A Matlab/Simulink Model of Piezoceramic Ring for Transducer Design

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Abstract – **In this paper a Matlab/Simulink model of thickness polarized piezoceramic rings, which is based on previously realized 3D matrix model of ring, is realized. Based on this Matlab/Simulink model it is able to compute all the relations between the input applied voltage and the output forces and velocities on every external surface. In order to compare the computed and experimental results, the input impedances for a piezoceramic rings with different dimensions are calculated and measured. Also, using the realized model the resonance and antiresonance frequencies for these rings are calculated and then compared with experimental results to validate the model. The proposed Matlab/Simulink model requires simpler implementation than mathematical model.**

Keywords – **Piezoceramic ring, Matlab/Simulink model, Resonance frequency characteristics, Input impedance.**

I. INTRODUCTION

Piezoelectric ceramics rings of different thicknesses and inner/outer diameters, especially lead zirconate titanate (PZT) rings, are widely used as active components in Langevine ultrasonicsandwich transducers for industrialapplications. In power applications of ultrasonic vibrations, piezoceramic rings are employed because of their good high electromechanical conversion efficiency.

Up to now various methods are proposed to model piezoceramic rings. Several one-dimensional (1D) models have been proposed to describe the principle modes of vibration of the piezoceramic rings in the thickness-extensional[1] and the radial modes [2].A three-dimensional (3D) approach is needed for proper modeling of piezoelectric transducer constructions with comparable lateral and thickness dimensions, such as e.g. sandwich transducers based on thick piezoelectric ceramic rings.Several 3D models also have been developed to analyze the piezoceramic rings[3,4]. Most of these3D models were based on simplified structures under simple loading (boundary) conditions.

In the present worka 3D Matlab/Simulink model to preview the behavior of a piezoceramic ring of any dimensions is proposed. The aim is to provide a simple and useful tool for the sandwich transducer design and optimization. The model is able to describe the composite vibrations both in the thickness and in the radial directions, and the coupling with the load and the backing.The piezoceramic ring is described, in the frequency domain, by previously realized approximated 3D piezoceramicring model [3]. Using the Matlab/Simulink model, the input impedance, as well as resonance and antiresonance frequencies are calculated and then compared to the experimental results.

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II. DESCRIPTION OF GOVERNING EQUATIONS

The typical piezoceramic ring geometry with outer radius a, inner radius b, thickness $2L$, and with completely metallized ring-shaped surfaces is shown in Fig. 1a. Every ring surface is loaded by acoustic impedance Z_i, where v_i and F_i are velocities and forces on those contour surfaces P_i (i=1,2,3,4).

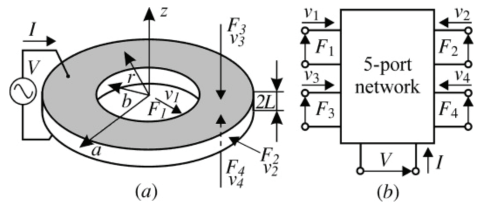

Fig. 1. Thickness-poled, electroded and loaded piezoceramic ring: (a)geometry and dimensions; (b) 5-port network representation

In paper [3],a 3D mathematical model is developed to describe the behavior of the thickness polarizedpiezoceramic ring.By means of this 3D model, the ring is modelled in the frequency domain as a five port system with one electrical and four mechanical ports, one for each external surface, by the following matrix form:

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ V \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{13} & z_{15} \\ z_{21} & z_{22} & z_{23} & z_{23} & z_{25} \\ z_{13} & z_{23} & z_{33} & z_{34} & z_{35} \\ z_{13} & z_{23} & z_{34} & z_{33} & z_{35} \\ z_{15} & z_{25} & z_{35} & z_{35} & z_{55} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ I \end{bmatrix} \quad (1)$$

Linear system of Eq. (1), which describes external behavior of the ring and relates the electrical (voltage V and current I) to the mechanical variables (forces F_i and velocities v_i) in frequency domain (Fig. 1b), is described in detail in paper [3]. The matrix elements are defined in Eq. (2), where $S=\pi(a^2-b^2)$ is ring area, $C_0=(\varepsilon_{33}^S S)/(2L)$ is the piezoceramic "clamped capacitance", c_{ij}^D are the elastic stiffness constants; ε_{33}^S is the clamped dielectric permittivity;$h_{ij}$are the piezoelectric tensor terms (i, j=1,2,3). $k_r = \omega / v_r$, $k_z = \omega / v_z$, $v_r = \sqrt{c_{11}^D / \rho}$ and $v_z = \sqrt{c_{33}^D / \rho}$ are the wave numbers and the phase velocities of the two uncoupled waves in the r and z directions respectively; ω is angular frequency; ρ is the piezoceramic density; J_1 and Y_1 are the first and the second kind of Bessel's functions of the first order.

