Analytical and Simulation Performance Result Analysis for Parallel M/M/1 Queuing System

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Abstract – **In this paper we display and compare the performance results, in terms of average system time, for a queuing system consisted of multiple M/M/1 subsystems. The analytical results are obtained by calculating the system time according to the commonly used formula for system time and the simulation results are obtained by simulating this system using MATLAB toolbox - SimEvents. The basic idea for this system is to simulate the behavior and performance of toll plaza. Therefore, the basic characteristics of the system are that the servers are identical and the switching of the entities through the queues is equiprobable. The point of this paper is to identify and calculate the deviation of the obtained results.**

Keywords – **Simulation, parallel M/M/1 queuing system, SimEvents, system time.**

I. INTRODUCTION

The inspiration for this paper came from real-life toll plazas. The area around the toll-plaza can be considered as a system of several M/M/1 queuing subsystems each one consisting of the traffic lane as a queue and the tollbooth as the server. Assuming that all drivers are rational, they always queue up to the shortest queue which results in queues of equal length before every tollbooth. The total incoming traffic flow (Φ) is evenly distributed and therefore the arrival rate in each subsystem depends on the number of tollbooths (N) which implies $\lambda = \Phi/N$. Another assumption is that all of the tollbooths apply identical manner of toll collection which means that they all have same service rate (μ). The formula for average system time in M/M/1 queuing system is [1]:

$$T = \frac{1}{\mu - \lambda} \qquad (1)$$

II. CONSTANTS

Our analytical results areobtained for constant values for the service rate and total traffic flow. The service rate μvaries

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depending on the technology that is used for toll collection. According to [2], the service capacity of several most used toll collection technologies are displayed in Table 1.

TABLE I
SERVICE CAPACITY OF TOLL COLLECTION TECHNOLOGIES

Person distributes change, issues receipt etc.	350 vph
Person distributes commuter tickets	500 vph
Automatic coin machine, regular coin only	500 vph
Automatic coin machine: some coins, tokens	650 vph
Mixed mode, any of the above plus ETC	700 vph

The arrival rate λdepends on the total traffic flow of the highway and the number of tollbooths. According to [3], the maximum traffic flow per lane is 2000 vph while the usual number of lanes can range from 1 to 6. The number of tollbooths is usually double from the number of lanes.

III. CALCULATIONS

We consider a system with μ=350 vph and Φ=1200 vph to be our sample system. By changing the number of tollbooths (N) and applying (Eq.1.) we obtained the analytical results displayed in Table 2. The system times are displayed in seconds.

TABLE II
ANALYTICAL RESULTS

N	4	5	6	7	8	9	10
T	72	32,73	24	20,16	18	16,62	15,65

It is obvious that by increasing the number of tollbooths, the arrival rate decreases, which causes lower system times.

