Concatenated "MMSE-Sequential Search" Algorithm for Multi User Detection in SDMA Uplink

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Abstract – **The concatenation of MMSE and sequential search is MUD method which combats the imperfect channel conditions and maintains low complexity at the receiver. In this work the combination of two methods is proposed, and it is regarded in Spatial Division Multiplexing Access scheme. The suppression of Multi Access Interference and MMSE error along with BER performance is studied in case of different number of users and receiving antennas.**

Keywords – **MUD, OFDM, MMSE.**

I. INTRODUCTION

Space Division Multiple Access (SDMA) based Orthogonal Division Multiplexing (OFDM) scheme has been presented as very attractive point of research recently. Considering every single antenna equipped user, communicating with the Base Station (BS) i.e. its multiple antennas, this uplink scheme can be regarded as Multiple Input Multiple Output (MIMO) model and inherits the benefits. Taking into consideration the OFDM approach, every single user transmits symbols within common frequency bandwidth as user differentiation is not maintained by allocating the users per different subcarriers. Once formed, OFDM symbols are transmitted over non-ideal channel that is approximated as flat slow Rayleigh fading channel in parallel with Additive White Gaussian Noise (AWGN).

A variety of multiuser detection (MUD) schemes were researched for user separation at the receiver, either linear or non-linear detectors. Minimum Mean Square Error (MMSE) MUD is a promising method to put up with the channel transfer function and restore the signal prior to the demodulation process. However, Multi Access Interference (MAI) caused by the imperfect channel condition can't be completely reduced by MMSE linear method and therefore, some non-linear methods are additionally invoked such as sequential search [1], genetic algorithms [2], parallel and successive interference cancellers [3] etc.

In this paper a potential sequential search (SS) is proposed to be concatenated with MMSE linear combiner in order to reduce the loss of information caused by MAI. This method was successfully proved to be functional in CDMA approach where MAI is the result of the non-ideal orthogonal spreading codes [4]. Moreover, modest complexity is maintained at the receiver in comparison with Maximum Likelihood (ML) non-linear detection which was found to give the best performance. Apart of the same cost function which is analyzed in both ML and SS, the latter tends to find the most accurate parallel combinations of users and compute the cost for every each of them. The minimum value of the cost drives up to the most probable combination that was sent by the users. Here in this paper, an iteration of this method is simulated and the performance of the system is estimated with or without the concatenation.

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II. SYSTEM MODEL

A. SDMA-MIMO-LFDMA channel model

Fig.1 shows the system model used in this paper where every user is equipped with single antenna, while the BS is equipped with multiple antennas. The number of users L and the number of receiving P antennas form SDMA-MIMO channel, essentially related with the PxL-dimensional matrix channel transfer function $\mathbf{H}_{PxL}$ in frequency domain.

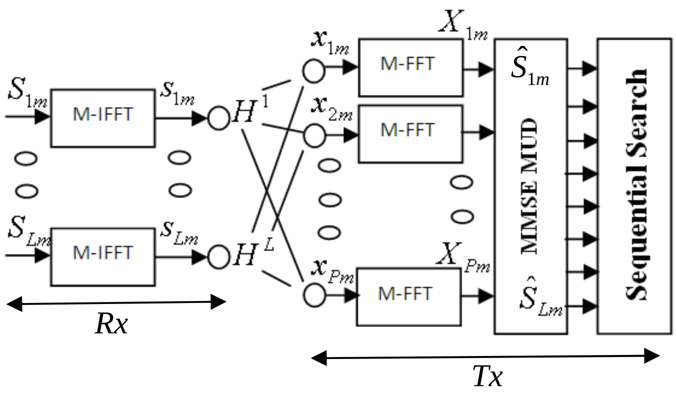

Fig.1 SDMA-MIMO Uplink Scheme

At every m-th subcarrier of the OFDM symbols received by the P-element receiver antenna array, the complex $Px1$-dimensional signal vector is formed as superposition of independently faded m-th subcarriers of the OFDM symbols associated with the L users and additionally distorted by AWGN. For every subcarrier the transfer function can be expressed as:

$$\mathbf{X}_m = \mathbf{H}\mathbf{S}_m + \mathbf{n} \quad (1),$$

where $\mathbf{X}_m$ is $Px1$ column vector that is constituted by symbols related to m-th subcarriers from every received OFDM symbol from P antennas and $\mathbf{S}_m$ is Lx1 column vector that is constituted by symbols related to m-th subcarriers from every transmit OFDM symbol per user. Px1-dimensional column vector $\mathbf{n}$ is the AWGN that exhibits a zero mean and a variance of σ_n^2. PxL-dimensional complex matrix $\mathbf{H}$ in frequency domain presents the channel transfer function. For example, the l-th column of the matrix:

$$\mathbf{H}^l = (\mathbf{H}_1^l ... \mathbf{H}_p^l), \quad l = 1,2...L \qquad (2),$$

represents the complex transfer function that is associated with the transmission paths from the l-th user's antenna to each element of the P-element receiver antenna array.

