New Teletraffic Loss System - Polya/G/n/0

Seferin Mirtchev[1], Rossitza Goleva[1], Georgi Balabanov[1] and Velko Alexiev[1]

Abstract – **In this paper we have proposed to use Polya and Pareto distribution to describe peaked arrival processes. With this arrival processes it is possible to analyze the real loss systems in telecommunication networks. We study a version of the "classical" Erlang's loss model M/G/n/0 called Polya/G/n/0. This is a full accessible loss system with Polya input flow (negative binomial distributed number of arrivals in a fixed time interval), general distributed service time and "n" servers. This model is evaluated by simulation with Pareto distributed inter-arrival time. An algorithm for calculation of the state probabilities and blocking are presented. It is shown that the variance of the input stream changes significantly the characteristics of the loss systems.**

Keywords – **Polya distribution, Pareto distribution, loss system, peaked flow.**

I. INTRODUCTION

The most common choice for telecommunication network design is based on the exponential assumption. Usual choice is the Poisson arrival of the calls or sessions and exponential service times. However, networks and applications of today generate a traffic that is bursty over a wide range of time scales. A number of empirical studies have shown that the network traffic is self-similar or fractal in nature.

There is no single traffic model that can efficiently capture the traffic characteristics of all types of networks. The study of traffic models for a specific environment has become a crucial and important task. Good traffic modelling is also a basic requirement for accurate capacity planning.

In [1] a careful overview of some of the widely used network traffic models is made, highlighting the core features of the model and traffic characteristics they capture. For heavy-tailed traffic it can be shown that the Poisson model underestimates the traffic. In case of high-speed networks with unexpected demand on packet transfers, Pareto distribution is a good candidate since the model takes into consideration the long-term correlation of packet arrival times.

In this work, we propose to use the Polya and Pareto distribution to describe the input flows. We study a version of the "classical" Erlang's loss model M/G/n/0 called Polya/G/n/0. This is a full accessible loss system with Polya input flow (negative binomial distributed number of arrivals in a fixed time interval), general distributed service time and "n" servers.

The Polya arrival process is peaked process. It is defined by two parameters – the mean value and the variance [2]. This way we present a basic study of the influence of the offered traffic's peakedness on the blocking probability.

The MMPP (Markov Modulated Poisson Process) traffic model is proposed in [3]. It accurately approximates the long range dependence characteristics of Internet traffic. The hidden Markov models (D-BMAP/D/1/k) is presented in [4] with an arrival process of frames and service process in the wireless channel. The method for analysis of the M/D/n is given in [5]. A simple accurate model for a multi-server central-queue system M/G/K when service requirements have heavy tails is considered in [6]. The bursty Internet traffic stream is studied in [7,8] as an ON/OFF model.

The main reason for studying the Polya/G/n/0 teletraffic system is that it can be used to analyze the real loss systems in telecommunication networks. We consider that the network analysis requires a technique that can represent any kind of traffic, and especially peaked.

II. POLYA ARRIVAL PROCESS

The Polya arrival process is a pure birth process with two parameters [9]. The probability $P_i(t)$ of i arrivals in an interval with duration of t seconds is given by

$$P_o(t) = (1+\beta\lambda t)^{-\frac{1}{\beta}}$$
$$P_i(t) = \binom{1/\beta + i - 1}{i}\left(\frac{\beta\lambda t}{1+\beta\lambda t}\right)^i P_o(t), \quad (1)$$

where $\lambda > 0$ and $\beta > 0$.

The Polya distribution is a variant of the negative binomial distribution. Its mean value (the average number of arrivals in an interval of length t) is

$$M(t) = \sum_{i=1}^{\infty} iP_i(t) = \lambda t\ . \quad (2)$$

This means that λ is an arrival rate.

The variance of the number of arrivals in an interval of length t is

$$V(t) = \sum_{i=0}^{\infty}[i - M(t)]^2 P_i(t) = \lambda t(1+\beta\lambda t)\ . \quad (3)$$

The peakedness of the Polya input flow is

$$z(t) = \frac{V(t)}{M(t)} = 1 + \beta\lambda t > 1\ . \quad (4)$$

When $\beta = 0$, $M(t) = V(t) = \lambda t$ i.e. it is a regular Poisson process. When $\beta = 1$ the Polya distribution is a geometric distribution.

