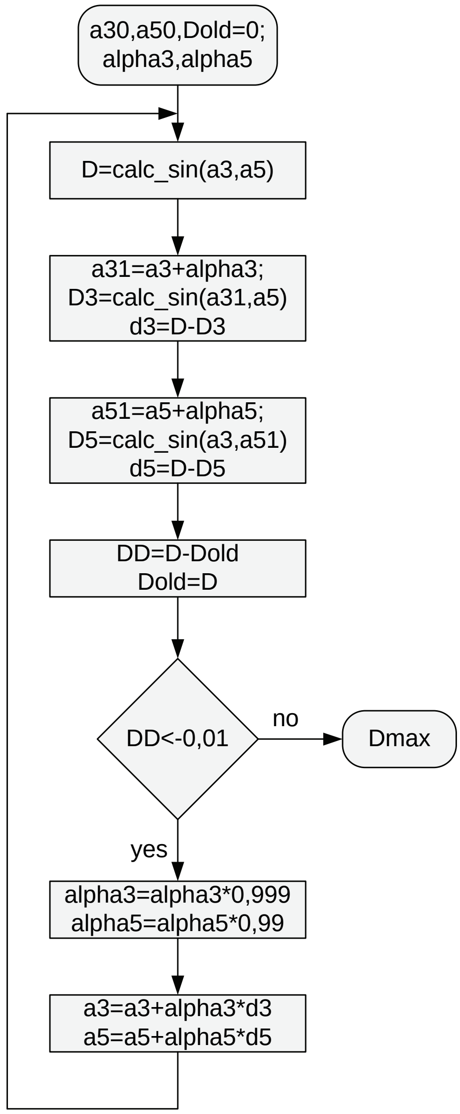

Fig.7 Block schema of the steepest ascent algorithm.

The path of steepest ascent algorithm is shown in Fig.7. The initial coefficients are a3=0.16564668, a5=0.00765671 and the calculated SFDR is D=68.1dB. The line search over the two dimentional area can be seen on fig.8. It can be seen that at the beginning of the search the start point in in the area with lower SFDRs. At the end of the algorithm, the last point defining the maximum SFDR is in the area with the highest values.

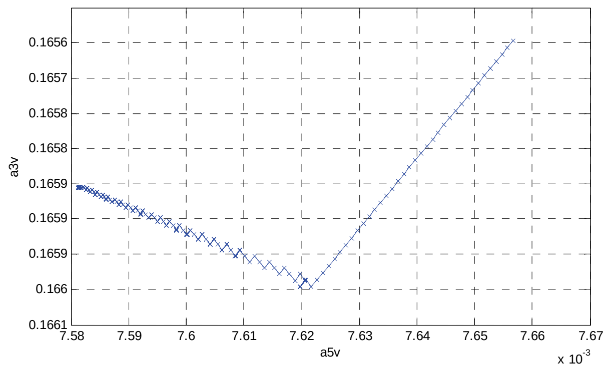

Fig.8. Path of steepest ascent algorithm.

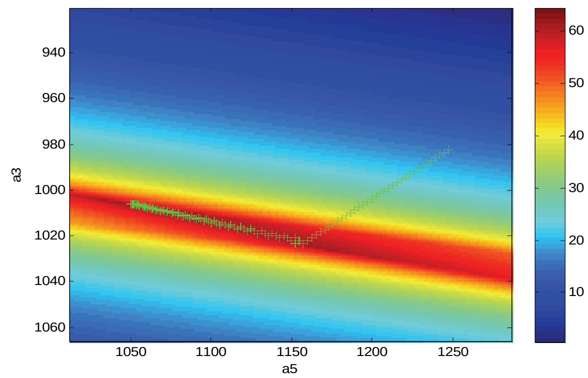

Fig.9. Path of steepest ascent over the search area.

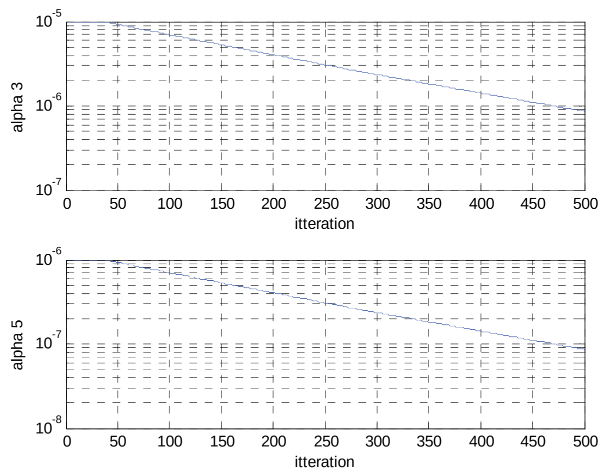

Fig.10 Change in the step sizes *alpha3* and *alpha5*.

Fig.10 represents the change in the step sized *alpha3* and *alpha5*. Because of the significants difference of the values of the two coefficient a3 and two different initial step sizes chould be assigned. In its initial state the steepest ascent technique is characterized by bigger step size and a constant direction. After approaching the area where the maximum is located, the step size begins to decrease; conversely the direction alternation is increased until the maximum is found.

