Fast Synthesis of High Order Microwave Filters by Coupling Matrix Optimization

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Abstract: This paper presents optimization method for synthesis of generalized high order microwave filters with arbitrary topology. The method utilizes local optimizer for coupling matrix determination. The synthesis procedure converges very fast as for a initial point is used a vector based on the Chebyshev all pole filter for the same degree of the filter. To validate the proposed synthesis method two numerical examples for resonant filters are computed. The frequency responses from the synthesis procedure and the theoretical responses show excellent agreement.

Keywords: microwave filter, Chebyshev filter, Nelder-Mead optimization, coupling matrix.

I. INTRODUCTION

Microwave coupled resonator filters play important role in the modern communication systems. The constraint RF/microwave spectrum requires high attenuation in the stop band and low insertion loss in the passband of the filters. These requirements can be met only by cross-coupled microwave filters, realizing attenuation poles on finite frequencies. Cross-coupled resonator filters allow using various topologies with variety of frequency responses. The microwave filter modelling is very important for the fast and accurate design.

Key point in the obtaining of the coupling matrix corresponding to the practical filter topology is to convert its transversal form to folded form using matrix rotations. Most of the matrix rotation sequences are given in [4]. It is noticed that this method for synthesis suffers from generality, because the matrix rotations cannot be derived for every one practical filter topology. Some of the matrix rotation sequences cannot converge in order to find the coupling matrix. Some of the disadvantages in this method are solved if arrow form of the coupling matrix is used [5] or Pfitzenmeir method is used [6].

In many practical cases, it is necessary to define the filter topology in order to satisfy some manufacturing or space requirements. In this case, the exact solution is hard to be found utilizing the conventional synthesis methods.

One possible general solution to the filter design for arbitrary topology is to apply direct local optimization over the coupling matrix with successive starting point. In the basic papers proposed optimization method for coupling matrix synthesis [7,8], the starting vector is set to arbitrary values. This makes the local optimization very unstable method for cost function minimization. Another method is to use global optimization method for finding the coupling matrix for certain filter topology.

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They perform robust optimization, no matter about the starting point. Unfortunately the global optimizers such as genetic or stochastic have very slow convergence to the cost function minimum.

This paper presents optimization method for synthesis of high microwave filters with arbitrary topology. The method uses Nelder-Mead local optimizer for coupling matrix determination. The synthesis procedure converges very fast as for a initial point is used a vector based on the Chebyshev all pole filter for the same degree of the filter. The cost function is based on amplitude of the transmission and reflection coefficient zeros and their values at the cut-off frequencies and the reflection coefficient maxima. To validate the proposed synthesis method two resonant filters are designed with asymmetrical responses. The frequency responses from the synthesis procedure and the theoretical responses show excellent agreement.

II. RESONATOR FILTER CHARACTERISTICS

The synthesis procedure starts with the low-pass prototype with normalized angular frequency of passband $\omega = 1$. The transfer and reflection coefficients may be expressed as a ratio of two N-th degree polynomials as follows:

$$S_{21} = \frac{P_N(\omega)}{E_N(\omega)}, S_{11} = \frac{F_N(\omega)}{\varepsilon E_N(\omega)} \quad (1),$$

where ω is real angular frequency and $\varepsilon = \left(1/\sqrt{10^{RL/10}-1}\right).\left(F_N(\omega)/P_N(\omega)\right)\Big|_{\omega=1}$, RL is the prescribed value of the return loss in dB, in the passband of the filter. It is assumed that all polynomials are normalized to their highest degree coefficient.

The method of computing the numerator of the reflection coefficient is outlined in [3].

$$F_N(\omega) = \frac{1}{2}\left(G_N(\omega) + G'_N(\omega)\right), \quad (2),$$

where both polynomials can be represented by two polynomials: $G_N(\omega) = U_N(\omega) + V_N(\omega)$ and $G'_N(\omega) = U_N(\omega) - V_N(\omega)$. Both polynomials $U_N(\omega)$, $V_N(\omega)$ can be arranged according to the Cameron's recursive procedure in [3]. Obviously the roots of $U_N(\omega)$ corresponds reflection zeros, and the roots of $V_N(\omega)$ correspond to the in-band reflection maxima.

