Algorithm for Modal Control of Dual-Mass Electromechanical System

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Abstract–**Created an algorithm for modal control of dual-mass electromechanical system which is composed of a DC motor, power electronic converter voltage and working machine. The created algorithm is based on discrete mathematical system description in state-space. It is provide fast and smooth acceleration of the working machine to set speed and current in the motor acceleration remains less than 2Inom.**

Keywords–**Algorithm for modal control, dual-mass electromechanical system, DC motor, power electronic converter, working machine, discrete systems, state space, MATLAB-implementation.**

I. Introduction

The structural scheme of dual-mass electromechanical system consisting of a power converter, a DC motor (at constant magnetic flux Φ=const) with moment of inertia J1 and working machine with an equivalent moment of inertia J2, is shown in Figure 1 [1, 4, 5].

Mathematical description of the structural scheme shown above can be obtained based on equations of the processes in the DC motor and dual-mass equations of mechanical part.

In this case, considering DC motors with parallel excitation, which has catalog data:

- ✓ $U_{nom}=220\ V;\ P_{nom}=0.3\ kW;$
- ✓ $n_{nom}=1000$ *tr/min;* $I_{nom}=2\ A;$
- ✓ $r_a+r_p=16.6\ \Omega$ *– resistance of the armature windings;*
- ✓ *N=3384 – number of active conductors;*
- ✓ *2a=2 – number of parallel branches;*
- ✓ $P_p=1$ *– number of pairs of poles;*
- ✓ $\Phi.10^{-2}=0.31\ Wb;$
- ✓ $N_{max}=2000$ *tr/min;*
- ✓ $J=J_1=0.042\ kgm^2;$
- ✓ *m=38.0 kg.*

The variables introduced in Fig. 1 have the following values:

- ✓ $R_a=20.8828\ \Omega;$
- ✓ $T_a=\dfrac{L_a}{R_a}=0{,}0126\ s$, **where**

$$L_a=\frac{r.U_{nom}}{P_{nom}.\omega_{nom}.I_{nom}}=0{,}2626\ H\ ;$$

- ✓ $r=0.25\Omega;\ \omega_{nom}=\dfrac{2pn_{nom}}{60}=104{,}7198\ rad/sec\ ;$
- ✓ $J_1=J=0{,}042\ kgm^2\ ;$
- ✓ $J_2=0{,}5J_1=0{,}021\ kgm^2\ ;$
- ✓ $cF=\dfrac{U_{nom}-R_aI_{nom}}{\omega_{nom}}=1{,}7020\ Vs\ ;$
- ✓ $M_{nom}=\dfrac{P_{nom}}{\omega_{nom}}=2{,}8648\ Nm\ ;$
- ✓ $c_{12}=0{,}5M_{nom}=1{,}4324\ Nm$ *- coefficient of hardness to elastic connection;*
- ✓ $M_{c1}=0{,}1M_{nom}=0{,}28648\ Nm\ ;$
- ✓ $M_{c2}=0{,}9M_{nom}=2.57832\ Nm\ .$

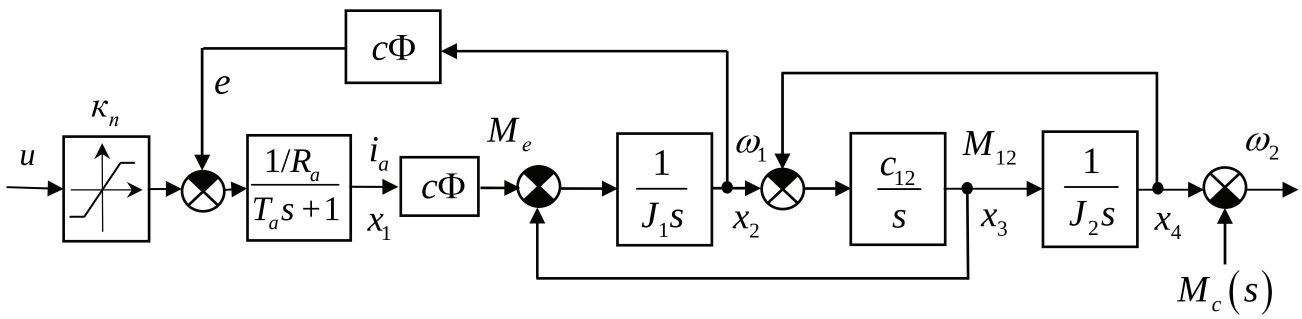

Fig. 1Structural scheme of dual-mass electromechanicalsystem

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The type of transition process of dual-mass electromechanical system is shown in Figure 2. It was simulating on the basis of the structural scheme of Figure 1.