B. Linear Detector – MMSE MUD

The OFDM symbols received at P antennas are transformed back to frequency domain by the M-dimensional Fast Fourier transform (M-FFT). Using the per-subcarrier approach, the column vector $\mathbf{X}_m$ is linearly combined with the aid of weight matrix $\mathbf{W}$, resulting in:

$$\hat{\mathbf{S}}_m = \mathbf{W}^H \mathbf{X}_m \qquad (3)$$

where $\mathbf{W}$ is PxL-dimensional matrix and the superscript H denotes the Hermitian transpose.

If we use (1) in case of avoiding IFFT and FFT operations, the last equation can be modified and invoked for the l-th user:

$$\begin{aligned}\hat{\mathbf{S}}_m &= \mathbf{w}^{lH}\mathbf{X}_m^l = \mathbf{w}^{lH}(\mathbf{H}\mathbf{S}_m + \mathbf{n}) \\ &= \mathbf{w}^{lH}\mathbf{H}^l\mathbf{S}_m^l + \mathbf{w}^{lH}\sum_{i=1,i\neq l}^{L}\mathbf{H}^i\mathbf{S}_m^i + \mathbf{w}^{lH}\mathbf{n} \qquad (4)\end{aligned}$$

where $\mathbf{w}^{lH}$ is the l-th column of the PxL - matrix $\mathbf{W}$.

Assuming the (4) and its components, the expression of undesired correlation matrix for the l-th user is:

$$\mathbf{R}_{a,I+N}^l = \mathbf{R}_{a,I}^l + \mathbf{R}_{a,N}^l = \sum_{i=1,i\neq l}^{L}\sigma_i^2\mathbf{H}^i\mathbf{H}^{iH} + \sigma_n^2\mathbf{I} \qquad (5),$$

where σ_i^2 is the variance of the interfering users contribution, σ_n^2 is the variance of the AWGN and $\mathbf{I}$ is the PxP identity matrix.

The quality of the linear detector can be measured by the Signal to Noise and Interference Ratio (*SINR*). This parameter is defined by the variances of the desired signal, interfering signals and noise signal. Moreover, MAI that is supposed to be eliminated by the concatenation of the linear detector with additional non-linear detecting method is mutually related to $\mathbf{R}_{a,I}^l$ and it appears in the following expression for *SINR*:

$$SINR^l = \frac{\sigma_s^{(l)2}}{\sigma_I^{(l)2} + \sigma_N^{(l)2}} = \frac{\mathbf{w}^{lH}\mathbf{R}_{a,S}^l\mathbf{w}^l}{\mathbf{w}^{lH}\mathbf{R}_{a,N+I}^l\mathbf{w}^l} \qquad (6)$$

Chanel estimation at the receiver is out of scope in this paper and therefore the channel matrix $\mathbf{H}$ is considered to be perfectly estimated. The linear combining using MMSE is method of finding the minimum mean square error of the cost function:

$$\Delta\mathbf{S}_m = \mathbf{S}_m - \mathbf{W}^H\mathbf{X}_m \qquad (7)$$

The optimum weight matrix that minimizes the mean square of the cost function is constituted by the channel matrix and noise variance:

$$\mathbf{W}_{\text{MMSE}} = (\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I})^{-1}\mathbf{H} \qquad (8)$$

Combining the receiving signal with (8) does not eliminate the MAI effect that basically depends proportionally on L. However, is very beneficial to deploy idealistic fading channel (flat and slow) because it eliminates the effect of Inter Sub-Carrier Interference (ICI). On the other hand, the perfect knowledge of the channel matrix at the receiver boosts up the performance of the linear detector and makes the process of MAI estimation straightforward.

C. Non-linear detector – Sequential search algorithm

In [4] this search algorithm was proposed in context of CDMA access and diversity at the receiver. Furthermore, the iterative form of the algorithm was proposed. In this paper, the same algorithm will be used and performed for QPSK modulated symbols. Additionally, the aim here is to prove that this concatenation of detectors is doable in context of SDMA-MIMO system model.