$$z_{11} = \frac{-4\pi L}{j\omega}\left\{c_{12}^D - c_{11}^D\left[1 - k_r b\left(A_1 J_0(k_r b) + B_1 Y_0(k_r b)\right)\right]\right\},$$

$$z_{22} = \frac{4\pi L}{j\omega}\left\{c_{12}^D - c_{11}^D\left[1 + k_r a\left(A_2 J_0(k_r a) + B_2 Y_0(k_r a)\right)\right]\right\},$$

$$z_{12} = \frac{-4\pi k_r b L c_{11}^D}{j\omega}\left[A_2 J_0(k_r b) + B_2 Y_0(k_r b)\right], \quad z_{13} = \frac{2\pi b c_{13}^D}{j\omega},$$

$$z_{21} = \frac{-4\pi k_r a L c_{11}^D}{j\omega}\left[A_1 J_0(k_r a) + B_1 Y_0(k_r a)\right], \qquad (2)$$

$$z_{15} = \frac{4\pi b L h_{31}}{j\omega S}, \; z_{23} = \frac{2\pi a c_{13}^D}{j\omega}, \; z_{25} = \frac{4\pi a L h_{31}}{j\omega S}, \; z_{35} = \frac{h_{33}}{j\omega},$$

$$z_{33} = \frac{c_{33}^D k_z S}{j\omega \,\mathbf{tan}(2k_z L)}, \quad z_{34} = \frac{c_{33}^D k_z S}{j\omega \,\mathbf{sin}(2k_z L)}, \quad z_{55} = \frac{1}{j\omega C_0}.$$

The constants in Eq. (2) are defined as:

$$\begin{aligned} A_1 &= \frac{Y_1(k_r a)}{j\omega\left[J_1(k_r b)Y_1(k_r a) - J_1(k_r a)Y_1(k_r b)\right]}, \\ A_2 &= \frac{Y_1(k_r b)}{j\omega\left[J_1(k_r b)Y_1(k_r a) - J_1(k_r a)Y_1(k_r b)\right]}, \\ B_1 &= \frac{J_1(k_r a)}{j\omega\left[J_1(k_r a)Y_1(k_r b) - J_1(k_r b)Y_1(k_r a)\right]}, \\ B_2 &= \frac{J_1(k_r b)}{j\omega\left[J_1(k_r a)Y_1(k_r b) - J_1(k_r b)Y_1(k_r a)\right]}. \end{aligned} \qquad (3)$$

With this model, external behavior of the ring and all the transfer functions of the ring can be easily computed, taking into account the interaction with the surrounding mediaand the coupling between the thickness (T) and radial (R) modes.

III. MATLAB/SIMULINK MODEL

This section presents how the mathematical model of the piezoceramic ring, described in previous section, is implemented in Matlab/Simulink.

Matlab/Simulink is interactive software which has been used recently as design and development environment for model implementation in various areas of engineering and scientific applications[5].The graphical representation of models in Matlab/Simulink is based on block communication diagrams. Internally, the model is split into smaller separate functions blocks.

In order to demonstrate the advantages of this process, the Matlab/Simulink model of a piezoceramic ring is developed.At this point, it is possible to encapsulate the whole model in a Simulink blocks.A general schematic of this model is presented in Fig. 2.

Eqs. (1), (2) and (3) fully describe the model of the piezoceramic ring, which has been used for the simulation.

Realized model of a piezoceramic ring consist of two main blocks (Fig. 2). The first block gives all elements of the matrix in Eq. (1) as well as all the required coefficients a customized set of Eq. (4), which has been obtained through a series of simple mathematical operations. Calculating of these elements is based on entered characteristics and dimensions of used ceramic.

