Multiuser IR-UWB System Performance

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Abstract – ***This paper analyzes a multiuser system performance under the Standard Gaussian Approximation - SGA. In particular it focuses on the analysis of the multiuser interference contribution. Simulations have been performed for a pulse position modulation and a pulse amplitude modulation for 5, 20 and 50 interfering users. The performances have been evaluated in terms of the error probability over the signal to noise ratio, assuming that the signal is transmitted through a AWGN channel.***

Keywords – **Ultra Wideband communication,** ***Standard Gaussian Approximation,*** **Probability of Error.**

I. INTRODUCTION

Ultra Wideband Technology (UWB) has been described as one of the most promising technologies during the last decade [1-3]. It offers the possibility of achieving higher rates for indoor systems with a reduced range of action due to the resistance encountered in multi-path environments. Moreover, this technology offers lower implementation cost and reduced power requirements then most of other technologies mentioned in the literature [1-2]. Taking into consideration the FCC regulations, a UWB is defined as being any signal in which the 3 dB bandwidth is at least 25% of the central frequency or any signal with a bandwidth larger than 500 MHz [1].

The UWB radio channels can use frequencies from 3.1 GHz to 10.6 GHz, using a frequency bandwidth larger then 7GHz, with the restrictions imposed by the spectral frequency-power masks given in standards [1]. Every radio channel may occupy a bandwidth of at least 500 MHz, in accordance to its central frequency. Regarding the multiple access techniques used, the original proposal for UWB was to use Time Hopping – TH combined with the Pulse Position Modulation – PPM. Later on, various modulation techniques have been used: PAM (pulse amplitude modulation), OOK (On-off keying) or other multiple access techniques such as DS (direct sequence) [2].

In this paper we will present the performances of two types of pulse modulation, in position and in amplitude (TH-PPM and TH-PAM). The communication channel will be an modeled as affected by Additive White Gaussian Noise (AWGN), while the UWB signal will be an Impulse Radio Ultra Wideband signal (IR-UWB). The IR-UWB transmitting method is the most common way of transmitting UWB signals. This type of signals are radiated pulses very short in time.

Several methods of evaluating the effects of the multi-user interference have been developed over the years [3,4]. This paper focuses on the Standard Gaussian Approximation (SGA) hypothesis, which models the effects of all the interferences as a Gaussian additive noise, with a uniform power spectrum distribution over all frequencies of interest. The hypothesis is very accurate for a large number of interfering users, and has very optimistically results for low values of bit rates or for a small number of interfering users.[

II. THE PERFORMANCES OF A MULTI-USER IR-UWB SYSTEM IN SGA HYPOTHESIS

In this section we will present the general simulation scenario and the analytical expressions of the probability of error for the two types of modulation which will undergo our analysis, PPM and PAM.

In the following work, the following assumptions are assumed to be valid:

1. All sources produce binary vectors **b,** b_k={0,1}, $\forall k$.
2. All sources use the same pulse code period frequency, T_s.
3. The spreading codes $c_k \in \{\pm 1\}, (\forall)k$ are independent and equally likely, with the same code period T_c.
4. For each transmission/reception path, a different code – known at the receiver - is used.
5. It is assumed that the base impulse has a limited duration, T_m and a symmetrical frequency shape.
6. Propagation is achieved on a channel with multi-paths. For a given user, n, the channel impulse response is a function of the path gain, $\alpha(n)$, and of the path time delay $\tau(n)$. Delays are considered to be independent and uniformly distributed within the $[0,T_s)$ interval. The channel impulse response is thus given by:

$$h^{(n)}(t) = \alpha^{(n)}\delta(t-\tau^{(n)}) \qquad (1)$$

7. The channel is affected only by AWGN, with the spectral power density of $N_0/2$(W/Hz)
8. Coherent single user correlation reception is implemented at the receiver for all users. The signal will be thus correlated with the user code and integrated over a bit period $T_b=N_sT_c$.