The method for mapping the modulated symbols with OFDM subcarriers was forced to maintain the fairness between the users [5] [6]. This means that Q, which is the number of modulated symbols per OFDM symbol, is equal per user. This leads to estimation of number of algorithm runs per OFDM symbol – Q. Hereafter, the algorithm specifications are described only for the n-th modulated symbol from every user, where $q = 1..Q$. If Q is equal to M (points of IFFT), the hardest case is encountered which is actually suitable for testing the efficiency of the non-linear algorithm in MAI environment.

The hard decision, made subsequent to MMSE MUD generates the input for the SS algorithm. For the q-th modulated symbols, the input combination can be expressed:

$$d_{Fq} = [d_{Fq1} ... d_{FqL}] \qquad (9),$$

where d_{Fqi}, $l = 1..L$ is demodulated combination of K bits, where K is assigned to be '2' in term of the QPSK modulation.

Having in mind the Hamming distance between two combinations:

$$M_d = \{ d : H_d(d_{Fq}, d_q) = 1 \} \qquad (10),$$

additional *LK* combinations are created based on the criteria.

Total of *LK*+1 combinations (including the initial one) are scope of the decision metric for SDMA-MIMO system, derived by the ML metric [3]:

$$\mathbf{\Delta}(\mathbf{S}) = \sum_{p=1}^{P} \mathbf{\Delta}_p(\mathbf{S}) \tag{11}$$

$$\mathbf{\Delta}_p(\mathbf{S}) = \left| \mathbf{X}_p - \mathbf{H}_p \mathbf{S} \right|^2 \tag{12}$$

The equation (11) solves the decision conflict so-called multiobjective optimization problem, since the optimization of the *P* metrics may result in more than one possible *L*-symbol solution. The equation (12) is the general expression of how the metric is calculated at every receiver. In (11) and (12), the parameter **S** is vector of *q*-th modulated symbols and has length of *L*. Once the needed symbols are extracted from the assigned subcarriers, the vector **S** can be formed and metric $\mathbf{\Delta}_p(\mathbf{S})$ can be calculated.

Solving (11) for every possible *LK*+1 combination will generate vector of *LK*+1 values, where the initial combination has index 1. From the theory of ML [3], the most likely combination that was sent is the one that leads to minimum value of the metric. Thus, the minimum is the factor in order to find the optimal combination from the pool. However, choosing the minimum value does not mean that the global minimum is attained. If the index with the minimum is not 1, than the optimal combination of the range becomes the initial input combination for the next iteration of the algorithm. As we stated above, in this paper the algorithm is broken at the first or third iteration and local minimum of the metric function is selected. Even with several iterations, this algorithm maintains the low computational complexity at the receiver in comparison to the extensive ML algorithm, based on full search.

III. Simulation Results

The algorithm is simulated in Matlab environment. The number of subcarriers assigned to particular user within OFDM symbol is equal per user and scheme without any user' differentiation is simulated. The modulation scheme is fixed to QPSK with Gray coding. The channel is modeled as AWGN with slow Rayleigh fading. Moreover, perfect channel estimation is assumed at the receiver.

These set options emphasize the MAI that depends only on the number of users. The other case which is more realistic for mobile channel is when fast fading (Doppler Effect) and frequency selectivity are taken into account. In that case MAI becomes more complex.

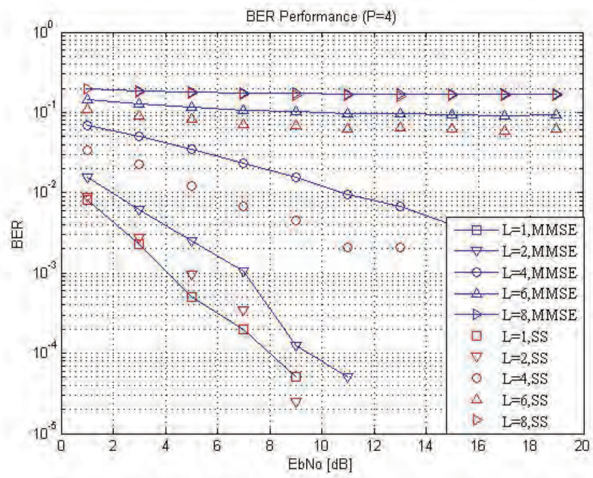

Fig.2 BER Performance – fixed P=4

Fig.2 shows the case of BER performance when the receiving antenna array has 4 elements. Both MMSE only and concatenation of MMSE and SS algorithms for MUD are simulated for different number of users. The parameter EbNo refers to the energy per bit to noise power spectral density ratio. When the case of single user is reviewed, we observe the lowest BER for the range of EbNo due to the lack of MAI effect. If L increases then the MAI will be obviously increased too due to the existing correlation between the users. This effect is inevitable and it is clearly shown on Fig.2 in case of L=2,4,6 and 8. All four multi-user scenarios has poorer performance than the single user scenario. The case L=8 has the worst performance and its curve for MMSE only MUD is on the top of the figure.

