Green's Function and Acoustic Standing Waves in Rectangular Loudspeaker Enclosures

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Abstract – **In this work are presented a theoretical analysis of acoustic standing wave and Green's function inside rectangular enclosures with rigid walls. The theory of room acoustics can be used for the analysis of sound field inside box of loudspeaker. Mathematical relations are presented for the calculation and researching of modal frequencies, standing sound waves and acoustics Green's function in a rectangular box. The results from the calculating and measuring modal frequencies and the box response are shown graphically and in a table.**

Keywords – **acoustic standing waves, Green's function, rectangular box of loudspeaker.**

I. Green's Function and Standing Wave

The Green's function for the sound field in a box with rigid walls can be defined with solutions to inhomogeneous Helmholtz differential equation [1, 2]:

$$\nabla^2 G(r,r_0) + k^2 \cdot G(r,r_0) = -\delta(r-r_0) \qquad (1)$$

where: G is Green's function [m^{-1}],

$k = \dfrac{\varpi}{c_0}$ - wave number [m^{-1}], c_0 - the speed of sound [m/s],

r -receiver position [m],

r_0 -source position [m],

δ - Dirac delta function,

$\nabla^2 = \text{div grad}$ - Laplacian.

Using the method of images, the walls of the box can be replaced by the image sources obtained by reflecting the source point (and all its images) towards the walls.

The Green's function is the sound pressure at one point, $r(x,y,z)$, generated by a (normalized) point source at another point, $r_0(x_0,y_0,z_0)$ [3, 4]:

$$G(k,r,r_0) = \frac{-1}{V}\sum_N \frac{\Lambda_N \cdot \Psi_N(r) \cdot \Psi_N(r_0)}{\left(k^2 - k_N^2\right)} \qquad (2)$$

where: Ψ_N - mode function [dimensionless].

The Ψ_N - functions are the corresponding functions:

$$\Psi_N(x,y,z) = \cos(k_x \cdot x)\cdot\cos(k_y \cdot y)\cdot\cos(k_z \cdot z) \qquad (3)$$

N - integer, represents the three integers n_x, n_y and n_z (all values are integers between 0 and ∞).

$$\sum_N = \sum_{n_x=0}^{\infty}\sum_{n_y=0}^{\infty}\sum_{n_z=0}^{\infty}$$

V - volume [m^3],

$V = l_x \cdot l_y \cdot l_z$, l_x, l_y, l_z - dimensions of rectangular box [m],

x, y, z - Cartesian coordinates [m],

The modal amplitude, Λ_N (normalizing factor), depending on the modal numbers, given by [1, 4]:

for oblique waves (three dimensional modes):

$\Lambda_N = 8$ for $n_x > 0$, $n_y > 0$ and $n_z > 0$,

for tangential waves (two dimensional modes):

$\Lambda_N = 4$ when one of n_x, n_y or n_z is zero,

for axial waves (one dimensional modes):

$\Lambda_N = 2$ when two of n_x, n_y and/or n_z are zeros.

Fig. 1 and Fig. 2 shows the magnitude of the Green's function in a box as a function of the frequency and the receiver's position.

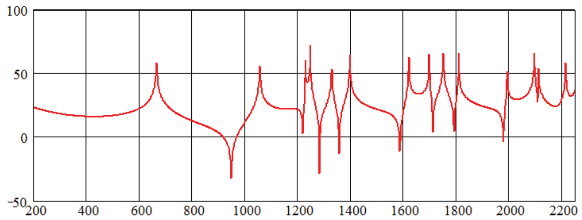

Fig. 1. Magnitude of the Green's function in a box as a function of the frequency.

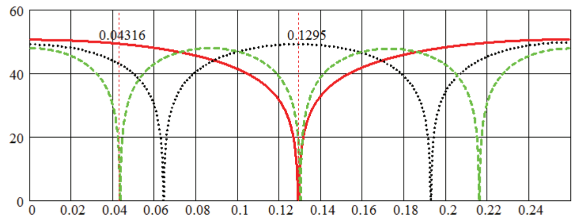

Fig. 2. Magnitude of the Green's function in a box as a function of the receiver position in x axis for natural frequency f_1=*664 Hz* - solid line, $f_5=2 . f_1$= *1328 Hz* - dot line and $f_{11}=3.f_1$= *1992 Hz* – dash line (l_x = 0.259, m).

