On-Line Identification of Time-Varying Systems

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Abstract – **A method for on-line identification of continuous linear time-varying systems is applied based on the block-pulse functions. A suitable m-file is proposed. Numerical examples are considered and the efficiency of the identification algorithm in case of noise influence is examined.**

Keywords– **On-line identification, Linear time-varying systems, Block-pulse functions.**

I. INTRODUCTION

Applications of the linear time-varying (LTV) systems include rocket dynamics, time-varying linear circuits, satellite systems, pneumatic actuators, ets. LTV structure is also often assumed in adaptive and standard gain-scheduled control systems. The motivation for this work is an investigation of a method for on-line identification of continuous LTV systems based on the block-pulse functions. The time-varying linear systems identification immediately in the continuous-time domain is a quite complicated problem. To avoid the direct measurements of time derivatives, the following identification methods have been proposed:

1. Methods based on Poisson moment functionals [5, 11];
2. Methods based on orthogonal functions and polynomials [3, 4, 6 - 10].

The second methods are non-recursive. The first methods can be applied for recursive identification, but the involved Poisson filter chains must be realized physically by analogue devices and the system structure may be complicated.

Z. Jiang and W. Schaufelberger [1] proposed a new method for recursive identification of continuous time-varying linear systems. Based on the relation between entries of block pulse integral operational matrices, regression equation models of continuous time-varying linear systems can be obtained and recursive algorithms developed for discrete time model identification can be applied to estimate the time-varying parameters without much modifications.

In this paper the recursive method above is applied and Matlab file is proposed. Numerical examples are considered.The efficiency of the identification algorithm in case of noise influence is examined andcorresponding conclusions are made.

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II. BLOCK PULSE ORTHOGONAL FUNCTION PROPERTIES

Before commencing consideration of the problem, some pertinent properties of block – pulse function (BPF) will be briefly reviewed.

A block – pulse function [2,3,4, 7] is defined over a time interval $t \in [0, T]$ as $\{B_i(t)\},\ i = \overline{1, m}$, where:

$$\left.\begin{aligned} & B_i(t) = 1, \qquad for\ t \in [0; T/m] \\ & B_i(t) = \begin{cases} 1, & for\ t \in [(i-1)T/m;\ iT/m] \\ 0, & elsewhere \end{cases} \\ & \qquad\qquad for\ i = 2, 3, ..., m \end{aligned}\right\}. \qquad (1)$$

They posses the orthogonal property

$$\int_0^T B_i(t).B_j(t).dt = \begin{cases} T/m, & if\ i = j \\ 0, & if\ i \neq j \end{cases}. \qquad (2)$$

A function y(t), absolutely integrable in the region $t \in [0, T]$, may be approximated to

$$y(t) \cong \sum_{i=1}^{m} y_i.B_i\ (t) = Y_M{}^T.B_M(t), \qquad (3)$$

where:

Y_M - block – pulse function coefficient vector of the function $y(t)$;

$$B_M(t) = [B_1(t)\ \ B_2(t)\ \ ...\ \ B_m(t)]^T.$$

The orthogonal function $B_M(t)$ has the property

$$\underbrace{\int_0^t ... \int_0^t}_{r-times} B_M(t).dt^r = P_M^r.B_M(t). \qquad (4)$$

where P_M is $(m \times m)$ integral operational matrix.

Applying the property (4) to $y(t)$ yields

$$\begin{aligned} & \underbrace{\int_0^t \int_0^t ... \int_0^t}_{r-times} y(t).dt^r = \\ & = \underbrace{\int_0^t \int_0^t ... \int_0^t}_{r-times} Y_M^T.B_M(t).dt^r = Y_M^T.(P_M)^r.B_M(t) \end{aligned} \qquad (5)$$

III. ON-LINE IDENTIFICATION OF SECOND ORDER TIME- VARYING LINEAR SYSTEM

Consider the following N-th order differential equation

$$\sum_{i=0}^{N} a_i(t) y^{(i)}(t) = \sum_{i=0}^{N} b_i(t) u^{(i)}(t) \qquad (6)$$

where:

$u(t),\ y(t)$ – system input and output respectively;

$a_i(t)$ and $b_i(t)$ - time functions which are described by n-th order power polynomials with constant coefficients, as follows:

$a_i(t) = a_{i,0} + a_{i,1}t + \ldots + a_{i,n}t^n$;

$b_i(t) = b_{i,0} + b_{i,1}t + \ldots + b_{i,n}t^n$.

The identification problem for continuous LTV systems described by the equation bellow is to determine $a_{i,j}$ and $b_{i,j}$ ($i = \overline{0, N}$; $j = \overline{0, n}$) from the system input $u(t)$ and output $y(t)$. This problem can be solved straightforward based on the block-pulse regression equation [1]

$$\sum_{i=0}^{N} \sum_{j=0}^{n} a_{i,j} z_{i,j,l} = \sum_{i=0}^{N} \sum_{j=0}^{n} b_{i,j} v_{i,j,l}\ , \qquad (7)$$

where $z_{i,j,l}$ and $v_{i,j,l}$ are linear combinations of the block pulse coefficients y_{l+r} and u_{l+r} ($r = \overline{0, N}$) respectively.