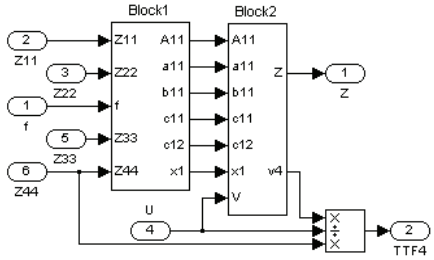

Fig. 2. Matlab/Simulink model of the piezoceramic ring

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ E_5 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ 0 & a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & 0 & b_{11} & b_{12} & b_{13} \\ 0 & 0 & 0 & c_{11} & c_{12} \\ 0 & 0 & 0 & c_{21} & c_{22} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ I \end{bmatrix} \qquad (4)$$

where is $E_5 = -A_{11} \cdot a_{11} \cdot b_{11} \cdot V$ and:

$$\begin{aligned} a_{i,j} &= A_{1,j+1} \cdot A_{i+1,1} - A_{1,1} \cdot A_{i+1,j+1} \quad (i, j = 1,2,3,4) \\ b_{i,j} &= a_{1,j+1} \cdot a_{i+1,1} - a_{1,1} \cdot a_{i+1,j+1} \quad (i, j = 1,2,3) \\ c_{i,j} &= b_{1,j+1} \cdot b_{i+1,1} - b_{1,1} \cdot b_{i+1,j+1} \quad (i, j = 1,2) \end{aligned} \qquad (5)$$

Since the calculation of coefficients in system of Eq. (4) requires model with a very complex structure, which includescomputing first and the second kind of Bessel's functions of the first order, the simplest way is to use embedded functions (*Embedded MATLAB Function block*). In this block,the functions that are not supported by Simulink, such as Bessel functions, is easy to declare with command *eml.extrinsic*.Also, since all the parameters to obtain these elements are previously defined, there is no risk of occurrence algebraic loops through this block.

The second part of the model use corresponding value obtained in the first block and solves the system of Eq. (4), and determines the input current I and input electrical impedance$Z=V/I$. The second block for model of a piezoceramic ring is shown in Fig. 3. The signal$x_1$is represented by the expression$x_1=c_{22}-c_{21}c_{12}/c_{11}$.

A brief overview of the internal structure of the others blocks is provided in Figs. 4 and 5. These figures contain the Simulink implementation of the mathematical models for determining of values for mechanical velocities on metalized ring-shaped surfaces (v_1 and v_2) and circular-curved surfaces (v_3 and v_4) of piezoceramic ring, respectively.

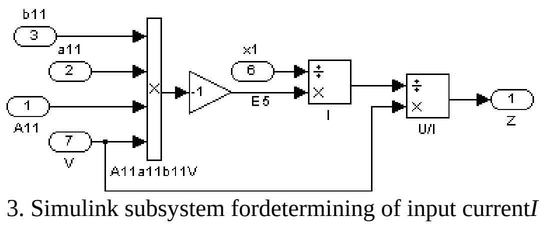

Fig. 3. Simulink subsystem fordetermining of input current$I$ by solving of equation system (4)

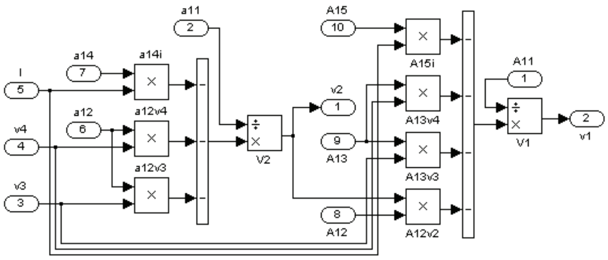

Fig. 4. Simulink subsystem for determining of values for mechanical velocities v_1 and v_2, on metalized ring-shaped surfaces of PZT ring

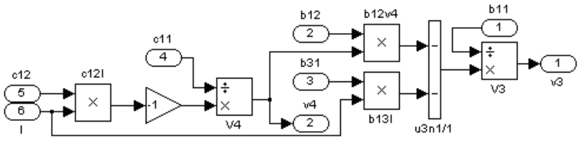

Fig. 5. Simulink subsystem for determining of values for mechanical velocities v_3 and v_4 on circular-curved surfaces of PZT ring