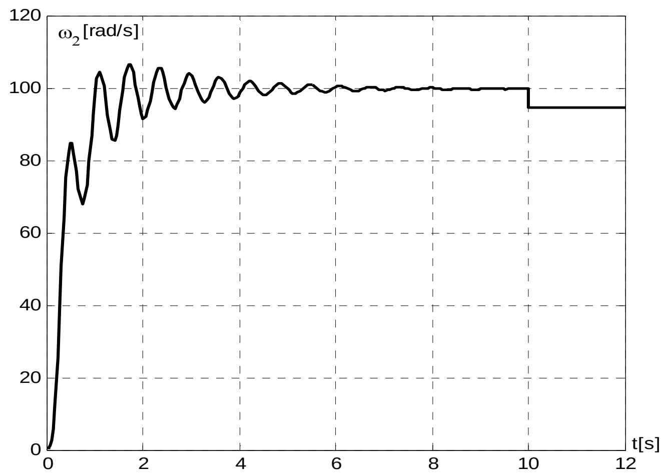

Fig. 2Transition process of dual-mass electromechanical system without modal control.

As a result of internal electromotive feedback, when input voltage is constant (u = *const*) the output speed is establishe. With increasing hardness of the elastic connection increases the frequency of fluctuations and reduces their amplitude. With increasing moment of inertia of the second mass fluctuations subside more slowly. When had appeared of disturbance (in the moment t = 10s) the speed of dual-mass system is lower.

II. Discrete Mathematical Model of Dual-mass Electromechanical System

The algorithm for modal control is going to synthesize based on discrete mathematical model of the dual-mass system in state-space. Therefore, it first is going to made mathematical description of the continuous system in state-space.

To be able the algorithm is using for practical purposes, elements of the state vector is going to chose real physical values: $x_1 = i_a$ - armature motor current; $x_2 = \omega_1$ - rotor speed the motor; $x_3 = M_{12}$ - mechanism elastic torque и $x_4 = \omega_2$ - speed of the working machine.

$$\left|\begin{aligned} \dot{x}_1 &= \frac{cF}{T_a R_a}\left(k_n u_{зад} - cFx_2\right) - \frac{1}{T_a}x_1; \\ \dot{x}_2 &= \frac{1}{J_1}\left(x_1 - x_3 - M_{c1}\right); \\ \dot{x}_3 &= c_{12}\left(x_2 - x_4\right); \\ \dot{x}_4 &= \frac{1}{J_2}\left(x_3 - M_{c2}\right). \end{aligned}\right. \qquad (1)$$

Based on (1) is obtained mathematical description of continuous dual-mass system in state-space:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t); \\ y(t) &= \mathbf{c}^{\mathrm{T}}\mathbf{x}(t) + du(t), \end{aligned} \qquad (2)$$

and the individual matrices are the following

$$\mathbf{A} = \begin{bmatrix} -1/T_a & -cF/T_a R_a & 0 & 0 \\ cF/J_1 & 0 & -1/J_1 & 0 \\ 0 & c_{12} & 0 & -c_{12} \\ 0 & 0 & 1/J_2 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} -79,3651 & -6,468451 & 0 & 0 \\ 40,52381 & 0 & -23,8095 & 0 \\ 0 & 1,4324 & 0 & -1,4324 \\ 0 & 0 & 47,619 & 0 \end{bmatrix};$$

$$\mathbf{b} = \begin{bmatrix} k_n/R_a T_a \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 65,8437 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

$$\mathbf{c}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}; \; d = 0.$$

Based on the mathematical description of the continuous system (2) mathematical description is made of discrete dual-mass system in state-space for a sample time $T_0 = 0.1\ s$:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k); \\ y(k) &= \mathbf{c}^{\mathrm{T}}\mathbf{x}(k) + du(k), \end{aligned} \qquad (3)$$

