Acoustic Standing Waves in Closed Cylindrical Enclosures

Ekaterinoslav Sirakov [1] and Hristo Zhivomirov[2]

Abstract – **The work presented in this paper provides a theoretical analysis of acoustic standing waves inside cylindrical enclosures with rigid walls. Mathematical relationships are given for the calculation and researching of modal frequencies and standing sound waves in a cylindrical box. The results from the calculating and measuring of modal frequencies and the box response are shown graphically and in a table.**

Keywords – **Acoustic standing waves, Closed cylindrical enclosures.**

I. Acoustic Standing waves

In the paper are discussed the acoustic processes in a closed cylindrical volume (Fig. 1). As a result of multiple reflections of the sound waves from the walls of the volume three-dimensional sound field arises [1], an example of which is given in Fig. 2. Depending on the shape, dimensions and their ratios in the enclosed volume fluctuations occur with a different set of natural frequencies [2, 3].

The acoustic processes in a closed cylindrical volume can be represented by the wave equation in cylindrical coordinate system (r , ϕ, z) [4]:

$$\nabla^2 p = c \cdot \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \phi^2} + \frac{\partial^2 p}{\partial z^2} \right] \quad (1)$$

where: ϕ - azimuth angle of the source.

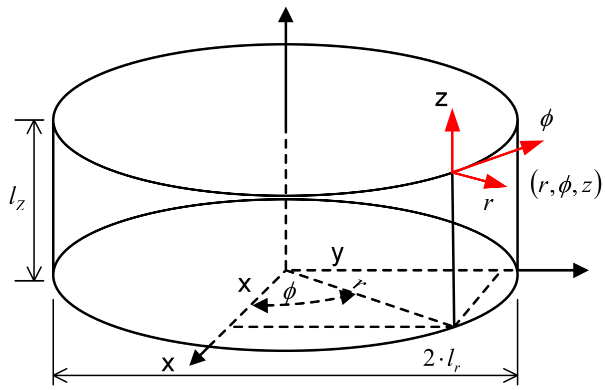

Fig. 1. Cylindrical acoustic volume [5]

[1]Ekaterinoslav Sirakov is with the Department of Communication Engineering and Technologies, Faculty of Electronics, Technical University-Varna, Studentska Street 1, Varna 9010, Bulgaria, E-mail: katiosirakov@abv.bg

[2]Hristo Zhivomirov is a Ph.D. student with the Department of Communication Engineering and Technologies, Faculty of Electronics, Technical University-Varna, Studentska Street 1, Varna 9010, Bulgaria, E-mail: hristo_car@abv.bg.

The solution of the wave equation is [6, 7]:

$$p = \begin{cases} \cos(j \cdot \theta) \\ or \\ \sin(j \cdot \theta) \end{cases} \cdot \cos\left(\frac{n_z \cdot \pi \cdot x}{l_z} \right) \cdot J_j \cdot \left(\frac{\lambda_{j,k} \cdot r}{l_r} \right) \quad (2)$$

$$n_Z, j, k = 0, 1, 2, \ldots$$

where: J_j - Bessel function.

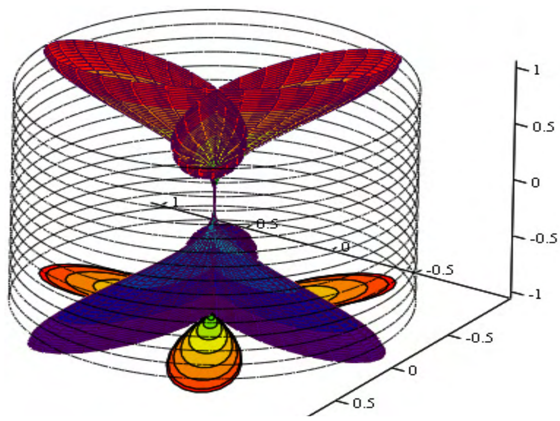

Fig. 2. Distribution of the magnitude of sound pressure in a cylindrical box: *j* axial sound wave, *f*=3.538 kHz, $j = 3,\ k = 0,\ \lambda_{j,k} = 4.201189$, $\cos(j \cdot \theta)$, mode (0, 3, 0)

The natural frequencies for the corresponding values of n_z, *j* and *k* can be found by [5]:

$$f = \frac{c}{2} \cdot \sqrt{\left(\frac{n_z}{l_z} \right)^2 + \left(\frac{\lambda_{j,k}}{\pi \cdot l_r} \right)^2}\ , \text{Hz} \quad (3)$$

$$n_Z, j, k = 0, 1, 2, \ldots$$

The natural frequency of the cylindrical box, calculated in accordance with mathematical dependence (3) is presented in Fig. 3 and Table I.

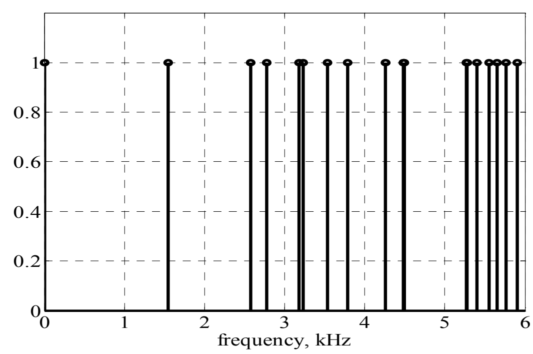

Fig. 3. Plot of mode distribution

TABLE I THE EIGHTEEN LOWEST NORMAL MODES AND THEIR NATURAL FREQUENCIES FOR A CILINDRICAL BOX WITH RIGID WALLS.