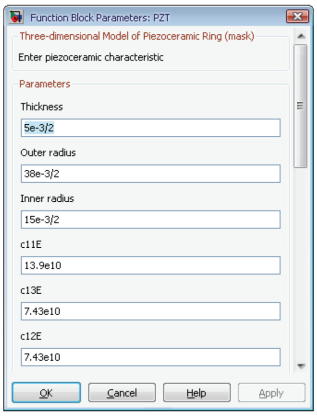

Fig. 6. Mask dialog block for the enter piezoceramic ring parameters

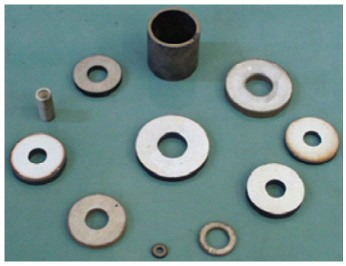

Fig. 7. Measured piezoceramic ring samples

Another powerful feature of the Simulink, called masking, is that it can simplify the use of the model by replacing many dialog boxes in a subsystem with a single dialog box. Instead of requiring the user of the model to open each block and enter piezoceramic parameter values, those parameter values can be entered on the mask dialog block and passed to the blocks in the masked subsystem. Fig. 6 illustrates how the mask dialog block for the piezoceramic ring looks like. The user has just to change the values of the parameters for different types of piezoceramic rings.

Loading the mechanical ports with the acoustical impedances of the surrounding media and by applying an *ac* voltage $V=V_0e^{j\omega t}$ to the electric port, it is possible to compute all the relations between the input applied voltage and the output forces and velocities on every external surface analytically, such as the electrical input impedance (V/I), the transmission (F_i/V) and the receiving (V/F_i) transfer functions.Our model can compute separate transfer functions for each external surface with arbitrary acoustic loads.

Thus development of the Matlab/Simulink model of the piezoceramic ring has been completed. The model provides numerous possibilities for the investigation ofpiezoceramic ringproperties. Verification of the created Matlab/Simulink model will be considered in the next section.

IV. Numerical and Experimental Results

The Matlab/Simulink model of the piezoceramic ring illustrated in Fig. 2, allows simulation of operation of the piezoceramic ring under different conditions. In order to obtain an experimental validation of the proposed model, the input electrical impedance of different piezoceramic rings are measured and compared with the computed results (if one assumes that the external medium is air).

Experimental input impedance curves of piezoceramic samples were measured with the frequency-sweeping apparatus (HP 4194A Network Impedance Analyzer). Seven samples of commercial PZT4 rings (Fig. 7) have been characterised. The PZT4 piezoceramic rings dimensions are given in Table I.

Table I
Piezoceramic Rings Dimensions

Sample	1	2	3	4	5	6	7
***2a* (mm)**	38	38	50	24	38	38	10
***2b* (mm)**	15	13	20	15	13	13	4
***2L* (mm)**	5	6.35	6.35	3	4	6	2

Fig. 8 shows a comparison between the measured and computed input electrical impedance for a 4th and 6th samplesof PZT4 rings. Computed results were carried out using PZT4 piezoceramic material constants [6].

As it can be seen, the form of the impedance curves is in accordance with experimental ones. According to piezoceramic rings dimensions, one of these vibrational modes is related to thickness mode (T) and the others are related to radial modes (R). Our model predicts with sufficient accuracy the first radial (R_1) and first thickness (T_1) modes, which are the mostly used in practical applications.

The resonance (f_p) and antiresonance (f_s) frequencies of piezoceramic rings are calculated with high accuracy. f_p is the frequency at which the electrical impedance of the ring reaches its minimum, and f_s is the frequency of minimum admittance. Using the proposed model, good agreement between simulated and experimental results for resonance and antiresonance frequencies is observed (Fig. 9). In this simulation up to four vibrational modes are presented for all piezoceramic rings.

As it is possible to see, the model is able to predict resonance and antiresonance frequencies with good accuracy both for the first radial mode, and for the first thickness mode. Because only these two modes are of relevance in the practical applications of piezoceramic rings as ultrasonic transducers, the model can be used as a simple and useful tool in transducer design and optimization. Difference between resonance and antiresonance frequencies obtained from simulation compared with experimental results in certain vibrational modes are occurrencesbecause this model includes only the thickness and radial resonant modes.

Results shown in Figs. 8 and 9 should not be used for very fine comparisons of measured and theoretical results, by using only typical values for the material constants [6]. The computed results can be improved by fitting the constants of the piezoceramic material. Further, some measured modes are not predicted by the model, probably because they are shear modes. However, general trends can be observed.