and the individual matrices are the following

$$\mathbf{A} = \begin{bmatrix} -0.0160 & -0.0331 & -0.1530 & -0.0016 \\ 0.2071 & 0.4235 & 1.7665 & -0.0146 \\ -0.0576 & -0.1063 & 0.1923 & 0.1036 \\ 0.0197 & -0.0291 & -3.4441 & 0.1777 \end{bmatrix};$$

$$\mathbf{b} = \begin{bmatrix} 0.3526 \\ 6.0168 \\ 0.0272 \\ 8.6667 \end{bmatrix};$$

$$\mathbf{c}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}; \; d = 0.$$

III. Algorithm for Modal Control of Dual-mass Electromechanical System

Algorithm which is presented below was developed based on algorithm for modal control in [3]. Modal control is realized by synthesizing the feedback vector **k**. The vector **k** is defined so that the poles of the closed-loop system to moved in a circle with specified radius η known as zone of stability.

Algorithm for modal control of dual-masssystem is shown in a structural form with the scheme of Figure 3 and is described step by step below.

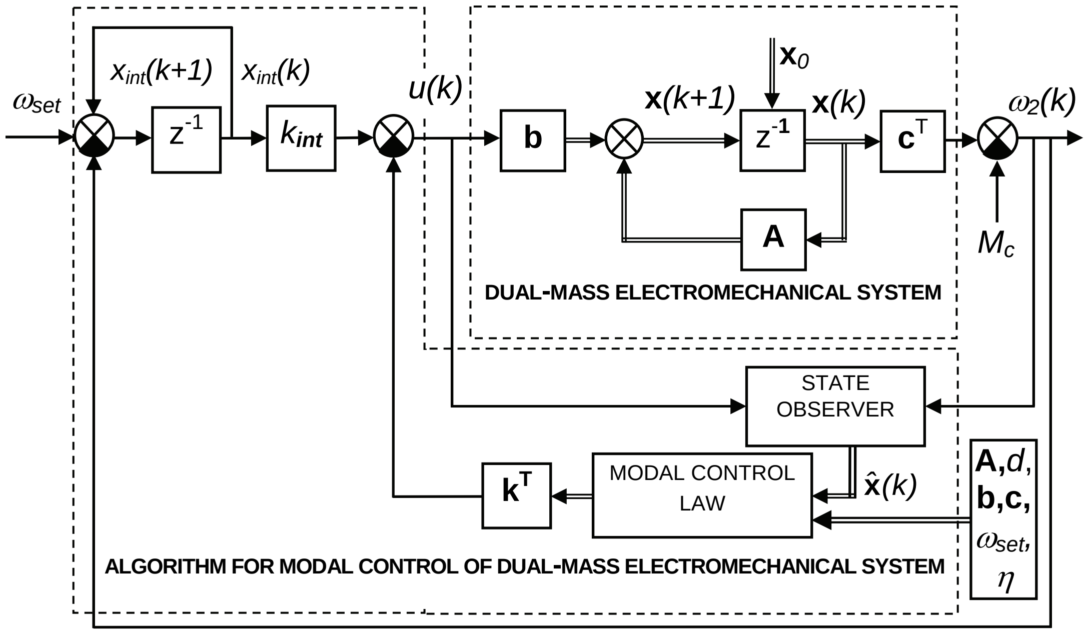

Fig. 3 Algorithm for modal control of dual-mass electromechanical system in a structural form.

Step 1 Introduction of: the matrix **A**, the vector **b**, the vector **c**, the scalar d and *zone of stability* η. Zone of stability specifies the radius of the circle in which to locate the poles of the closed system;

Step 2 Check order of the system-n through the length of the vector **b**;

Step 3 Determination the eigenvalues of the matrix **A** ($\lambda_1, \lambda_2, \ldots, \lambda_n$) and corresponding to them own vectors $\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_n$;

Step 4 Formation the matrix of own vectors **Q**
$$\mathbf{Q} = \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \cdots & \mathbf{q}_n \end{bmatrix}; \mathbf{q}_i = \begin{bmatrix} q_{i1} & q_{i2} & \cdots & q_{in} \end{bmatrix}; \quad i = 1,2,\ldots,n;$$

Step 5 Generation of n-dimensional vector ξ, of random numbers with normal distribution between 0 and 1
$$\boldsymbol{\xi} = \begin{bmatrix} \xi_1 & \xi_2 & \cdots & \xi_n \end{bmatrix}^T;$$