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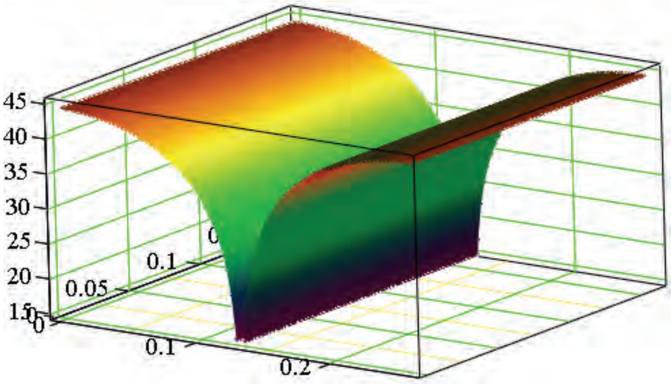

Fig. 3. The distribution of magnitude of sound pressure in a rectangular box in *x* axial sound wave, f=664 Hz, of mode (1, 0, 0).

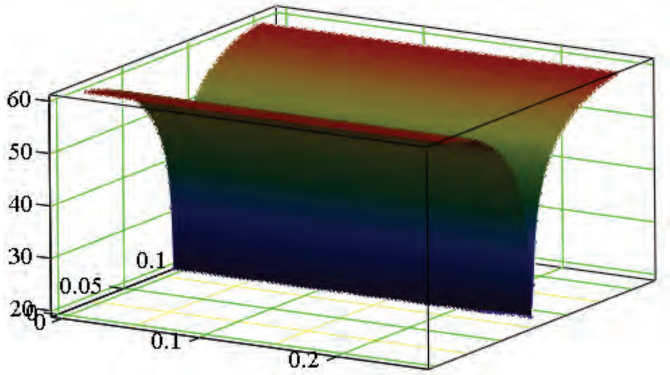

Fig. 4. The distribution of magnitude of sound pressure in a rectangular box in *y* axial sound wave, *f=1055 Hz*, of mode (0, 1, 0).

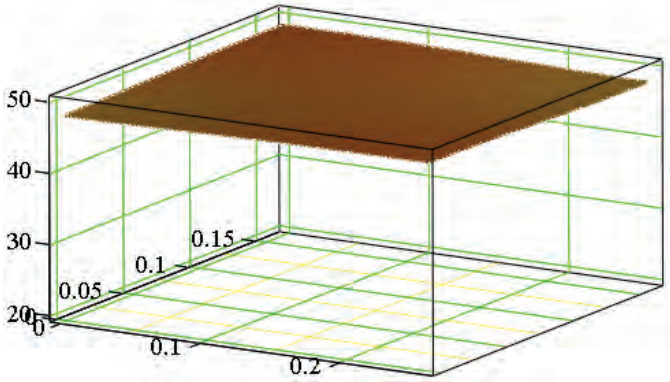

Fig. 5. The distribution of magnitude of sound pressure in a rectangular box in z axial sound wave, *f*=1229 *Hz*, of mode (0, 0, 1).

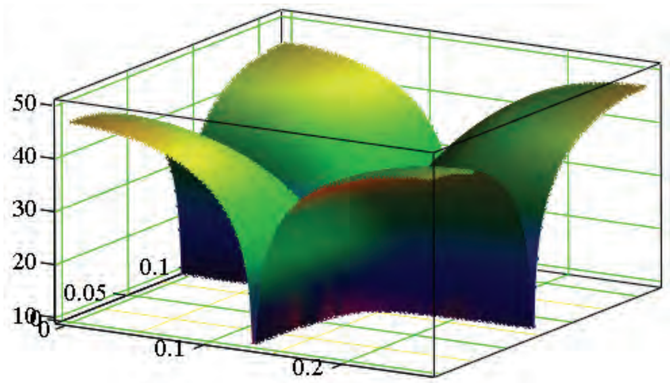

Fig. 6. The distribution of magnitude of sound pressure in a rectangular box in x, y tangential sound wave, f=1247 Hz, of mode (1, 1, 0).