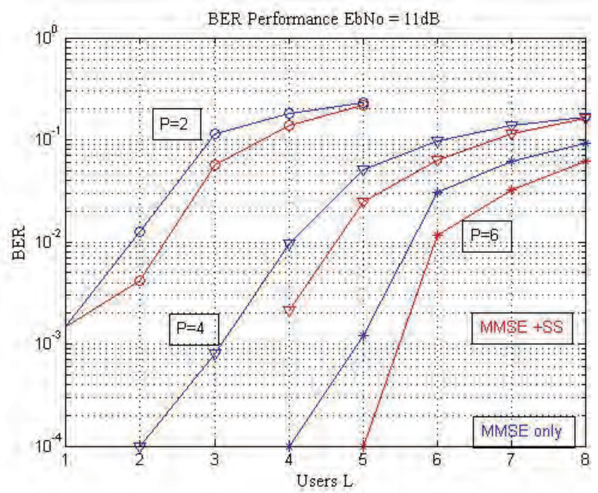

Fig.3 – BER performance, fixed EbNo=11dB

On the same figure, the effect of the concatenation of MMSE and SS MUD algorithms is presented as red symbols below every curve. The note here is that only three iterations have been invoked for every single simulated case. This fixed number of iterations can give us the real possibilities of this non-linear method. Obviously, the number of iterations is chiefly and proportionaly related to the number of users and therefore, fixing the errors for cases with lower L is more significant rather than fixing for higher L. In this context, as can be seen on the figure, the curve for L=2 is closer to the single user case where MAI is not encountered.

Fig.3 is based on fixed EbNo to 11dB and the number of receiving antennas is a variable. Considering P, three cases are shown on the figure such as P=2,4 and 6. As stated previously, this scheme inherits the MIMO benefits and that is why the increasing of receiving antennas will gradually decrease the bit errors. Again, it can be concluded from these cases that MMSE SS MUD algorithm is more efficient when less users are communicating with the BS. Hence, less users, better quality. Summing up, the approaches to rich better quality when L is relatively high are either increasing of P or the number of iteration. The first approach is the matter of physical presence. The latter, will drastically increase the computational complexity of the receiver. The trade-off between two must be considered when receiver is designed.

A. Computational complexity of the algorithm

The computational complexity of each non-linear detector is defined as the average number of calculations of the metric function (11) and that is determined by the number of sums and multiplications made. In [4] SS algorithm has been compared to the Genetic and Optimum algorithm. In this case the number of computations depend on the number of combinations to be checked for minimum metric. They have been calculated $LK+1$ per iteration and per modulated symbol, where K was fixed to 2 because of the QPSK modulation. On the other hand, (11) is mutually dependant on (12) which means the number of receiving antennas play important role when the number of sums is considered in (12).

IV. Conclusion

The combination of linear detector MMSE and non-linear method SS in context of SDMA-MIMO system is practically doable and leads to some improvements on BER performance. This was shown in this paper either with only few iterations invoked of the SS algorithm. More iterations can be invoked and eliminate MAI in multi-user scenarios. The computational complexity of the proposed sub-optimal algorithm depends on the number of iterations set, but, however, it is lower in comparison to the ML optimal algorithm. Apart from the MAI elimination, the presented system model inherits the MIMO benefits and therefore, the more receiving antennas are implemented, the fewer bits are mistaken.

V. References

[1] I. G. Iliev, M.Nedelchev, Performance Analysis of a suboptimal multi-user detection algorithm, ICEST 2007, Macedonia, pp.531 – 534.

[2] K.Yen and L.Hanzo, Hybrid genetic algorithm based multi-user detection schemes for synchronous CDMA system, in Proceedings of the IEEE Vehicular Technology Conference (VTC), (Tokio, Japan), May 15-18.

[3] L.Hanzo, M.Muster, B.J. Choi and T. Keller, OFDM and MC-CDMA for Broadband Multi-User Communications, WLANs and Broadcasting, Piscataway, NJ:IEEE Press, New York: Willey, 2003

[4] I. G. Iliev, M.Nedelchev, Antenna Diversity multi User Detection Algorithm for Synchronous CDMA System, ICEST 2009, Bulgaria, pp67-80

[5] H. G. Myung, D. Goodman, Single Carrier FDMA, a new interface for Long Term Evolution, Willey, 2009.

[6] Anja Sohl, Anya Klein, Comparison of Localized, Interleaved and Block Interleaved FDMA in terms of pilot multiplexing and channel estimation, EUSIPCO 2007, Poland