Step 6 Determination of vector eigenvalues of the closed system
$$\boldsymbol{\mu} = \begin{bmatrix} \eta \\ \eta \\ \vdots \\ \eta \end{bmatrix} - \left| \left(\begin{bmatrix} 0.5 \\ 0.5 \\ \vdots \\ 0.5 \end{bmatrix} - \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{bmatrix} \right) 0.1 \right| = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix};$$

Step 7 Determination of the vector
$$\mathbf{g} = \mathbf{Q}^T \mathbf{b} = \begin{bmatrix} g_1 & g_2 & \cdots & g_n \end{bmatrix}^T;$$

Step 8 Calculation of the elements of the vector **d**
$$d_i = \frac{\prod_{j=1}^{n} \left(\lambda_i - \mu_j \right)}{g_i \prod_{j=1, j \neq i}^{n} \left(\lambda_i - \lambda_j \right)}, \quad i = 1,2,\ldots,n;$$
$$\mathbf{d} = \begin{bmatrix} d_1 & d_2 & \cdots & d_n \end{bmatrix}^T;$$

Step 9 Defining the vector of feedback
$$\mathbf{k}^T = \mathbf{d}^T \mathbf{Q} = \begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix};$$

Step 10 Determination of the innovation vector addition **f** and matrix of the state observer **F**
$$\mathbf{f} = \mathrm{acker}\left(\mathbf{A}, \mathbf{c}, \mathbf{w} \right), \ \mathbf{w} = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}^T;$$
$$\mathbf{F} = \mathbf{A} - \mathbf{f}\,\mathbf{c}^T;$$

Step 11 Estimation the current state vector $\hat{\mathbf{x}}(k)$
$$\hat{\mathbf{x}}(k+1) = \mathbf{F}\hat{\mathbf{x}}(k) + \mathbf{b}u(k) + \mathbf{f}y(k), \ \hat{\mathbf{x}}(0) = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}^T, \ k = 0,1,2,\ldots;$$

Step 12 Calculation of the integral variable x_{int}
$$x_{int}(k+1) = x_{int}(k) + \omega_{set} - \omega_2(k), \ \ k = 0,1,2,\ldots;$$

Step 13 Calculation of control voltage
$$u(k) = \begin{bmatrix} k_{int} & -\mathbf{k}^T \end{bmatrix} \begin{bmatrix} x_{int}(k) \\ \hat{\mathbf{x}}(k) \end{bmatrix}, \quad k = 0, 1, 2, \ldots;$$

Step 14 Repeating steps 11, 12 and 13 during the control process.

IV. MATLAB-Implementation of Algorithm for Modal Control

```
clc,clear
%State Space Modal Control for DC Motor ***********
omega_set=100; stab_zone=0.27; k_int=0.02; Mc=2.57832;
takt_disturb=80; endprocess=120;
%*********************************************
A=[-0.0297   -0.0544    0.1308   -0.0094;
    0.3408    0.6087   -1.7352    0.1446;
    0.0493    0.1044    0.5422   -0.1204;
    0.1180    0.2891    4.0035    0.6867];
b=[ 0.6496;   2.5117;   0.1632;   0.2463];
c=[ 0; 0; 0; 1]; d=0;
n=length(b);
[Aq,Az]=eig(A');
ERROR(1:n,:)=Aq(:,1:n).'*A-Az(1:n,1:n)*Aq(:,1:n).';
if abs(max(ERROR))<0.0001
   q(1:n,:)=(Aq(:,1:n)).'; z=diag(Az);
```

```
if max(abs(z))>stab_zone
      mu=(stab_zone-(0.5-randn(1,n))*0.1);
end
    p=[q(1:length(q(:,1)),:)*b];
   dm(1:length(z))=0;
    Pden=1;
    Pnum=1;
for i=1:length(z)
for j=1:length(z)
            Pnum=Pnum*((z(i)-mu(j)));
if i~=j
            Pden=Pden*((z(i)-z(j)));
end
end
      DEN=p(i)*Pden;
      dm(i)=-(Pnum/DEN);
      Pden=1;
        Pnum=1;
end
   k=dm*q;
else
   error('Discrepancy between eigenvalues and own
vectors')
end
if abs(imag(k))<0.001
   K=real(k);
end
w=zeros(n,1); f=(acker(A',c,w))'; F=A-f*c';
takt=1
x_int(takt)=0; xo(:,takt)=zeros(n,1);
xr(:,takt)=zeros(n,1); y(takt)=0; disturb=0;
key1=1;
while key1==1
   takt=takt+1
   x_int (takt)=x_int (takt-1)+y_point-y(takt-1);
   u(takt)=[ki K]*[x_int (takt); xo(:,takt-1)];
   xo(:,takt)=F*xo(:,takt-1)+b*u(takt)+f*y(takt-1);
   xr(:,takt)=A*xr(:,takt-1)+b*u(takt);
if takt>takt_disturb
       disturb=Mc;
end
   y(takt)=c'*xr(:,takt)-disturb
   I(takt)=xr(1,takt);
if takt>=endprocess
       key1=0;
end
```