It can be easily found that the transfer coefficient may be expressed in the following way[3]:

$$S_{21}^2(\omega) = \frac{1}{1+\varepsilon^2 C_N^2(\omega)} \quad (3)$$

where $C_N(\omega)$ is the filtering function. For general Chebyshev characteristics, the filtering function is in the form:

$$C_N(\omega) = \cosh\left(\sum_{n=1}^{N} a\cosh(x_n)\right) \quad (4),$$

where $x_n = \dfrac{\omega - 1/\omega_n}{1 - \omega/\omega_n}$, where ω_n is the angular frequency of the prescribed transmission zero.

In order to obtain the coupling matrix, it is necessary to consider the equivalent circuit of general coupled resonator filter shown on Fig.1.

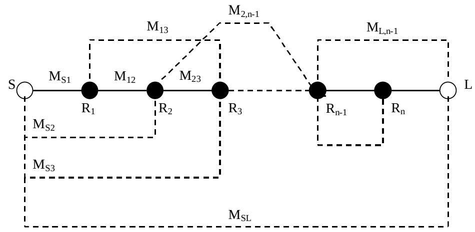

Fig.1. General coupled resonator filter

The equivalent circuit consists of *N* series coupled resonators with frequency independent couplings M_{ij} ($i \neq j$), between the *i-th* and *j-th* resonators. The circuit is driven by voltage source *E* with internal normalized resistance $R_1 = 1$ and loaded to normalized impedance $R_2 = 1$. The resonant frequency of each resonator f_{0i} is represented by the self-coupling coefficient M_{ii} and the center frequency of the filter. The transmission and reflection coefficients of a lossless filter of *N*-th order depend only of the coupling matrix $|M|$ (7):

$$S_{21} = -2j[A]^{-1}_{N+2,1}, \qquad S_{11} = 1 + 2j[A]^{-1}_{11}, \quad (6),$$

where $[A] = -j[R] + \omega[W] + [M]$, and $[R]$ is $(N+2)\times(N+2)$ matrix, which elements are zeroes except $R_{11} = R_{N+2,N+2} = 1$. $[W]$ is a $(N+2)\times(N+2)$ matrix, where the main diagonal elements are unity except $W_{11} = W_{N+2,N+2} = 0$. All remaining elements of $[W]$ are zeroes. $[M]$ is the coupling matrix, symmetrical around the main diagonal.

III. Synthesis Of Microwave Filter With Coupling Matrix Optimization

The cost function used in the optimization process is based on the zeroes and poles of the filtering function C_N , assuming that the number of poles is *P* and zeroes *N* [8]:

$$Cost = \sum_{i=1}^{N}\left|S_{11}(\omega_{zi})\right|^2 + \sum_{i=1}^{P}\left|S_{21}(\omega_{pi})\right|^2 + \left(\left|S_{11}(\omega=-1)\right| - \frac{\varepsilon}{\sqrt{\varepsilon^2+1}}\right)^2 + \left(\left|S_{11}(\omega=1)\right| - \frac{\varepsilon}{\sqrt{\varepsilon^2+1}}\right)^2 + \sum_{i=1}^{N-1}\left(\left|S_{11}(\omega=\omega_{mi})\right| - \frac{\varepsilon}{\sqrt{\varepsilon^2+1}}\right)^2 \quad (7).$$