A. TH-PPM

The binary signal, TH-PPM, transmitted by user n can be written as [3,4]:

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$$s_{TX}^{(n)}(t)=\sum_{j=-\infty}^{\infty}\sqrt{E_{TX}^{(n)}}\,p_0(t-jT_s-c_j^{(n)}T_c-a_j^{(n)}\varepsilon) \quad (2)$$

where $p_0(t)$ is the normalized base impulse, $E_{TX}^{(n)}$ is the energy transmitted by each pulse, $c_j^{(n)}T_c$ is the time shift imposed by the TH code, $c_j^{(n)}$ is the j-th TH code sequence used by user n, and T_c is the chip duration. Each TH code is a sequence of N_p identically and independently distributed random variables, each of them with a probability of $1/N_h$, and with values within $[0,N_h\text{-}1]$ interval, where N_h is the cardinality of the TH code. In order to identify the users, each of them will be assigned a specific TH code in such a way to avoid collision at the receiver. The term $a_j^{(n)}\varepsilon$ the time-shift introduced by the data; ε is the specific PPM time delay and a_j is the binary value assigned to the jth pulse for user n. The binary vector $\boldsymbol{a}$ represents the output of a $(N_s,1)$ repetition coder, that receives as input the binary vector $\boldsymbol{b}$, meaning that N_s pulses carry the information of one bit. The binary vector $\boldsymbol{a}$ length is the length of $\boldsymbol{b}$ time N_s.[1,3,4,5]

Assuming that the channel is modeled by Eq. (1), in the presence of AWGA noise the received signal can be written as

$$r(t)=\sum_{n=1}^{N_u}\sum_{j=-\infty}^{\infty}\sqrt{E_{RX}^{(n)}}\,p_0(t-jT_s-c_j^{(n)}T_c-a_j^{(n)}\varepsilon-\tau)+n(t) \quad (3)$$

where N_u is the number of users and E_{RX} is the energy of each transmitted pulse at the receiver. [1,3,4,5].

Referring to user (1) and assuming that the receiver is perfectly synchronized, such that the time delay is accurately known at the receiver and can be assumed 0, for simplicity, the received signal can be written as [4]:

$$r(t)=r_u(t)+r_{mui}(t)+n(t) \quad (4)$$

Next, focusing our analysis on the bit interval T_b and taking into account the symmetry of the system, the analysis can be performed within $[0,T_b]$ interval. The $r_u(t)$ and $r_{mui}(t)$ contribution can be written for $t\in[0,T_b)$ as[4]:

$$r(t)=\sum_{j=0}^{N_S-1}\sqrt{E_{RX}^{(1)}}\,p_0(t-jT_s-c_j^{(1)}T_c-a_j^{(1)}\varepsilon) \quad (5)$$

$$r_{mui}(t)=\sum_{n=1}^{N_U}\sum_{j=-\infty}^{\infty}\sqrt{E_{RX}^{(n)}}\,p_0(t-jT_s$$

$$-c_j^{(n)}T_c-a_j^{(n)}\varepsilon-\tau^{(n)}) \quad (6)$$

In the decision process performed at the reception, the correlation output at reception is thus given by [2,4,5]:

$$Z=\int_0^{T_b}r(t)m(t)dt \quad (7)$$

,where $m(t)$ is the correlation mask upon reception defined by:

$$m(t)=\sum_{j=0}^{N_S-1}v(t-jT_s-c_j^{(1)}T_s) \quad (8)$$

$$v(t)=p_0(t)-p_0(t-\varepsilon) \quad (9)$$

Combining Eqs. (7) and (9) we obtain [4]:

$$Z=Z_u+Z_{mui}+Z_n \quad (10)$$

Under the SGA hypothesis, Z_{mui} and Z_n represents random Gaussian processes with a 0 mean and a variance of σ_{mui}^2 and σ_n^2 respectively. The average bit error rate Pr_b can be written as [3,4,5]:

$$\Pr_b=\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{2(\sigma_n^2+\sigma_{mui}^2)}}\right)$$

$$=\frac{1}{2}erfc\left(\sqrt{\frac{\left((SNR_n)^{-1}+(SIR)^{-1}\right)^{-1}}{2}}\right) \quad (11)$$