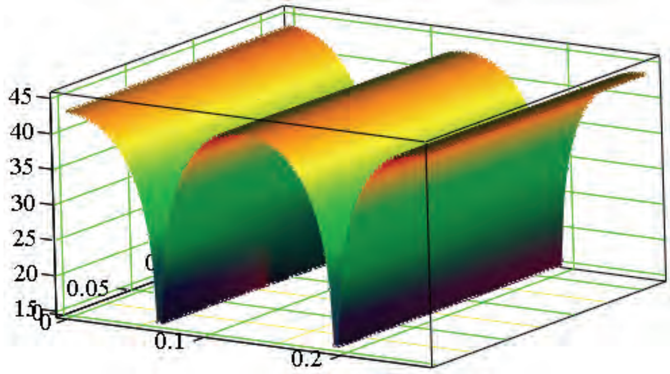

Fig. 7. The distribution of magnitude of sound pressure in a rectangular box in *x* axial sound wave, f=1328 Hz, of mode (2, 0, 0).

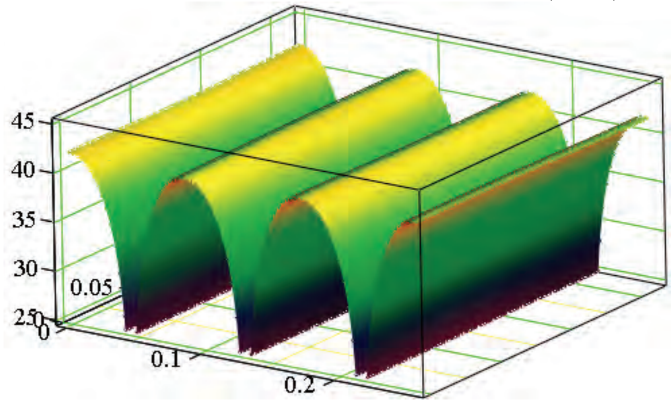

Fig. 8. The distribution of magnitude of sound pressure in a rectangular box in *x* axial sound wave, f=1992 Hz, of mode (3, 0, 0).

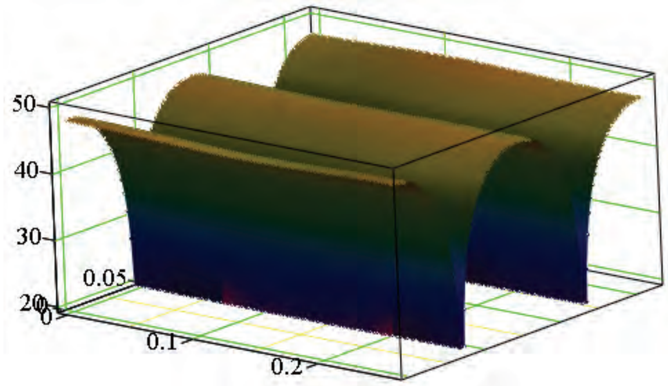

Fig. 9. The distribution of magnitude of sound pressure in a rectangular box in *y* axial sound wave, *f=2110 Hz*, of mode (0, 2, 0).

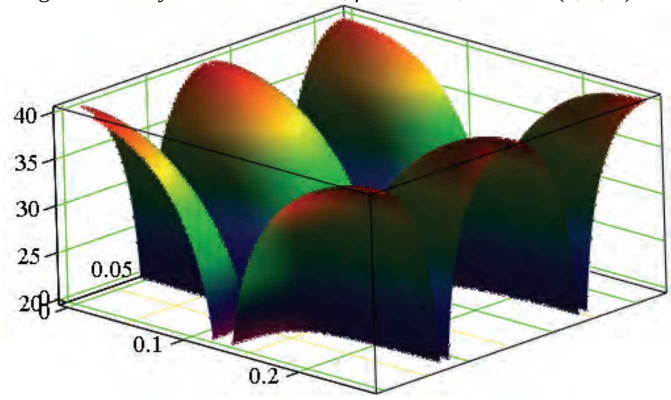

Fig. 10. The distribution of magnitude of sound pressure in a rectangular box in x,y tangential sound wave, *f=2212 Hz*, of mode (1, 2, 0)

