Doppler Fading Effects on OFDM Transmissions

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Abstract – **This paper presents the performances of different Dopler fading models for a OFDM communication. The simulations were carried out over a AWGN channel with a BPSK modulation, and the performances have been evaluated in terms of probability of error for each model.**

Keywords – **Fading, Doppler Effect, OFDM, MATLAB.**

I. INTRODUCTION

Radio propagation for mobile communications, especially in dense urban areas or in indoor environment, is characterized by severe multipath phenomenon that causes fading and distortion effects, leading to inter-symbol interference (ISI) [1].

The fading is one of the major factors affecting the performance of such systems [1,2]. The large-scale fading is given by path loss and shadowing effects of buildings or prominent terrain contours. The small-scale fading is the common reference to the rapid changes in signal amplitude and phase. In this work, the combined effects of large- and small-scale fading are considered.

Multi-carrier communication is a way to increase bandwidth without amplifying the noise in the signal on account of the frequency selectivity fading that affects the channel. This type of communication implies sending several narrower band signals (called subcarriers) instead of a single broadband one. The signals are multiplexed in frequency and they are transmitted together to the same receiver, on the same radio link. By sending M signals in parallel on the same radio link we increase the rate of transfer by M times. In the same time, the impact of frequency selectivity fading depends on the bandwidth of each subcarrier [1].

An extended band involves higher transfer rates, especially for the descending connection. As the symbol rate for each subcarrier is much smaller that the initial symbol rate, the effects of delayed scattering, for example ISI (Inter Symbol Interferences), considerably decrease and reduce the complexity of the equalizer at the receiver [1.2].

II. TYPES OF DOPPLER FADING THAT AFFECT THE RADIO CHANNEL

A. Multipath Channel

The performances of any wireless communication system is strongly affected by the multipath phenomenon, very common in dense urban areas, when the transmission path between the transmitter and the receiver is severely obstructed by buildings, trees or other objects.

The variation of the signal in the communication systems can be caused by the short and/or long term fading. Short term fading includes multi-path fading (Rayleigh, Rice fading) and the Doppler fading [2].

The multi-path fading determines whether a channel is flat or selective in frequency, while the Doppler fading divides channels into slow fading channels and fast fading ones. While the multi-path fading causes a scattering of the pulse in time, the Doppler fading causes a scattering in frequency [2,3].

B. The Doppler Fading

The multi-path fading (Rayleigh, Rice) does not take into consideration the possible movements of the emitter and/or of the receiver. If the emitter/receiver is mounted on a moving vehicle the Doppler fading occurs. If a signal is emitted with the f_0 frequency, then the spectrum of the received signal will broaden and it will contain spectral components from f_0-f_d to f_0+f_d, where f_d is the Doppler deviation given by [4]:

$$f_d = \frac{v f_0 \cos(\theta)}{c} \tag{1}$$

where v is the velocity of the emitter/receiver, and θ is the angle between the emitter/receiver forward velocity and the line of sight from the emitter to the receiver.

If the band which is occupied by the useful signal is wider than the Doppler bandwidth, the Doppler scattering will cause no problem either for emission or reception. In this case we are dealing with a channel affected by a slow fading. On the other hand, if the band is smaller than the Doppler bandwidth, the movement produces a fast variation of the channel during the length of the pulse. Thus the fading is considered to be fast.

In the following paragraphs we will analyze in detail some Doppler power spectrum models from the point of view of their applicability and that of the theoretical expressions of the power spectral density (PSD) of fading processes [2,4,5].

The **Jakes Doppler** power spectrum model is applied to a mobile receiver. It is called the classic model and it is built based on the following hypothesis: propagation of radio

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waves parallel to the ground; at receiver, the angle of arrival is uniformly spread in [-π,π]; the receiver antenna is omnidirectional.[3,4].

The baseband normed Jake spectrum is:

$$S_j(f) = \frac{1}{\pi f_d \sqrt{1-(f/f_d)^2}}, |f| \le f_d \quad (2)$$

where f_d is the maximum Doppler deviation.

It has been proven that in a 3-D isotropic scattering environment, in which the arrival angles are uniformly distributed in the elevation plan and in that of the azimuth, the total PSD for the θ angle of elevation and the α azimuth angle is: $p_{\theta,\alpha}(\theta,\alpha) = \frac{\sin\theta}{4\pi}, 0 \le \theta < \pi, 0 \le \alpha < 2\pi$, the theoretical spectrum is flat.[2,3,4].