The bit energy of the received signal, E_b, can be obtained by calculating the energy of the useful components at the output of the receiver for all N_s pulses that form a bit. Therefore

$$E_b=E_{RX}^{(1)}N_s^2(1-R_0(\varepsilon))^2 \quad (12)$$

,where $R_0(t)$ is the autocorrelation function of the base impuse $p_0(t)$ pulse. In the presence of the thermal noise, the signal to noise ratio (SNR) can be written as:

$$SNR_n=\frac{N_sE_{RX}^{(1)}}{N_0}(1-R_0(\varepsilon))=\frac{E_b^{(1)}}{N_0}(1-R_0(\varepsilon)) \quad (13)$$

Regarding the signal to interferences ratio (SNI) it can be written as

$$SIR=\frac{E_{RX}^{(1)}N_s^2(1-R_0(\varepsilon))^2}{\frac{1}{T_s}N_s\sigma_M^2\sum_{n=2}^{N_u}E_{RX}^{(n)}}=\frac{(1-R_0(\varepsilon))^2\gamma_R}{\sigma_M^2}\frac{1}{R_b\sum_{n=2}^{N_u}\frac{E_{RX}^{(n)}}{E_{RX}^{(1)}}} \quad (14)$$

By combining Eqs. (13) and (14) and replacing them in Eq. (11) we obtain the average bit error rate [3,4,5]:

$$\Pr_b=\frac{1}{2}erfc\left(\sqrt{\frac{1}{2}\left(\left(\frac{E_b^{(1)}}{N_0}(1-R_0(\varepsilon))\right)^{-1}+\left(\frac{(1-R_0(\varepsilon))^2\gamma_R}{\sigma_M^2R_b\sum_{n=2}^{N_u}\frac{E_{RX}^{(n)}}{E_{RX}^{(1)}}}\right)^{-1}\right)^{-1}}\right) \quad (15)$$

C. TH-PAM

The TH-PAM signal can be analyzed by following a similar procedure as the one presented above. The binary signal transmitted by user n can be written as [6]:

$$s_{TX}^{(n)}(t) = \sum_{j=-\infty}^{\infty} \sqrt{E_{TX}^{(n)}}\, a_j^{(n)} p_0(t - jT_s - c_j^{(n)} T_c) \qquad (16)$$

The signal at the output of the correlator at reception has the same expression as in Eq. (7) with a correlation mask $m(t)$ defined as [1,2,6]:

$$m(t) = \sum_{j=0}^{N_S - 1} p_0(t - jT_s - c_j^{(1)} T_c) \qquad (17)$$

With the same decision criterion as in the above paragraph, the error probability, Pr_b obtained in Eq. (11) is still valid.

The expressions for the signal to noise ratio (SNR) in the presence of the thermal noise, as well as the signal to interference ratio (SIR) can be written as:

$$SNR_n = \frac{N_s E_{RX}^{(1)}}{\frac{N_0}{2}} = \frac{2E_b^{(1)}}{N_0} \qquad (18)$$

$$SIR = \frac{E_{RX}^{(1)} N_s T_s}{\sigma_M^2 \sum_{n=2}^{N_u} E_{RX}^{(n)}} = \frac{\gamma_R}{\int_{-T_M}^{T_M} R_0^2(\tau)d\tau} \frac{1}{R_b \sum_{n=2}^{N_u} \frac{E_{RX}^{(n)}}{E_{RX}^{(1)}}} \qquad (19)$$

Therefore, the probability of error, Pr_b, is given by [6]

$$\Pr_b = \frac{1}{2} erfc\left(\sqrt{ \frac{1}{2} \left(\left(\frac{2E_b^{(1)}}{N_0} \right)^{-1} + \left(\frac{\gamma_R}{R_b \sum_{n=2}^{N_u} \frac{E_{RX}^{(n)}}{E_{RX}^{(1)}} \int_{-T_M}^{T_M} R_0^2(\tau)d\tau} \right)^{-1} \right)^{-1} } \right) \qquad (20)$$

III. Numerical Results and Simulations

As far as the numerical results are concerned we will analyse the performances of a IR-UWB system in the presence of the multi-user interferences (MUI). First, we will evaluate the probability of error, Prb, in the case of using the TH multiple access technique both for a PPM binary modulation and for a PAM one. In both circumstances Prb will be estimated in accordance with the theoretical results obtained in Sections II.A. and II.B.