Simulation study is done through the above shown implementations of MATLAB. Results of the study is shown in Figure 4.

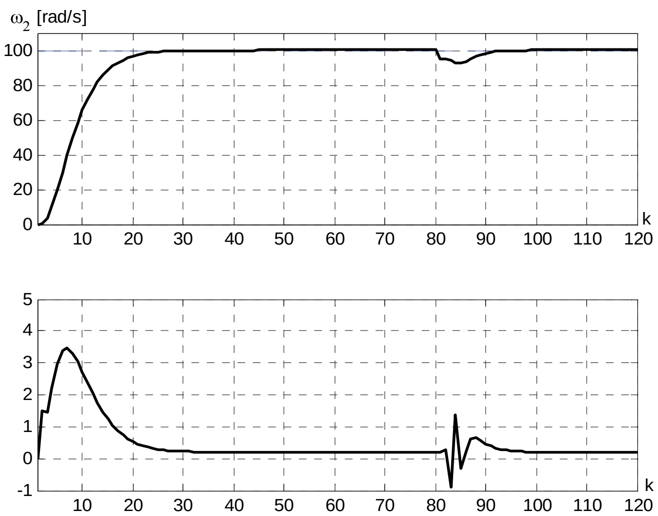

Fig. 4Results of the simulation study with MATLAB-implementation.Above-the output speed, below-armature current.

V. Conclusion

The results, shown in Figure 4 were obtained in assigned speed of working machine ω_{set}=100 rad / s; integrating factor $k_{int}=0.02$; and poles of the closed system are located in a circle of radius = *stab_zone=0.27*. In *k=80* (8 th second) appears disturbance in the form of torque-Mc of the shaft of the working machine.

If you compare results of Figure 2 and Figure 4 shows that in the control of the proposed algorithm, acceleration is smooth and three times faster. There are no fluctuations which are caused by elastic connections. Inrush current exceeds nominal only *1.9* times ($I_{in} = 1.9 I_{nom}$). Integral component in the law for control eliminated the disturbance and return the system back to set speed.

From the experiments it became clear that the algorithm is highly sensitive to the location of the poles of the closed-loop system (vector $\boldsymbol{\mu}$) and the value of the coefficient of integration k_{int}. Here $\boldsymbol{\mu}$ and k_{int} are determined experimentally. Theoretical problem of determining $\boldsymbol{\mu}$ and k_{int} is not solved.

VI. References

[1] Mikhov M.R., T.T.Georgiev, Optimal Modal Control of Dual-Mass Electromechanical Systems, Proceedings EDS'08 IMAPS CS International Conference; Invited Papers Brno, Czech Republic, September 10-11, 2008, pp.47-55.

[2] Nikolov N., Algorithm for Synthesis of Modal Adaptive State Controller, International Conference "AUTOMATICS AND INFORMATICS'07", proceedings vol.1, p.I-13÷I-16, october 3-6, Sofia, 2007.

[3] Nikolov N, Study of algorithms for synthesis of modal adaptive state controllers, dissertation-PhD, TechnicalUniversity of Varna, Bulgaria, 2009.

[4] Башарин А.В., В.А.Новиков, Г.Г. Соколовский, Урпавление электропроводами, Ленинград, Энергоиздат, 1982г.

[5] Ключев В.И., Теория на електрозадвижването, София, Техника, 1989.

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