The baseband normed **flat Doppler** spectrum is:

$$S_p(f) = \frac{1}{2\pi f_d}, |f| \le f_d \quad (3)$$

The next model corresponds to the multi-path components with high rates of delay in the UHF communications. It has also been put forward in the case of the high frequency (HF) channels as well as for the aeronautical channels with a VHF band [3,4].

The baseband normed **Jake Gaussian** spectrum is:

$$S_g(f) = \frac{1}{\sqrt{2\pi\sigma_g^2}} e^{-\frac{f^2}{2\sigma_g^2}} \quad (4)$$

where σ_g is the standard deviation.

The **Jakes bi-Gaussian** model is built from two Gaussian spectrums which are shifted in frequency. This is used for modeling the long echoes which can occur in urban areas and hilled terrains [2,3,4].

The baseband normed Jakes bi-Gaussian spectrum is :

$$S_{bg}(f) = A_{bg}\left[\frac{C_{g1}}{\sqrt{2\pi\sigma_{g1}^2}} e^{-\frac{(f-f_{g1})^2}{2\sigma_{g1}^2}} + \frac{C_{g2}}{\sqrt{2\pi\sigma_{g2}^2}} e^{-\frac{(f-f_{g2})^2}{2\sigma_{g2}^2}}\right] \quad (5)$$

where σ_{g1} and σ_{g2} are the standard deviations , f_{g1} and f_{g2} are the central frequencies, C_{g1} and C_{g2} are the power gains, and $A_{bg} = \frac{1}{C_{g1}+C_{g2}}$ is the norming coefficient. If $C_{g1} = 0$ or $C_{g2} = 0$, we can obtain a Gaussian Doppler spectrum which is shifted in frequency. In case both central frequencies are 0 and the standard deviations are equal, the result is a Gaussian Doppler spectrum.

As we have mentioned beforehand, the Jakes Doppler spectrum is build based on the fact that the angle of arrival at the mobile receiver is uniformly distributed, and the spectrum covers the $[-f_d, f_d]$ frequencies. If the angles are not uniformly distributed, the spectrum does not cover this interval, a fact which occurs in the case of a directional antenna. This type of spectrum is called restricted. The spectrum will also be considered symmetrical in order to obtain a real impulse response [4].

The baseband normed **restricted Jakes Doppler(rjakes)** spectrum is:

$$S_{rj}(f) = \frac{A_{rj}}{\pi f_d \sqrt{1-(f/f_d)^2}} \quad (6)$$

where $0 \le f_{d,min} \le |f| \le f_{d.max} \le f_d$ and the norming factor defined as:

$$A_{rj} = \frac{\pi/2}{\sin^{-1}\left(\frac{f_{d,max}}{f_d}\right) - \sin^{-1}\left(\frac{f_{d,min}}{f_d}\right)} \quad (7)$$

where $f_{d,min}$ și $f_{d.max}$ are the minimum and maximum positive frequencies for which the spectrum is non-zero. These frequencies can be determined from the PSD of the angle of arrival.

The restricted Jakes Doppler spectrum was considered to be symmetrical. The asymmetric spectrums occur in the case of directional antennae, of aeronautical channels and of the satellite mobile radio channels. Taking an asymmetrical spectrum in consideration, the pulse response will be complex.

The baseband **asymmetrical restricted Jakes Doppler(ajakes)** spectrum is expressed analytically [2,4]:

$$S_{aj}(f) = \frac{A_{aj}}{\pi f_d \sqrt{1-(f/f_d)^2}} \quad (8)$$

where $, -f_d \le f_{d,min} \le f \le f_{d.max} \le f_d$ and the norming factor defined as:

$$A_{aj} = \frac{\pi}{\sin^{-1}\left(\frac{f_{d,max}}{f_d}\right) - \sin^{-1}\left(\frac{f_{d,min}}{f_d}\right)} \quad (9)$$

where $f_{d,min}$ și $f_{d.max}$ are the minimum and maximum positive frequencies for which the spectrum is non-zero.

The round spectral power density is approximated by the measured PSD of a scattering component, while taking into consideration a wireless channel of 2.5 GHz. In this case, the PSD representation is also influenced by the frequency of the central carrier [1,2,3,4].

The baseband **rounded normed Doppler** spectrum is defined as follows:

$$S_r(f) = C_r\left[a_0 + a_2\left(\frac{f}{f_d}\right)^2 + a_4\left(\frac{f}{f_d}\right)^4\right] \quad (10)$$

where $|f| \le f_d$ and the norming factor defined as:

$$C_r = \frac{1}{2f_d\left[a_0 + \frac{a_2}{3} + \frac{a_4}{5}\right]} \quad (11)$$

We can notice that the rounded Doppler spectrum is a polynomial in frequency function, of the fourth order, in which only even exponents appear. The real numbers - a_0, a_2, a_4 are the coefficients of the polynom. In the IEEE 802.16 standard the following values are used: $a_0 = 1,\ a_2 = -1.72,\ a_4 = 0.785$.