We compared the performances for PPM and PAM in three different scenarios, using 5, 20 and 50 interference signals. The results are presented in Figs 1, 2 and 3. The transmitted signal has a rate of 20 Mbit/s. The used pulse is given and has the shape of the second Gaussian derivative with a shaping factor of 0.25ns. In the case of PPM the timeshift is ε=0.5 ns.

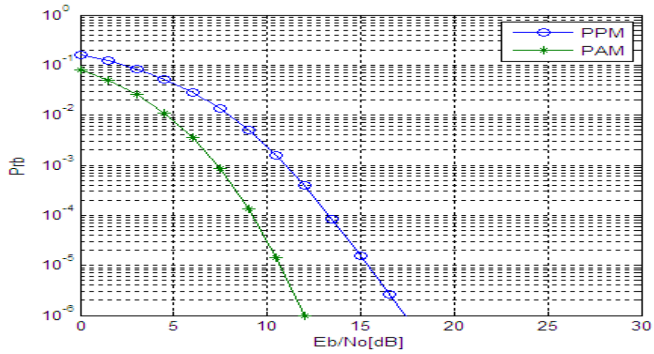

Fig. 1 Probability of error for 5 users and PAM and PPM

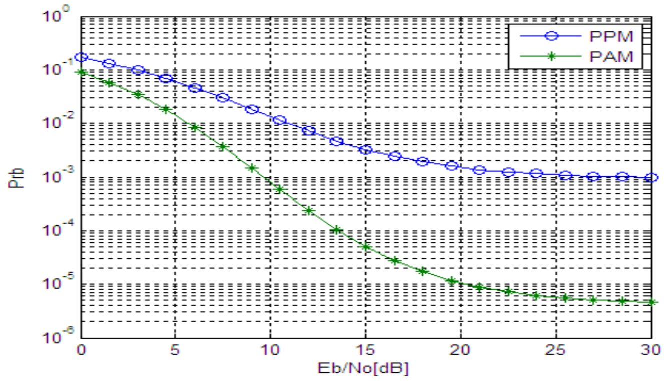

Fig. 2 Probability of error for 20 users and PAM and PPM

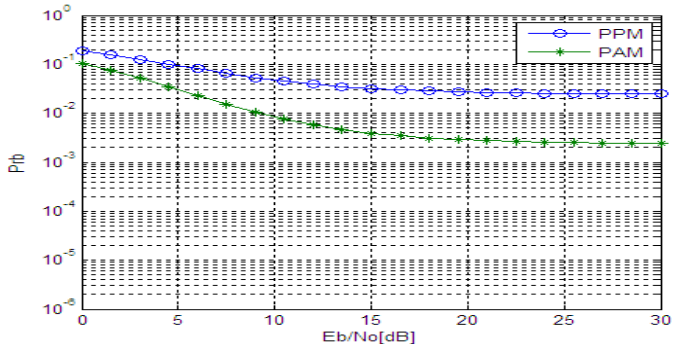

Fig. 3 Probability of error for 50 users and PAM and PPM

From these figures we can extract the following conclusions. First, in Fig. 1 we can notice that the multi-user interference term can be neglected, because the probability of error decreases as the SNR increases. We can say that, if the number of users is small enough, only the thermal noise affects the probability of error. The E_b/N_0[dB] distance between PAM and PPM is approximately 3dB, for a $Pr_b=10^{-2}$, and it increases for the lower values of Pr_b. From Figs. 2 and 3 we can observe that the probability of error Pr_b tends to a constant value, as the signal to noise ratio increases, showing the fact that, for large E_b/N_0, the system performances are dominated by the multi-user interference. We can, moreover, identify two regions: for low values of E_b/N_0, Pr_b is determined mostly by the thermal noise, in which case we can improve the performances of the system by increasing the transmission power; for high Eb/N_0 ratios, the systems performances trends asymptotically to a constant value and does no longer depend on Eb/N_0. In this case the performances are dominated by the multi-user interference effects.