The results using the inicial coefficient mentioned above over 500 iterations are – the new coefficients are

a_3=0.16585470, a_5= 0.00758128 and the resulting SFDR is D=91.3dB. Thus the SFDR of the polynomial ot 5^{th} order (2) is increased with approximately 3dB.

IV. Comparison between Steepest Ascent Method and GAO

A comparison between the two algorithms is made.

The steepest ascent algorithms searches for the minimum of an N-dimensional function in the direction of the positive increment with a step size $alpha3_k$ and $alpha5_k$ at iteration k adjusted so that function value is maximized along the direction by a line search technique. At the current simulations the search is made over a complex plane of $4*10^6$ points for 500 iterations, generating 3 new sets of polynomial coefficient at each iterations.

GAO is a directed random search technique modeled on the natural evolution/selection process toward the survival of the fittest. The genetic operators deal with the individuals in a population over several generations to improve their fitness. The individuals are compared to chromosomes and are represented by a string of binary numbers. The step size remains the same. At the current simulations the search is made over a plane of 2^{48} points for 100 iterations with 90 individuals. One individual corresponds to a set of two polynomial coefficients a_3 and a_5.

Three sets of coefficients for each algorithm ensuring SFDR of 91dB are presented in Table I.

Table I
Comparison between Steepest Ascent and GAO

Comparison	a3	a5	D[dB]
Steepest	0.16585281	0.00758103	91.2
Ascent	0.16585478	0.00758132	91.3
	0.16586387	0.00758484	91.4
GAO	0.16585968	0.00758353	91.4
	0.16585782	0.00758302	91.4
	0.16585784	0.00758302	91.4

The points are shown in Fig.11, where the SFDR obtained by the ***polyfit*** function is also presented with 'o'. The SFDR of the coefficients of steepest ascent algorithm is presented by '+' and the SFDR of the GAO coefficients is presented by '*'.

The marked points of the two algorithms are approximately close to one another. The difference in the SFDR is about 0.1dB. An exception is the SFDR obtained by the ***polyfit*** function where the SFDR differs with around 3 dB.

It can be seen that the steepest ascent is faster and also so accurate as the GAO. But there is a possibility if the algorithm comes across a local maximum to stop at it and not continue to search for the global one.

On the other hand the slower GAO can more accurately find the global maximum in a complex area with a lot of local extremes.

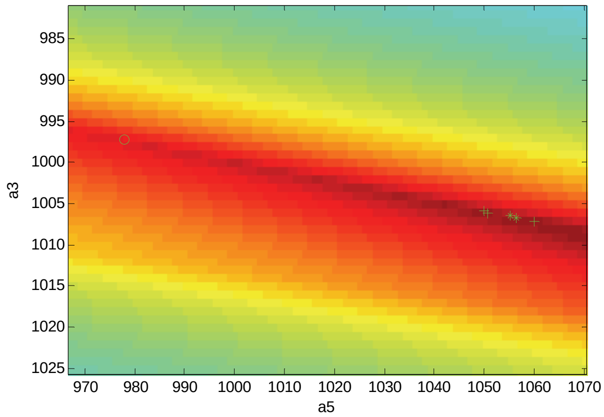

Fig.11. Positions of the resulted points on the search area

V. Conclusion

The optimization of a 5^{th} order polynomial approximation of the sine wave was considered. Taking into account the sinus odd symmetry and rounding the first coefficient of the polynomial the number of mathematical operations necessary for the computation of the sine wave is reduced to 2 multiplications and 2 addition subtractions.

A comparison between two optimization algorithms is made. The advantage of the steepest descent algorithm is the lesser search time. GAO is slower but more accurate than the first one. Thus the results for the both algorithms are increasing the SFDR of the synthesized signal with about 3dB.

References

[1] B. Boyanov, "*Single table direct digital synthesis of signals, based on dsPIC digital signal controllers*", II International Congress on MEEMI 2005, Varna, Bulgaria, 2005.

[2] M. Abramowitz and I. A. Stegun. *Handbook of mathematical functions with formulas, graphs and mathematical tables. Applied Math. Series* 55. National Bureau of Standards, Washington, D.C., 1964.

[3] J.-A. Pineiro, S. Oberman, J.-M. Muller, and J. Bruguera, "*High-Speed Function Approximation Using a Minimax Quadratic Interpolator*", IEEE Transactions on Computers, vol. 54, No. 3, March 2005, pp. 304-318.

[4] N. Brisebarre, J.-M. Muller, and A. Tisserand. "*Computing machine-efficient polynomial approximations*" ACM Trans. Mathematical Software, 32(2): 236–256, June 2006.

[5] S. W. Smith, "*The scientist and engineering guide to Digital Signal processing*",California Technical Publishing, 2002

[6] *Foundations of Genetic Algorithms 2*, Edited by L. Darrel Whittley, Morgan Kaufmann Publishers, San Mateo, CA, 1993.

[7] K. S. Tang, K. F. Man, S. Kwong and Q. He, "*Genetic algorithms and their Applications*", IEEE Signal Processing Magazine, vol. 13, No. 6, pp. 22-37, Nov. 1996.

[8] B. G. Boyanov, M. Iv. Shotova, "*Optimization of Polynomial Approximation in Series Based Software Direct Digital Synthesis of Signals*", ICEST 2010, Ohrid, Macedonia, pp191-194, June 2010.