III. ORTHOGONAL FREQUENCY-DIVISION MULTIPLEXING – BASIC PRINCIPLES

The Orthogonal frequency-division multiplexing(OFDM) transmission is a multi-carrier type of transmission.

A complex OFDM signal, x(t), during the $mT_u<t<(m+1)T_u$ interval, can be written as follows [5,6,7]:

$$x(t)=\sum_{k=0}^{N_c-1} x_k(t)=\sum_{k=0}^{N_c-1} a_k^{(m)} e^{j2\pi k\Delta f t} \qquad (12)$$

where $x_k(t)$ is the k-th subcarrier modulated with the frequency of $f_k=k\Delta f$ and a_k is the modulator symbol for the k-th subcarrier in the m-th order OFDM time interval and T_u is the useful symbol duration [1,5,7].

The OFDM transmission is based on blocks, which means that during the length of every OFDM symbol, N_c modulating symbols are transmitted in parallel. The symbols can pe modulated using one of the following modulations: BPSK, QPSK, 16QAM sau 64QAM.

At any rate, for a channel with time dispersion, the orthogonality between the subcarriers will be partially or definitely lost. Consequently, in the case of a channel with time dispersion there will be both inter-symbol interferences in the same subcarrier and interferences between different subcarriers. In order to solve this problem and to make the OFDM signal impervious to the time dispersion that occurs within the channel, the cyclical prefix is introduced. The insertion of the cyclical prefix consists of copying the last part of the OFDM symbol and introducing that at the start of the symbol. Once this cyclical prefix is inserted, the OFDM signal increases from T_u to T_u+T_{cp}, where T_{cp} is the length of the cyclical prefix, but also there occurs a decrease in the symbol rate. If at the receiver end, the correlation is performed during the $Tu=1/\Delta f$ interval, then the orthogonality between the subcarriers will be kept even if there is a channel with a time dispersion, with the condition that the duration of the dispersion is lesser than the length of the cyclical prefix [1,2,6].

Due to its features and to the choice of an adequate spacing, Δf, between subcarriers, the OFDM allows for a less complex implementation from the point of view of the calculus efficiency by using the Fast Fourier Transform (FFT) [1,2].

IV. PERFORMANCE ANALYSIS

Simulations were carried out over an Additive white Gaussian noise (AWGN) multipath channel, with Ricean/Rayleigh fading and different types of Doppler fading presented in II.B. For the OFDM communication, the number of subcarriers is 52, with a BPSK modulation and a frequency spacing of 312,5 kHz . The bandwidth is 20MHz, the useful symbol duration is 3,2 µs and a cycling prefix of 0.8 µs. The length of the FFT algorithm used is 64.

We are focusing our interest on the variation of the bit probability of error for the different types of Doppler fading presented in II.B. First of all a AWGN channel with Rayleigh fading is analyzed. The Doppler frequency used in the simulations is 10Hz, 50Hz and 100Hz. Thus, the following variations of the bit probability of error are obtained.

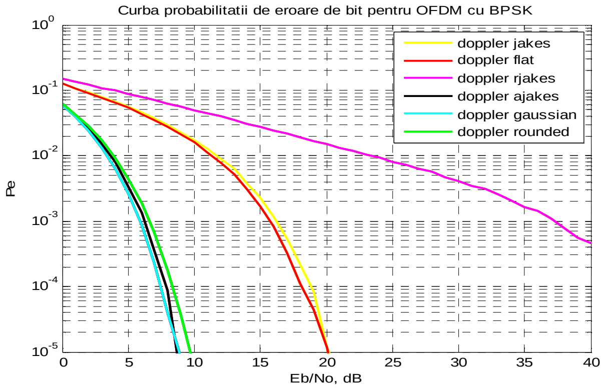

Fig. 1. Probability of error for a AWGN, Rayleight fading channel, and the Doppler frequency of 10Hz

For a Doppler deviation of 10Hz, we can notice in figure 1 that the probability of error(P_e) reaches its minimum value of 10^{-5} for E_b/N_0=8-10 dB in the case of rounded, Akajes, Gaussian type of Doppler scattering. For a flat type of Doppler and a Jakes one, P_e is minimum when E_b/N_0= 20 dB, and then the error increases significantly for a Rjakes type of Doppler.