[1]Seferin Mirtchev, Rossitza Goleva, Georgi Balabanov and Velko Alexiev are with the Faculty of Telecommunications at Technical University of Sofia, 8 Kl. Ohridski Blvd, Sofia 1000, Bulgaria, E-mails: stm@tu-sofia.bg, rig@tu-sofia.bg, gbalabanov@ieee.org, velko.alexiev@gmail.com

III. MODEL DESCRIPTION

Let us consider a multi servers loss system Polya/D/∞ with a Polya input stream with arrival rate λ, constant service time τ and infinite number of servers. This queueing system is a non-Markovian model or renewal process (Fig. 1).

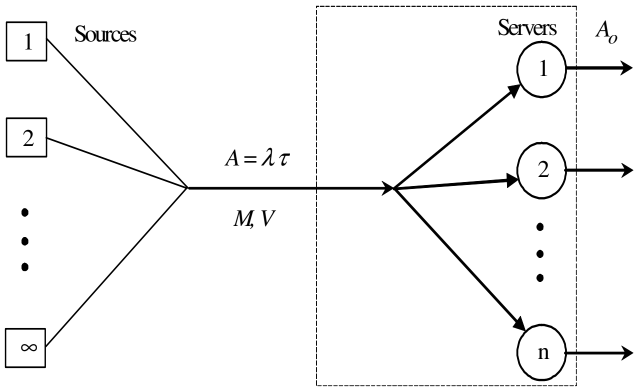

Fig. 1. *Generalised queueing model with peaked input flow.*

To study this system, we consider two epochs (points of time) t and $t + \tau$ at a distance of τ. Every customer being served at epoch t has left the server at epoch $t + \tau$. The numbers of the customers arriving during the interval $(t;\ t+\tau)$ are still in the system at epoch $t+\tau$ (being served) and they are described whit Polya distribution as the arrival process.

For this model with Polya arrival process the offered traffic is equivalent to the average number of call attempts in service time:

$$A = \lambda\tau,\ erl. \tag{5}$$

The offered traffic is equal to the carried traffic because there are no losses and delays and it is called intended traffic.

The states probability of the system under the assumption of statistical equilibrium can be calculated by means of Polya distribution because we choose arbitrary epoch to observe the system:

$$\begin{aligned} P_o &= (1+\beta A)^{-\frac{1}{\beta}} \\ P_i &= \binom{1/\beta + i - 1}{i}\left(\frac{\beta A}{1+\beta A}\right)^i P_o \end{aligned} \tag{6}$$

The number of busy channels at a random point of time is thus Polya distributed with both mean value equal to the offered traffic A and different variance which depends from the chosen parameter β.

We still assume the same arrival process. The number of channels is now limited so that n is finite. The number of states becomes n+1. In this case we get the truncated Polya distribution:

$$P_i = \frac{\binom{1/\beta + i - 1}{i}\left(\frac{\beta A}{1+\beta A}\right)^i}{\sum_{k=0}^{n}\binom{1/\beta + k - 1}{k}\left(\frac{\beta A}{1+\beta A}\right)^k} \tag{7}$$

The name truncated is due to the fact that the solution may be interpreted as a conditional Polya distribution Pi(| i≤n). The fact that we are allowed to truncate the Polya distribution means that the relative ratios between the state probabilities are unchanged. We prove this fact by simulation.

IV. TRAFFIC CHARACTERISTICS OF POLYA/D/N/0 SYSTEM

Knowing the state probabilities, we are able to find all performance measures defined by state probabilities.

Time Congestion

The probability that all n channels are busy at a random point of time is obtained from system equations when i = n:

$$B_t = \frac{\binom{1/\beta + n - 1}{n}\left(\frac{\beta A}{1+\beta A}\right)^n}{\sum_{k=0}^{n}\binom{1/\beta + k - 1}{k}\left(\frac{\beta A}{1+\beta A}\right)^k} \tag{8}$$

Carried traffic

By definition the carried traffic is:

$$A_n = \sum_{i=1}^{n} i\,P_i,\ erl. \tag{9}$$

Utilization

Traffic carried by one channel**:**

$$\rho = A_n / n,\ \%. \tag{10}$$

Traffic congestion

The ratio of traffic lost and intended traffic:

$$B_a = \frac{A - A_n}{A} \tag{11}$$

There are different time and traffic congestion because the arrival process is peaked.