In the cost function ω_p are the prescribed transmission zeros, ω_z are the zeros of the reflection coefficient, and ω_m are the in-band maxima frequencies. In most papers concerning the optimization of the coupling matrix the last term of the cost function is missing. Because of the high order of the filter, the value of the transmission coefficient at the prescribed zeros ω_p is comparable to the precision of the computer. This make the optimization process hard to converge at the global minimum of the cost function. The global minimum is the Chebyshev solution for the microwave filter. As the values of the second term of the cost function needs to be weighted, in order to achieve comparable values to the other terms of the cost function. Obviously there will come up a problem with the choice of the weighted constant. For each filter topology and frequency response, a different constant will be necessary. One possible solution for the problem with the weights is to make each term of the cost function in logarithmic scale with no weight coefficient. Another solution is to add to the cost function another term equalizing the reflection coefficient at its maxima to the ripple factor ε . In this case the cost function contains all possible constraints for the filter response. The zeros for the transmission coefficient S_{21} are set at the prescribed frequencies. The reflection coefficient must be zero at the frequencies ω_z , equal to $\varepsilon/\sqrt{\varepsilon^2+1}$ at the normalized cut-off frequencies $\omega_{cut-off} = \pm 1$ and equal to $\varepsilon/\sqrt{\varepsilon^2+1}$ at the frequencies ω_m at the minimum of the cost function. The cost function may be modified with respect to the transmission coefficient at the frequencies ω_m . At these frequencies S_{21} must be equal to ε , but the cost function will not be changed in its character.

In this way it is possible to formulate the local optimization problem for obtaining the coupling matrix.

The starting point for optimization of the coupling matrix is very important for the reaching of the global minimum of the cost function (7). Having on mind that a local optimizer is used, the starting vector should be close to the target value in order to assure a fast convergence of the method. One of the possible starting coupling matrices is to set all self-coupling couplings to zero ($M_{ii} = 0$) and all direct couplings to 1. The cross-coupling coefficients are all set to zero. The second possible starting coupling matrix is to use classical Chebyshev filter from the same order. All self- and cross-couplings are set to zero.

The investigation of the problem of high order filter design two numerical designs are investigated.

III. Numerical Results

For verification of the optimization method presented in this paper, it is applied to an asymmetric resonator filters.

A. Asymmetric 9 Resonator Passband Filter

The first numerical example is 9-th order CT filter sharing common resonator. This filter is of Chebyshev type and it has return loss more than 20dB in the passband. The transmission coefficient prescribed zeros are placed on normalized frequencies $\omega_p = [-1.8,\ -1.4,\ 1.3,\ 1.6]$ .The coupling diagram of the synthesized filter is shown on Fig.2.

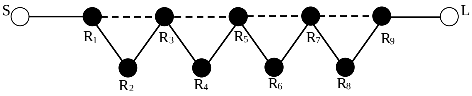

Fig.2. Coupling scheme of an asymmetric 9 pole filter

The reflection and transmission zeroes are calculated and summarized in Table 1.

Table 1. Poles and zeros of asymmetric nine resonator filter

№	Reflection zeros	Transmission zeros	Roots of V_N
1	*-j0.9880*	*-j1.8*	*-j0.9514*
2	*-j0.8888*	*-j1.4*	*-j0.7984*
3	*-j0.6795*	*j1.3*	*j0.9566*
4	*j0.9893*	*j1.6*	*j0.8166*
5	*j0.8998*		*j0.5654*
6	*j0.7051*		*-j0.5331*
7	*j0.4006*		*j0.2164*
8	*-j0.3629*		*-j0.1752*
9	*j0.0211*		

The initial point for the coupling matrix elements for the optimization procedure is to set the values of the all pole nine resonator Chebyshev filter $M_{S1} = M_{9L} = 0.9876$, $M_{12} = M_{89} = 0.9168$, $M_{23} = M_{78} = 0.5870$, $M_{34} = M_{67} = 0.5480$, $M_{45} = M_{56} = 0.5372$.