IV. SIMULATIONS

The simulation results are gained from a simulation model built in SimEvents. SimEvents extends MATLAB toolbox Simulink with tools for discrete-event simulation of the transactions between components in a system architecture [4]. Our model is consists of several interconnected blocks from the SimEvents Block Library (Fig.1.). The simulation of the events starts with a Time-Based Entity Generator which
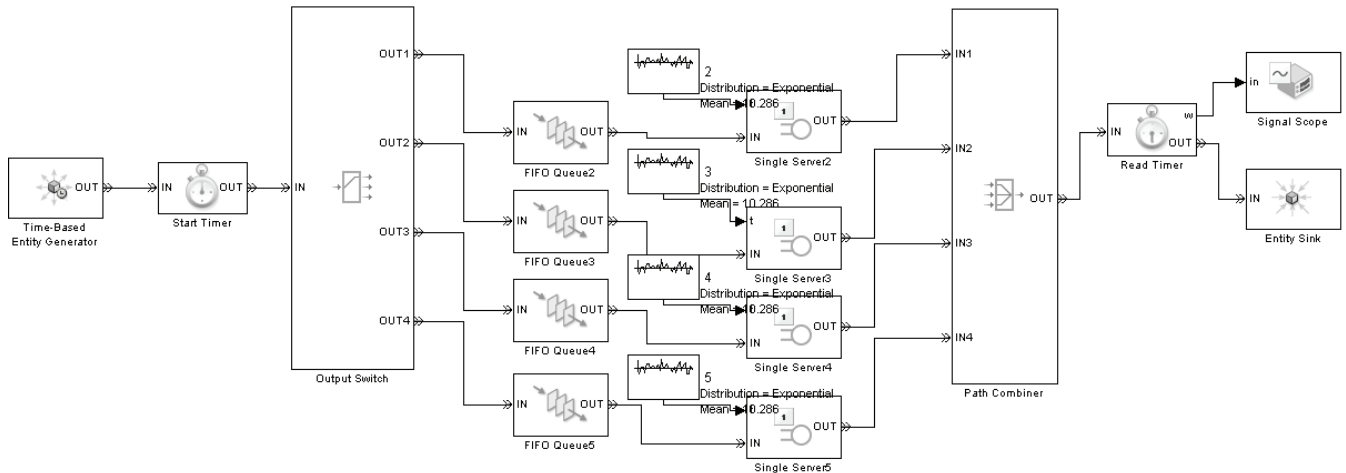
Fig. 1. The simulation model

generates entities every 3 time units (seconds) based on 1200 vph traffic load. The entities are then equiprobable distributed using an Output switch block through the FIFO Queue blocks and then conducted to the associating Single server block. Each of the single servers blocks has service time that corresponds with the 350 vph service rate of the tollbooth. The number of the existing Queue/Server pairs in the model depends on the number of tollbooths the simulation is taken for. After the completion of the service, the entities are combined with a Path Combiner block and then destroyed into an Entity sink block. The system time is measured by using Start timer block positioned before the Output switch and Read timer block positioned after the Path combiner block. The visual results of the plot acquired with the signal scope blockfor simulation with duration of 30000 time units are displayed in (Fig.2.) to (Fig.5.).
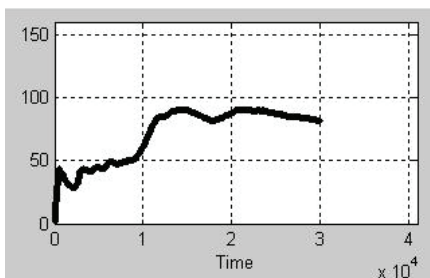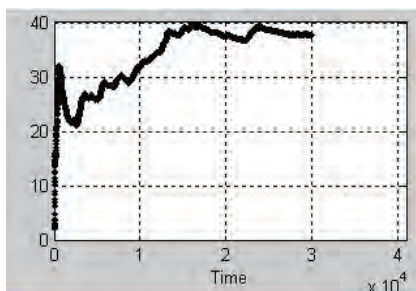

Fig. 2. Simulation results for N=4
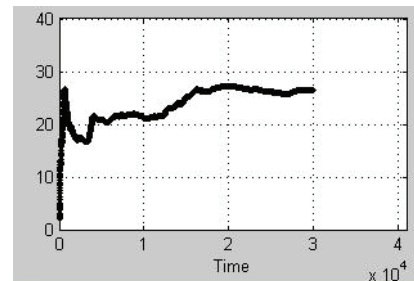

Fig. 3. Simulation results for N=5

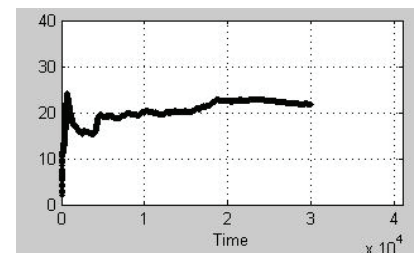Fig. 4. Simulation results for N=6

Fig. 5. Simulation results for N=7

V. CONCLUSION

It is obvious that the simulation results tend to get close to the values obtained by analytical calculations. That means that this simulation model is authentic because it verifies the analytical results.

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