Insensitivity

A system is insensitive to the holding time distribution if the state probabilities of the system only depend on the mean value of the holding time.

It can be shown with Cox distribution that loss formula (11), which above is derived under the assumption of constant service time, is valid for arbitrary service time distributions. The state probabilities for both the Polya distribution and the truncated Polya distribution only depend on the service time distribution through the mean value which is included in the offered traffic A. We prove this property by simulation.

It can be shown that all classical loss systems with full accessibility are insensitive to the holding time distribution [5].

V. SIMULATION WITH PARETO DISTRIBUTION

The fundamental relationship between the number and interval representations is given by Feller-Jensen's identity:

$$P\{N_t < n\} = P\{T_n \geq t\}, \quad n = 1,2,3,... \tag{12}$$

where: Nt is the random variable for the number of calls arrived in time interval t

Ti is the random variable for the time interval until there has been n arrivals.

Simulations are the main tools for studying the performance of telecommunication networks and we will evaluate the Polya/D/∞ and Polya/G/n/0 queue using simulation.

The family of Generalized Pareto Distributions (GPD) has three parameters: the location parameter μ, the scale parameter σ and the shape parameter ξ. If we choose the location parameter $\mu = 0$ then the cumulative distribution function of the GPD is:

$$F(x) = 1 - \left[1 + \frac{\xi x}{\sigma}\right]^{-\frac{1}{\xi}} , \tag{13}$$

where $\sigma > 0$ and $\xi > 0$.

The mean value of the generalized-Pareto distribution is:

$$M_p = \frac{\sigma}{1-\xi} . \tag{14}$$

When $\xi < 1$ the mean value is finite.

The variance of the Generalized Pareto Distribution is:

$$D_p = \frac{\sigma^2}{(1-\xi)^2(1-2\xi)} . \tag{15}$$

When $\xi < 0.5$ the variance is finite.

The probability density function of the GPD is:

$$f(t) = \frac{1}{\sigma}(1+\lambda)\left(1 + \frac{\xi x}{\sigma}\right)^{-1/\xi - \xi} \tag{16}$$

We choose these substitutions

$$\sigma = \frac{1-\beta}{\lambda(1+2\beta)}; \quad \xi = \frac{\beta}{1+2\beta} . \tag{17}$$

Therefore, we receive another form of the mean value of the generalized-Pareto distribution:

$$M_p = \frac{1}{\lambda} \tag{16}$$

The mean value is the average inter-arrival time for our study. The parameter λ is the average call arrival intensity.

The variance of the Generalized Pareto Distribution is:

$$D_p = \frac{1+2\beta}{\lambda^2} \tag{17}$$

With these substitutions for every positive value of λ and β the mean end variance of he GOD will be finite.

Random number generation

Many programming languages do not yet recognize the Pareto distribution. In the field of telecommunications, the Pareto distribution is widely used to estimate the inter-arrival and service times.

One can easily generate a random sample from Pareto distribution by using inverse distribution function. Given a random variable U with uniform distribution on the unit interval (0;1), the random variable x is Pareto-distributed.

$$x = \frac{\sigma\left(U^{-\xi} - 1\right)}{\xi} \tag{18}$$

We have developed a real time trace simulation algorithm for evaluating the state probabilities of the Polya/D/∞ system and make a comparison with M/D/∞. The simulation results have shone that with Pareto distribution inter-arrival time the state probabilities of the Polya/D/∞ system have the same mean e variance as the Polya distribution. When there are truncations (Polya/D/n/0) or we change the service time distribution (exponential and Pareto) the state distribution by simulation is identical with by Polya's formula (7) and (6).

VI. NUMERICAL RESULT

In this section, we give numerical results obtained by a Pascal program on a personal computer. The described methods are tested over a wide range of arguments.

Figure 2 illustrates the stationary probability distribution in a loss system Polya/G/n/0 with a Polya input stream with different peakedness *z*, 15 erl offered traffic and 50 servers. It is seen that when the peakedness increases the probability than all servers are busy increase vastly. Figure 3 shows the traffic congestion as a function of the intended traffic for various values of the peakedness when the number of the servers is 30. The peaked input flow increases the congestion with several orders. Figure 4 presents the utilisation of the servers (the carried traffic for one server) as function of the intended traffic intensity with different peakedness of the Polya input flow when the number of the servers is 20. The peakedness of the arrival stream decreases the utilisation end in the same time increases the congestion.