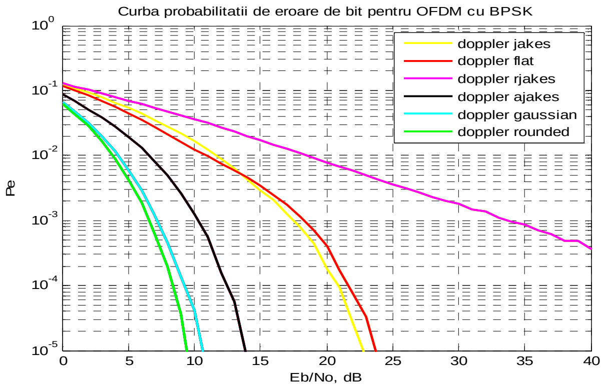

Fig. 2. Probability of error for a AWGN, Rayleight fading channel, and the Doppler frequency of 50Hz

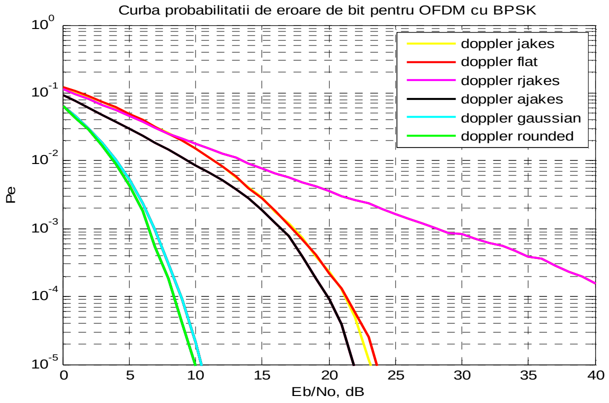

Fig. 3. Probability of error for a AWGN, Rayleight fading channel, and the Doppler frequency of 100Hz

In figure 2 the most favorable cases from the point of view of the bit error probabilities are obtained for the rounded and Gaussian type of Doppler, while the most unfavorable situation is that of the channel which is affected by the Rjakes Doppler. For the Ajakes Doppler, the proportion between the bit energy and the power spectral density reaches the value of 13,8 dB, increasing until 22,8 dB and 23,7 dB in the case of the channel which is affected by the Jakes Doppler, flat respectively. Modifying the Doppler deviations to 100Hz, the only major difference that occurs in figure 4.25 is noticeable around the value of the E_b/N_0 proportion that increases by 8 dB, from 13,8 dB to 21,8 dB, in the case of the channel affected by the Ajakes Doppler.

V. CONCLUSION

In this paper we compared the performances of several Doppler fading models, presented in II.B, for an OFDM communication. Thus, for lower Doppler frequencies we obtained a better probability of error than for higher ones. Furthermore, in the case of Rayleight fading, for the same probability of error, E_b/N_0 is at minimum in the case of the rounded Doppler model and at a maximum for the Rjakes Dopller model. For Rician fading we observed the minimum for E_b/N_0 is still for the rounded Doppler model but the maximum is, in this case for the Jakes Doppler model.

Therefore, both for Rician and Rayleight fading the more robust Doppler model is the rounded Doppler model. The worst model, in terms of probability of error is the Rjakes respectively the Jakes Doppler model.

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REFERENCES

[1] Andreas F. Molisch, *Wireless Communication Second Editions*, Wiley & Sons.,2011.

[2] Franz Hlawatsch , Gerald Matz , *Wireless Communications Over Rapidly Time-Varying Channels* **,** Elsevier 2011.

[3] R. H. Clarke, “A statistical theory of mobile-radio reception”, Bell Sys.Tech. J., vol. 47, no. 6, pp. 957–1000, July-Aug. 1968.

[4] P. M. Shankar, *Fading and Shadowing in Wireless Systems*, Springer 2011.

[5] Haiying Zhu, Bouchard, L., Boucher, L. , *Performance of OFDM based wireless LAN system under Doppler over Rayleigh fading*, Communication Technology Proceedings, International Conference on. Vol.2,pp 1234 - 1237 , 2003.

[6] K.A.Hamdi, Unified Error-Rate Analysis of OFDM over Time-Varying Channels, Wireless Communications, IEEE Transactions on, Vol.10, No. 8,pp. 2692 – 2702, 2011.

[7] L. Noor A. , Anpalagan ;S. Kandeepan. *SNR and BER derivation and analysis of downlink OFDM systems with noisy fading Doppler channels,* Signal Processing and Its Applications, 9th International Symposium on, pp 1-4, 2007.