The characteristic frequencies f_N of these standing waves are given by:

$$k_N^2 = \left(\frac{2 \cdot \pi \cdot f_N}{c_0}\right)^2 = k_x^{\ 2} + k_y^{\ 2} + k_z^{\ 2}$$

(4)

The constant k_N can be represented with its up on x , y and z

$$k_x = \frac{n_x \cdot \pi}{l_x} \ , \qquad k_y = \frac{n_y \cdot \pi}{l_y}, \qquad k_z = \frac{n_z \cdot \pi}{l_z}$$

The modal frequency of a rectangular box is given by Eq. (4) which can be rewritten as:

$$f_N = \sqrt{f_x^2 + f_y^2 + f_z^2}$$

$$f_N = \frac{c_0}{2} \cdot \sqrt{\left(\frac{n_x}{l_x}\right)^2 + \left(\frac{n_y}{l_y}\right)^2 + \left(\frac{n_z}{l_z}\right)^2} \qquad (5)$$

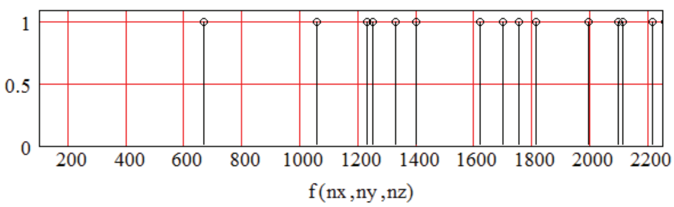

Fig. 11. Plots of mode distribution [5].

TABLE I
THE FOURTEEN LOWEST NORMAL MODES AND THEIR NATURAL FREQUENCIES FOR A BOX WITH RIGID WALLS AT THE SPEED OF SOUND 344 m/s [5].

№	mode	n_x, n_y, n_z	Frequency, Hz
1	x axial	1, 0, 0	664
2	y axial	0, 1, 0	1055
3	z axial	0, 0, 1	1229
4	x, y tangential	1, 1, 0	1247
5	x axial	2, 0, 0	1328
6	x, z tangential	1, 0, 1	1397
7	y, z tangential	0, 1, 1	1620
8	x, y tangential	2, 1, 0	1696
9	x, y, z oblique	1, 1, 1	1750
10	x, z tangential	2, 0, 1	1809
11	x axial	3, 0, 0	1992
12	x, y, z oblique	2, 1, 1	2095
13	y axial	0, 2, 0	2110
14	x, y tangential	1, 2, 0	2212

A graphical representation of the theoretical sound pressure distribution of the *x* axial mode 1, 0, 0 in a model rectangular box was show in Fig. 3.

The sound pressure is zero in the vertical plane at the center of the box and maximum at its ends. The distribution of the sound pressure in the *xy* plane at *y* and *z* axial modes, respectively, is shown in Fig. 4 and Fig. 5.

Three-dimensional representations of the sound pressure distribution in a rectangular box for a tangential mode are represented in Fig. 6 (1, 1, 0) and Fig. 10 (1, 2, 0).

From Figs. 3 to 10 can be noted that sound pressure is maximum at the corners for all modes.

II. ENCLOSURE RESPONSE MEASUREMENTS IN MODEL BOX

The characteristics of the sound pressure in a rectangular loudspeaker enclosure, measured with Realtime Analyzer [6] application software, are presented in graphical form in Fig.12.