The relation between $z_{i,j,l}$ and y_{l+r}, and relation between $v_{i,j,l}$ and u_{l+r} respectively have the forms:

$$z_{i,j,l} = \sum_{s=0}^{\min(i,j)} \left((-1)^s \binom{i}{s} \left[\sum_{k=0}^{N} \sum_{r=0}^{N-k} (-1)^k \binom{N}{k} \frac{j!\,h^{N-i+j}}{(N-i+j+1)!} p_{N-i+s,\,j-s,\,l+r,\,l+N-k} y_{l+r} \right] \right) \qquad (8)$$

$$v_{i,j,l} = \sum_{s=0}^{\min(i,j)} \left((-1)^s \binom{i}{s} \left[\sum_{k=0}^{N} \sum_{r=0}^{N-k} (-1)^k \binom{N}{k} \frac{j!\,h^{N-i+j}}{(N-i+j+1)!} p_{N-i+s,\,j-s,\,l+r,\,l+N-k} u_{l+r} \right] \right) \qquad (9)$$

Equation (8) can be written as:

$$z_{i,j,l} = \sum_{s=0}^{\min(i,j)} (-1)^s \binom{i}{s} \frac{j!}{(j-s)!} x_{s,l} \qquad (10)$$

where

$$x_{s,l} = \sum_{r=0}^{N} y_{l+r} \left[\sum_{k=0}^{N} (-1)^k \binom{N}{k} \frac{(j-s)!\,h^{N-i+j}}{(N-i+j+1)!} p_{N-i+s,\,j-s,\,l+r,\,l+N-k} \right] \qquad (11)$$

By substitution u_{l+r} for y_{l+r} and $v_{i,j,l}$ for $z_{i,j,l}$ the value $v_{i,j,l}$ can be given in the same way. The values $x_{s,l}$ (eq. (11)) can be computed from the N-th order difference of entries in each row of the submatrix and from the block-pulse coefficients of y(t). The value $z_{i,j,l}$ is a linear combination of the obtained values $x_{s,l}$. After all the terms of $z_{i,j,l}$ and $v_{i,j,l}$ are obtained, the block-pulse regression equation (7) can be constructed, and the algorithms for discrete-time model identification can be applied to the time-varying linear system without much modification.

Consider a system modelled by the following time- varying ordinary differential equation

$$\frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_2(t) y(t) = b_0(t) u(t)\,, \qquad (12)$$

where:

$a_2(t) = a_{2,0} + a_{2,1}t + a_{2,2}t^2$;

$b_0(t) = b_{0,0} + b_{0,1}t$.

Given a record of output y(t) and input u(t) signals, shown in Fig.1, the problem is to estimate the unknown parameters a_1; $a_{2,0}$; $a_{2,1}$; $a_{2,2}$; $b_{0,0}$; $b_{0,1}$ of the system (12).

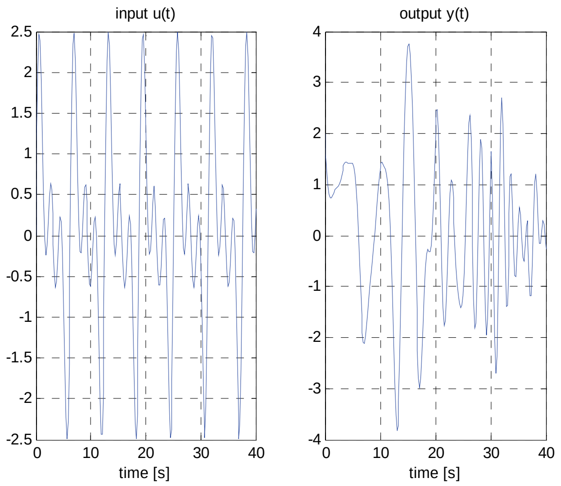

Fig. 1. Output and input signals

M – file is created in Matlab based on considered algorithm. The results obtained from the recursive estimation are given in Fig. 2.

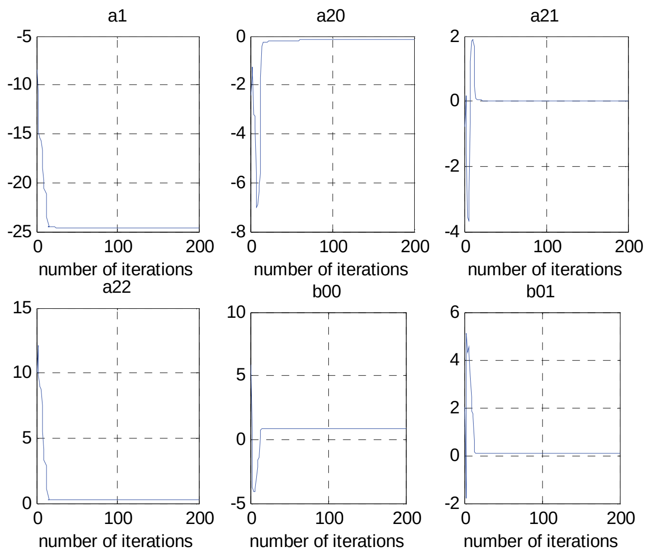

Fig. 2. Values of the unknown parameters

As givev from Fig.2 the unknown parameters after small number of iterations are obtained. The output of the model Ym(t) and the prediction error E are given in Fig.3.