№	mode	n_z, j, k	frequency, kHz
1	j axial	0, 1, 0	1.551
2	j axial	0, 2, 0	2.573
3	z axial	1, 0, 0	2.774
4	z, j tangential	1, 1, 0	3.178
5	k axial	0, 0, 1	3.227
6	j axial	0, 3, 0	3.538
7	z, j tangential	1, 2, 0	3.784
8	z, k tangential	1, 0, 1	4.256
9	j axial	0, 4, 0	4.479
10	j, k tangential	0, 1, 1	4.491
11	z, j tangential	1, 3, 0	4.497
12	z, j tangential	1, 4, 0	5.268
13	z, j, k oblique	1, 1, 1	5.278
14	j axial	0, 5, 0	5.404
15	z axial	2, 0, 0	5.548
16	j, k tangential	0, 2, 1	5.648
17	z, j tangential	2, 1, 0	5.761
18	k axial	0, 0, 2	5.909

II. ENCLOSURE RESPONSE MEASUREMENTS IN MODEL CLOSED CYLINDRICAL BOX

The measured characteristics of the sound pressure in cylindrical enclosure with the application software Realtime Analyzer [8] are presented in graphical form in Fig. 4.

Measurements were made in a cylindrical box with dimensions: height 6.2 cm, diameter 13 cm and wall thickness 0.1 cm. The program allows the data from the measured values of sound pressure in dB to be stored in tabular and text format for further analysis.

To examine the modal structure of the enclosure box response at the center of the volume was measured.

III. CONCLUSION

In the cylindrical acoustic volumes as with rectangular [2] "axial" and "tangential" natural frequencies can be defined. The z-axial natural frequency for $j = k = 0$ is given in Table I, № 3 and № 15. When $n_z = 0$ by analogy with the rectangular speaker enclosure z, ϕ-tangential natural frequencies can be defined – Table I, № 10 and № 16. If $n_z = 0$ and $j = 0$ the sound is focused along the axis of the cylinder, the sound wave propagates radially and the natural frequencies are r-axial – Table I, № 5 and № 18. When $n_z = 0$ and $k = 0$ the natural frequencies can be called ϕ-axial [4] (perpendicular to z and r) as shown in Table I, № 1, 6, 9 and № 14.

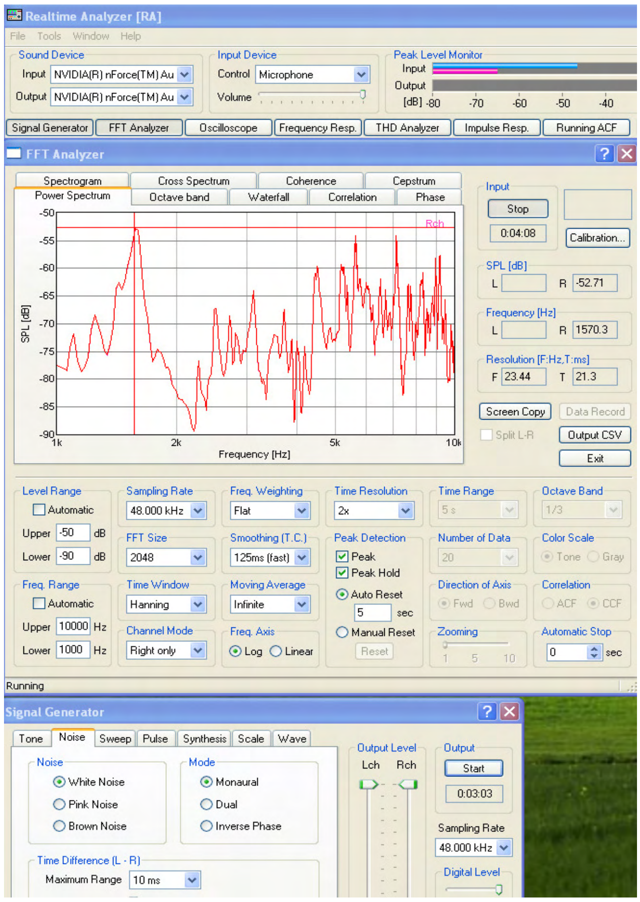

Fig. 4. The measured sound pressure in the closed cylindrical enclosure (dimensions: height 6.2 cm, diameter 13 cm and wall thickness 0.1 cm)

REFERENCES

[1] Ekaterinoslav Sirakov, Hristo Zhivomirov, Boris Nikolov, “Green’s Function and Acoustic Standing Waves in Rectangular Loudspeaker Enclosures”, ICEST 2011, Proceedings of Papers, pp. 721-724, Nis, Serbia, 2011.

[2] Екатеринослав С. Сираков, “Собствени резонансни честоти на правоъгълно озвучително тяло”, Национална конференция с международно участие „Акустика 1”, Варна, 2009, Списание „Акустика”, год. XI, бр.11, 2009 г., стр. 110-118.

[3] Екатеринослав Сираков, Борис Николов, Любомир Камбуров, “Влиянието на съотношението на размерите на правоъгълен акустичен обем върху разпределението на собствените честоти”, Национална конференция с международно участие „Акустика 1”, Варна, 2010, Списание „Акустика”, год. XII, бр.12, 2010 г., стр. 94-103.

[4] Philip M. Morse, *Vibration and Sound*, New York, McGraw-Hill, 1936.

[5] Екатеринослав Сираков, “Акустичен модел на цилиндрични затворени обеми”, Национална конференция с международно участие „Акустика 1”, Варна, 2011.

[6] Frank Fahy, *Foundations of Engineering Acoustics*, Academic Press, San Diego, 2005.

[7] István L. Vér, Leo L. Beranek, *Noise and Vibration Control Engineering: Principles and Applications*, Wiley, 2006.

[8] http://www.ymec.com/products/dssf3e/