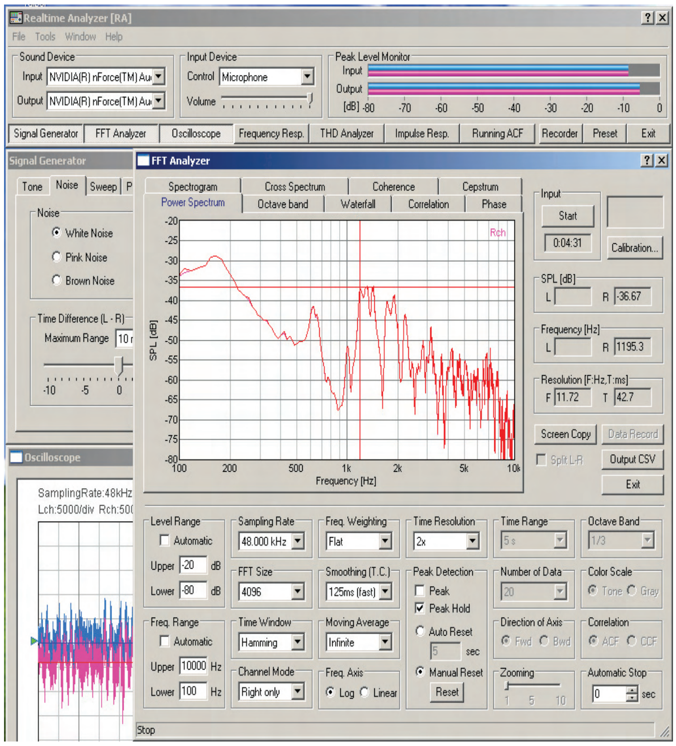

Fig. 12. The magnitude measured sound pressure [6] in the rectangular loudspeaker enclosure (dimensions: width 18 cm, height 27 cm, depth 15.4 cm and wall thickness of 0.85 cm) with a loudspeaker type BK 138 A4 TM (mechanical resonance frequency 90 Hz ± 20Hz, and bandwidth to 15 kHz)

A PC with an adequate sound card is necessary for the measurement. Measuring microphone must be small in size (e.g. electret microphone) and must be placed where extreme values of sound pressure for the characteristic frequencies are expected. For example, if the microphone is placed in a corner of the box, a peak sound pressure will be expected (Figs. 3, 4, ..., 10) due to the box's specific frequencies.

Measurements were made with broadband loudspeaker type BK 138 A4 TM with mechanical resonance frequency of 90

Hz ± 20Hz, and bandwidth of 15 kHz [7], mounted in a rectangular box with dimensions: width 18 cm, height 27 cm, depth 15.4 cm and wall thickness of 0.85 cm.

The program allows the data from the measured values of sound pressure in dB to be stored in tabular and text format for further analysis. The natural frequency of the rectangular box, calculated in accordance with mathematical dependence (5) is presented in Fig. 11.

To examine the modal structure of the bare enclosure, box response at a corner was measured.

Frequency response of sound pressure, measured in the frequency domain from 20 Hz to 100 Hz corresponds to a closed box loudspeaker system (high pass filter).

Local maximum for frequencies around 664 Hz corresponding to the first order own frequency (Table I, № 1 and Fig. 2) in *x* axial wave (1, 0, 0). The second natural frequency 1.043 kHz (Table I, № 2) in *y* axial wave (0, 1, 0) raised slightly the slope of this characteristic.

The peak frequencies of the response curve agree very well with the modal frequencies (Table I) from normal mode theory.

The few modes missing in the measured response are mostly the degenerate modes.

III. Conclusion

Reflected or standing waves may be present in the box of a loudspeaker due to internal reflections [8].

The method – Green's function theory used in room acoustics can be applied to the analysis of a loudspeaker box.

The comparison of experimental results (Fig. 12) with a specific theoretical sound box shows the influence of resonant frequencies on their own characteristics and the possibility to use the established theory of rectangular rooms.

The results obtained in this work can be used for theoretical analysis, design and production of boxes for loudspeakers.

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References

[1] Waig T. Chu, ASHRAE Research Project 624-RP, "Laboratory Methods of Testing for Rating Low Frequency Sound Emission of Air Conditioning Equipment", 1993.

[2] Finn Jacobsen, *The sound field in a reverberation room*, Acoustic Technology, Department of Electrical Engineering, Technical University of Denmark, Note 31261, 2010

[3] Philip M. Morse, *Vibration and Sound*, New York, McGraw-Hill, 2nd. ed., 1948.

[4] Yu Luan, Finn Jacobsen, "A method of measuring the Green's function in an enclosure", Journal of the Acoustical Society of America, vol. 123, issue. 6, pages: 4044-4046, 2008

[5] Екатеринослав С. Сираков, "Собствени резонансни честоти на правоъгълно озвучително тяло", Национална конференция с международно участие „Акустика 1", Варна, Технически Университет-Варна, Катедра „Радиотехника", 2009.

[6] http://www.ymec.com/products/dssf3e/

[7] http://www.vissokogovoriteli.plc.bg/

[8] Glen Ballou, *Electroacoustic Devices: Microphones and Loudspeakers*, Focal Press, 2009