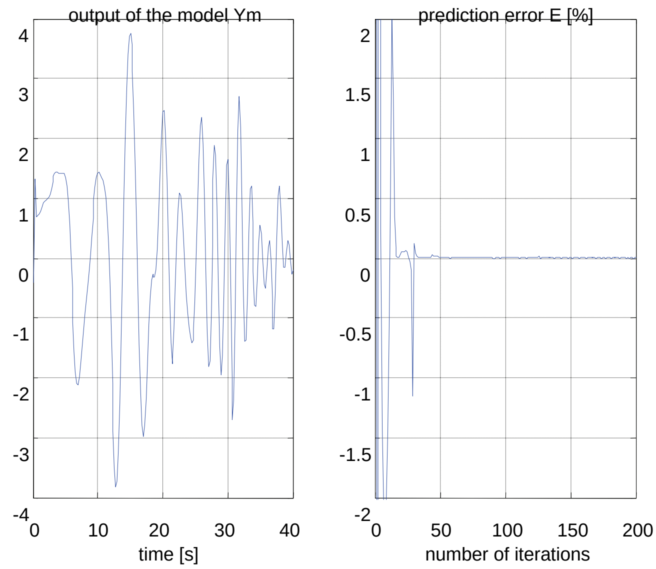

Fig. 3. Model output and prediction error

An example shows that the method is simpler and more efficient in comparison with other methods for solving the same problem.

Then the efficiency of the identification algorithm in case of noise influenceis examined. Therefore the signaly(t) with independent zero – mean white Gaussian noise iscorrupted. First the ratio nose-to-signal q= 1% is used.The values of the unknown parameters $a_1; a_{2,0}; a_{2,1}; a_{2,2}; b_{0,0}; b_{0,1}$ are given in Fig.4.

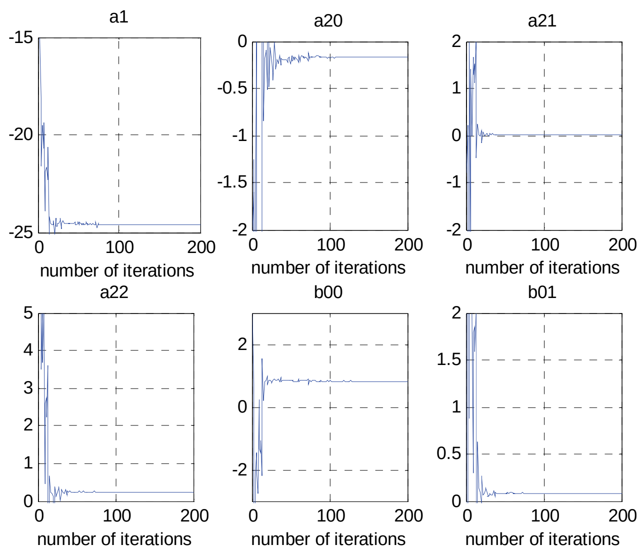

Fig. 4 Values of the unknown parameters
(ratio nose-to-signal q= 1%)

Then the signaly(t) with independent zero – mean white Gaussian noise with q= 10% iscorrupted. The values of the unknown parameters areshown in Fig.5.

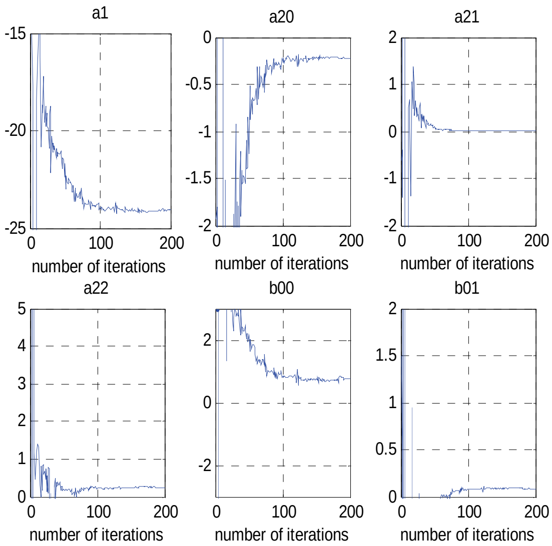

Fig. 5.Values of the unknown parameters (ratio nose-to-signal q= 10%)

IV. Conclusion

A comparatively new method for on-line identification of continuous linear time-varying systems is applied based on the block-pulse functions. A suitable m-file is proposed. Numerical examples are considered and the noise immunity of considered algorithm is investigated. Comparing with other methods for solving the same identification problem, no analogue devices, no large computations in the data preparation stage and no initial values are involved in this method. Therefore the estimation procedure is much simpler.

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