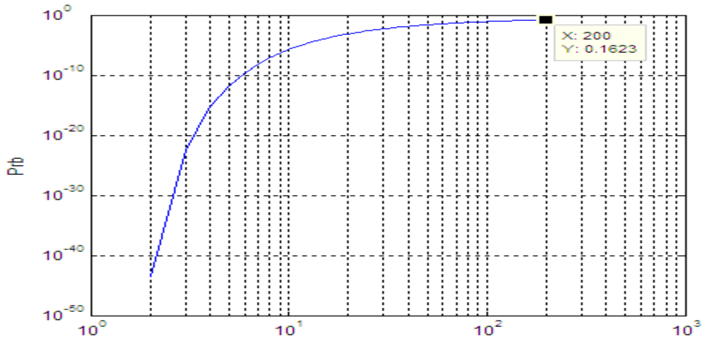

Fig. 3 Probability of error for 1: 200 users

As the number of users increases, the Pr_b is asymptotically limited to a higher value. If the number of users increses up to 50, the probability of error decrease only to 2 $10^{-3,}$ in the PAM case, and to 2.5 $10^{-2,}$ in the PPM case. All graphs shows that PAM is slightly more robust then PPM with respect to Pr_b performances.

As it can be seen in Figs. 2 and 3, Pr_b increases with the augmentation of the number of users. In order to evaluate the performances of the system with the number of users increases. we represented graphically Pr_b as a function of number of users. Thus, in Fig. 4 we can see that in the presence of more than 200 users, the error probability is very high, with a magnitude order of 10^{-1} , 10^{-2}, and the system is dominated by M.U.I.

In order to validate the theoretical results, we simulated the UWB receiver in the presence of multi-user interference. We have been particularly interested in how precise the SGA hypothesis used for error probability estimation is fulfilled. We simulated a system with 7 and 10 users. Each user generates stream of data with a bit period *Tb*=18ns, leading to a bit rate of 55.55Mbit/s. Every bit period is organized in 3 frames with a duration *Ts*=6ns, meaning 3 pulses are transmitted for every bit. Each frame is then divided into 6 slots with a length of T_c=1ns. All users transmit with the same format of the signal. The results are shown in Fig. 5.

We can notice that the theoretical model used for the evaluation Pr_b underestimates the effect of MUI. The error probability obtained based on the simulation is larger than the one the SGA hypothesis predicted. In the 7 user case the difference between the error probability from the theoretical model and the simulated one is larger than in the 10 user case:

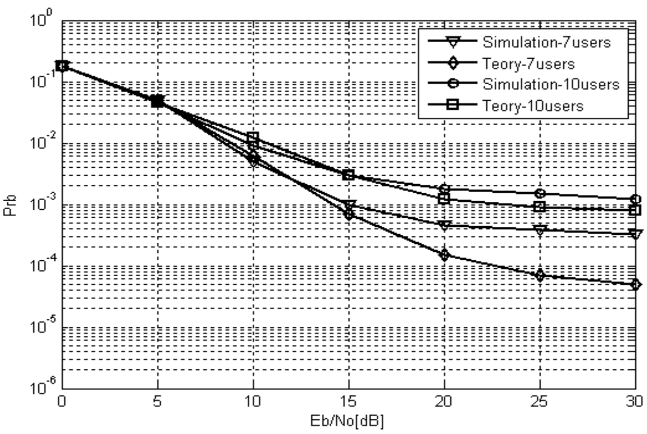

Fig. 4 Comparasion between theoretical ans simulation results for 7 and 10 users

IV. Conclusion

In this paper we have evaluated the performances of an IR-UWB system in the presence of multi-user interferences and in the Standard Gaussian Approximation hypothesis. We have noticed that the multi-user interferences influence on the performances of such a system is important only if the number of users is large enough. We have also noted that the PAM modulation is far more robust than the PPM one, achieving error probabilities ten times smaller at the same E_b/N_0 ratio. As the number of users increases, the Pr_b is asymptotically limited to a higher value.

Next, investigating the validity of the SGA hypothesis in comparison with simulation results, we observed that, for a relatively low number of users (up to 10) the results obtained based on simulation are better with respect to the error probability then the ones obtained under the SGA assumptions, showing that the formulas developed represents an upper limit for the Pr_b.

In a future work we will focus on increasing the number of users in the simulation, developing simulations under different sets of parameters and checking other hypotheses, like the multi-user interference model based on package collision or the chip-synchronous hypothesis.

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