It is shown that the influence of the variance of the input stream over the performance measures is significant and can be easily evaluate by Polya and Pareto distribution.

VII. Conclusion

In this paper the Polya distribution is used to describe the peaked arrival processes in telecommunication networks. A basic generalised multiple loss teletraffic model Polya/G/n/0 with peaked input flow, generalise service time and full accessibility is investigated. All performance measures of interest are estimated. The idea is based on the Polya distribution as an analytical continuation of the Poisson distribution and the classical M/G/n/0 system.

The proposed approach provides a unified framework to model peaked teletraffic in real telecommunication systems. Numerical results and subsequent experience have shown that this approach is accurate and useful in analyses of traffic systems and especially in Quality of Service and performance parameters estimation.

The importance of this multiple loss teletraffic system with peaked input stream comes from its ability to describe behaviour that is to be found in complex real time queueing systems. It is a general traffic system which is important in designing telecommunication networks.

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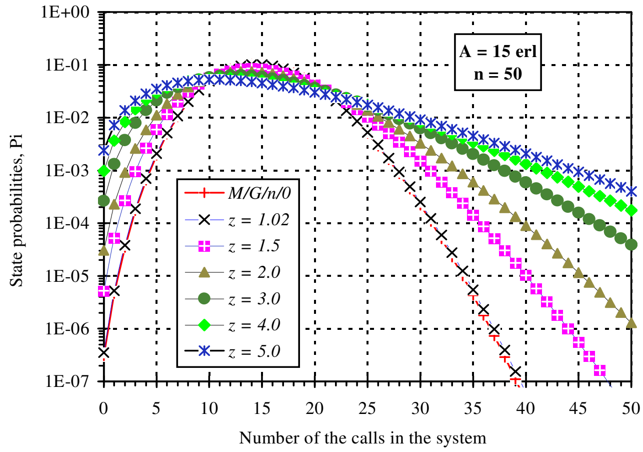

Fig. 2. Stationary probability distribution

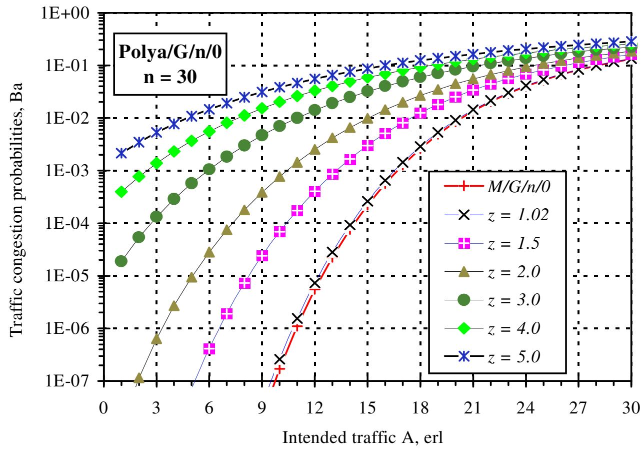

Fig. 3. Traffic congestion as a function of the intended traffic for various values of the peakedness.

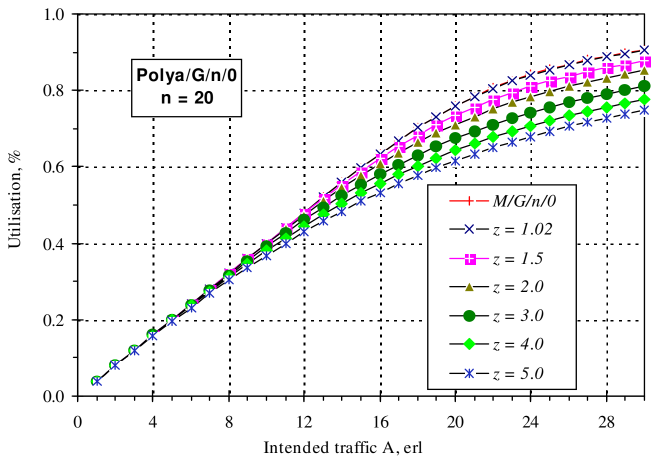

Fig. 4. Utilization of servers