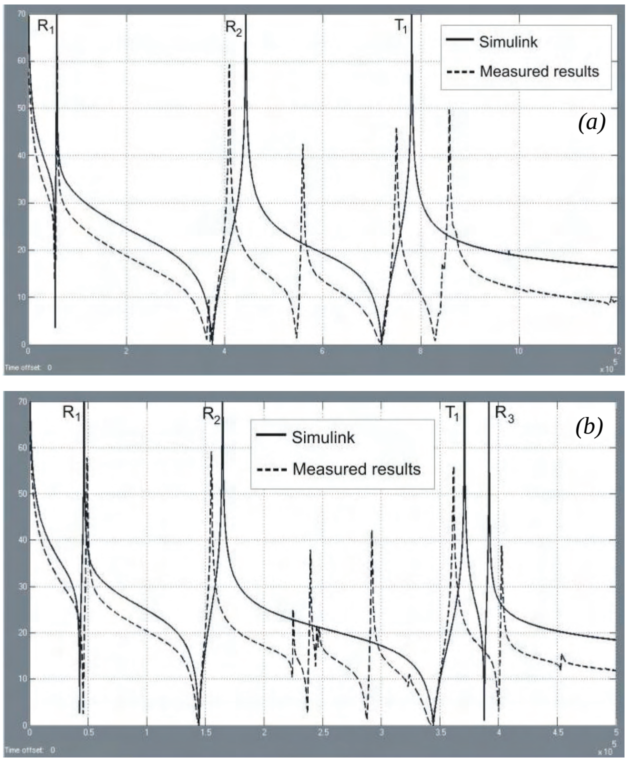

Fig. 8. Simulated and experimental input electrical impedance versus frequency for the4th sample (a)and 6th sample(b)

V. CONCLUSION

In this paper an accurate piezoceramic ring model valid to any diameter to thickness ratio is realized and demonstrated in Matlab/Simulink for a typical PZT rings. This model taking the interaction with the surrounding media into account is able to compute all the ring transfer functions, such as the input electrical impedance.External behavior of the piezoceramic ring is described in frequency domain by a system with four mechanical ports (one for each external surface) and one electrical port.

The piezoceramic rings with different dimensions are analyzed using the developed model. The comparison experimental and theoretical results are quite good and validate the new design approach. Firstly, the electrical input impedance of samples with different dimensions was computed by the model and compared with experimental results, obtaining a good agreement. After that, the resonance and antiresonance frequencies are calculated using the model and then compared to the experimental results. Such comparison also shows satisfactory agreement.

The Matlab/Simulink model gives a very good prediction of the piezoceramic ring behavior, although only the electrical impedance is studied when the ring is without outer load. It can be applied successfully to design and optimize ultrasonic sandwich transducers for industrial applications.In a future work our aim is to improve the performance of the model in order to obtain a reliable tool for more complex Langevine ultrasonic sandwich transducer design.

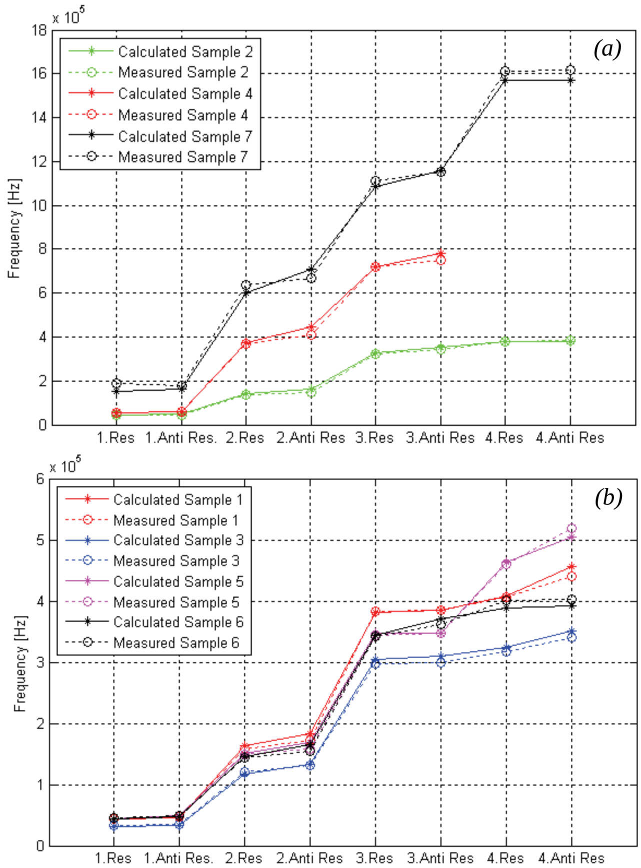

Fig. 9. Comparison between calculated and experimental resonance and antiresonance frequencies for the samples 2, 4 and 7 (a) and samples 1, 3, 5 and 6 (b)

Slightly modified, the realized model is also applicable to piezoceramic disks. Therefore, it is a useful tool for transducer manufacturers and material scientists.

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