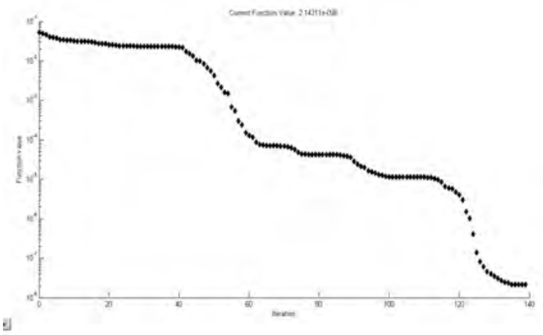

Fig.3 Cost function value during optimization for CT nine resonator filter

All self coupling and cross coupling coefficients are set to zero. The number of the independent values of the coupling matrix is 23.

After 139 iterations for the optimization coefficient, the optimization procedure converges. The values of the cost function vs the number of iterations is shown on Fig.3. The initial value of the cost function is 0.51 and the end value is $2.143.10^{-8}$. The optimization process stopped because of reaching local minimum of the cost function (7). The final coupling matrix is:

$$M = \begin{bmatrix} 0 & 0.9842 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0012 & 0.7589 & -0.2868 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7589 & 0.4095 & 0.5306 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2868 & 0.5306 & -0.0498 & 0.4665 & 0.2788 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4665 & -0.5535 & 0.4462 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2788 & 0.4462 & 0.0035 & 0.4844 & 0.2049 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4844 & -0.4152 & 0.5049 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2049 & 0.5049 & -0.0656 & 0.4767 & -0.4048 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4767 & 0.5722 & 0.7031 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4048 & 0.7031 & 0.0012 & 0.9842 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The frequency response of the designed filter, calculated according to (6) and the coupling matrix derived in the optimization process, is shown on Fig.4. It is clearly seen that the normalized cut off frequency is $\omega_c = \pm 1$, while the transmission zero frequencies are exactly at $\omega_p = -1.8,\ -1.4,\ 1.3,\ 1.6$ .The maximum value of the return loss is with the prescribed value of *-20dB.*

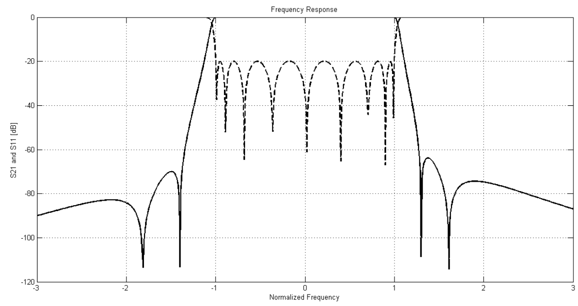

Fig.4 Frequency response of nine resonator filter with asymmetric response. Solid line-S_{21}, dashed line- S_{11}

B. Cascaded Quadruplet and Triplet Resonator Passband Filter of 10-th Order

The 10-th order resonator filter is formed by cascade connection of two trisections and one quadruplet section between them (CQT filter). Each trisection realizes one prescribed transmission zero and the quadruplet section realizes two prescribed symmetrical transmission zeros. The filter is of Chebyshev type and it has maximum return loss of -20dB. The transmission zeroes are placed on frequencies $\omega_p = [-1.2,\ \pm 2,\ 1.6]$ .The coupling scheme of the filter is shown on Fig.5.

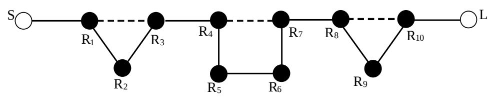

Fig.5. Coupling scheme of CQT filter of 10-th order

The roots of the polynomials in the numerator and denominator in (1) are shown in Table2. The starting point for the optimization process is based on the Chebyshev coupling matrix elements $M_{S1} = M_{10L} = 0.9854$, $M_{12} = M_{9,10} = 0.8130$, $M_{23} = M_{89} = 0.5839$, $M_{34} = M_{78} = 0.5444$, $M_{45} = M_{67} = 0.5321$, $M_{56} = 0.5321$.

Table 2. Poles and zeros of CQTfilter

№	**Reflection zeros**	**Transmission zeros**	**Roots of V_N**
1	*j0.9892*	*-j2*	*j0.9566*
2	*j0.9018*	*-j1.2*	*j0.8244*
3	*j0.7246*	*j1.6*	*j0.6032*
4	*-j0.9915*	*j2*	*-j0.9655*
5	*-j0.9201*		*-j0.8537*
6	*-j0.7647*		*-j0.6530*
7	*-j0.5203*		*-j0.3696*
8	*j0.4625*		*J0.3060*
9	*-j0.2057*		*-j0.0342*
10	*j0.1385*		

The number of the independent values of the coupling matrix is 24 The optimization process converges fast in 238 iterations of the optimizer with end cost function value $1.64519.10^{-7}$. Fig. 6 shows the cost function value with respect to the iterations.

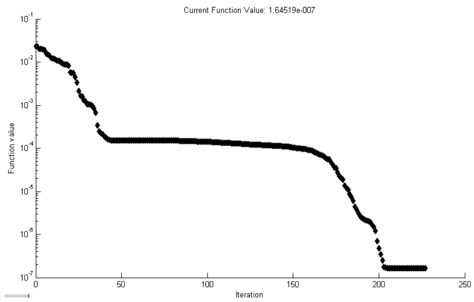

Fig.6 Cost function value for asymmetric five resonator filter

The coupling matrix derived in the optimization process is given by (8). The corresponding frequency response calculated by the coupling matrix and Eq.(6) is shown on Fig.7.

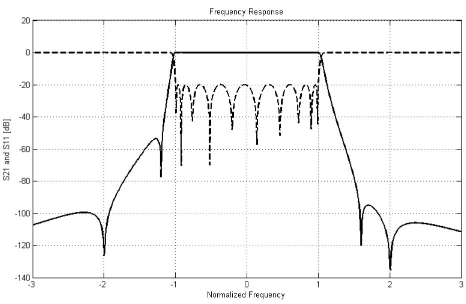

Fig.7 Frequency response of 10-th order CQT filter with asymmetric response. Solid line-S_{21}, dashed line- S_{11}

As it is clearly seen from Fig.7, the transmission zeros are placed on the prescribed values The maximum value of the reflection coefficient is -20dB. Both presented examples show fast convergence of the cost function to a local minimum. In both cases this local minimum is found to be a global minimum corresponding to general Chebyshev filter. In both cases the starting point for the optimization process was the coupling matrix of classic Chebyshev filter. Starting from random initial point leads to a local minimum not corresponding to Chebyshev filter.

IV. Conclusion

This paper presents optimization method for synthesis of microwave filters with arbitrary topology of high order. The method uses local optimization method for coupling matrix determination. The synthesis procedure converges very fast as for an initial point is used a vector based on the Chebyshev all pole filter for the same degree of the filter. To validate the proposed synthesis method two resonant filters are designed with asymmetrical responses. Both presented examples show fast convergence of the cost function to a local minimum. In both cases this local minimum is found to be a global minimum corresponding to general Chebyshev filter. The frequency responses from the synthesis procedure are within the expectations and found to be consistent with the theoretical responses and given filter specifications.

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$$M = \begin{bmatrix}
0 & 0.9832 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.9832 & -0.0019 & 0.6244 & -0.5149 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.6244 & 0.7140 & 0.4020 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.5149 & 0.4020 & -0.0802 & 0.5442 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5442 & -0.0293 & 0.5266 & 0 & -0.0427 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.5266 & -0.012 & 0.5671 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.5671 & 0.002 & 0.5267 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.0427 & 0 & 0.5267 & 0.0157 & 0.5421 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5421 & 0.0563 & 0.5093 & 0.3348 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5093 & -0.4770 & 0.7368 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3348 & 0.7368 & -0.0019 & 0.9832 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9832 & 0
\end{bmatrix} \quad (8).$$