Some Integral Characteristics of MRC Receiver in Nakagami-*m* fading Environment

Hana Stefanovic[1], Dejan Milic[2], Dimitrije Stefanovic[3], Srdjan Milosavljevic[4]

Abstract – **In this paper Maximal Ratio combining (MRC) technique over Nakagami-*m* fading channel is described and some statistical characteristics of received signal envelope are analyzed. For analytical and numerical evaluation of system performance, the probability density functions (pdf) of received signal envelope after MRC are analyzed like particular solutions of corresponding differential equation, while the existence of singular solution is considered and analyzed under different conditions.**

Keywords – **Nakagmi-*m* fading channel, MRC receiver, probability density function (pdf), singular solution.**

I. INTRODUCTION

Many modern communication systems, like wireless cellular systems, operate in environments that are interference and bandwidth limited, where propagation characteristics are more complicated and multipath-induced fading and shadowing are a common problem [1]. A great number of channel models have been proposed to describe the statistics of the amplitude and phase of multipath faded signals [2]. As the result of multipath reception, the mobile antenna receives a large number of reflected and scattered waves. The rapid fluctuations of the instantaneous received signal power due to multipath effects are usually described with Rayleigh, Rician, Nakagami-*m*, Nakagami-*q* or Weibull model [1]. This paper discusses the case of Nakagami-*m* distribution, which models radio transmission in urban areas [3] where the random fluctuations of the instantaneous received signal power are very frequent and fast. It is shown that Nakagami-*m* distribution can model different propagation conditions, providing more flexibility and higher accuracy in matching some experimental data in comparison with the commonly adopted distributions [4]. Nakagami-*m* distribution is suitable for describing statistics of mobile radio transmission in complex medium such as the urban environment [5].

In the case of all distributions considered in the statistical theory of telecommunications, the pdfs of the received signal envelope are functions of several variables [1-3]. In the process of determination and analysis of integral characteristics, one of the variables is treated as a parameter, while the others are set to certain constant values of interest in practice. In this way, one obtains the received signal pdf curves family. The analysis of the position of the maximums for curves family can be performed analytically, using the first derivative of function, and also numerically. In such a case the same procedure is repeated for the situation where the second variable is treated as a parameter, and the others are set to constant values. This process gives a new family of curves, while the maximum position is determined by a new envelope. In papers [6-10] the position of maximums and the analytical expression for the integral characteristic of Nakagami-*m* distribution of received signal envelope are determined. Some integral characteristics of Rayleigh, Weibull and Rician distribution are presented in [11-13].

In order to combat multipath fading and shadowing effects, the complex receiver structures, using complicated synchronization schemes, demodulators, symbol estimators and diversity and MIMO techniques, are often applied [1-3]. An efficient method for mitigating fading effects by using multiple receiver antennas is called space diversity [3]. The main goal of space diversity techniques is to improve transmission reliability without increasing transmission power and bandwidth while increasing channel capacity. There are several types of space combining techniques that can be generally performed depending on the amount of channel state information (CSI) available at the receiver. The most frequently used schemes are selection combining (SC), equal-gain combining (EGC) and maximal ratio combining (MRC). EGC involves co-phasing of the useful signal in all branches, and summing of the received signals from all antennas. By co-phasing, all the random phase fluctuations of the signal that emerged during transmission are eliminated. For this process it is necessary to estimate the phase of the received signal, so this technique requires all of the amount of the CSI on received signal, and separate receiver chain for each branch of the diversity system, increasing the complexity of system. MRC output is a weighted sum of co-phased signals from all branches, also requiring all of the amount of CSI. Unlike pervious, SC technique processes only one of the diversity branches. Generally, SC selects the branch with the highest signal-to-noise ratio (SNR), that is the branch with the strongest signal [3], assuming that noise power is equally distributed over branches. In this paper we analyze MRC technique, which is chosen because of its benefits when compared to other techniques.

The remainder of this paper is as follows. After this Introduction, some characteristics of Nakagami-*m* fading

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channel model and MRC receiver operating over it are presented in Section II. The procedure of determining the integral properties in the case of MRC reception is described in Section III. For a fixed value of the fading depth parameter and average signal power, the pdf of signal envelope after MRC, depending on the received signal level, is analyzed for different number of diversity branches. Then the average signal power is treated as parameter, while the values of the received signal level and fading depth parameter are set to the constant values. In that case, the pdf of signal envelope after MRC is analyzed also for different number of diversity branches. In such a way, two series of families of curves are obtained and for each of them equation of the envelope of curves maximums is considered. In both cases these envelopes are straight lines, in the logarithmic scale, whose direction coefficients and values on ordinate-axis are determined analytically and numerically. Also, in both cases, the differential equations describing the complete dynamics of signal transmission process are determined, whereby the envelopes of the pdf curves families represent their singular solutions. Finally, some conclusions are given in Section IV.

II. General properties of Nakagami distribution

A. Nakagami-m Fading Channel Model

Since the Nakagami-*m* random process is defined as envelope of the sum of 2*m* independent Gauss random processes, the Nakagami-*m* distribution is described by pdf [3-4]:

$$p_z(z,\Omega)=\frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m z^{2m-1}\cdot\exp\left(-\frac{m}{\Omega}z^2\right), z>0, m\geq\frac{1}{2} \quad (1)$$

where z is the received signal level, $\Gamma(\cdot)$ is Gamma function, m is the fading depth parameter (fading figure), defined as:

$$m=\frac{\mathrm{E}^2[z]}{\mathrm{Var}[z^2]} \quad (2)$$

while Ω is the average signal power:

$$\Omega=\mathrm{E}[z^2] \quad (3)$$

The analytical expression for the envelope of pdf curves family maximums is determined [6-9] depending on the relevant parameters, like fading depth parameter and average signal power. The obtained results show that the position of the maximums of these functions is uniquely determined by envelope's equation, regardless of the values of the other parameters.

B. MRC Receiver over Nakagami-m Fading Channel

MRC is the optimal combining scheme, regardless of fading statistics, but it requires knowledge of all channel fading parameters [3]. In the case of MRC diversity technique, received signal envelope is described by [14]:

$$p_{MRC}(z,\Omega)=\left(\frac{m}{\Omega}\right)^{mM}\frac{z^{mM-1}}{\Gamma(mM)}\cdot\exp\left(-\frac{m}{\Omega}z\right) \quad (4)$$

where M presents the number of diversity branches.

III. Analytical and numerical results

Graphic presentation of dependence of received signal pdfs after MRC, versus received signal level, for fixed values of the fading depth parameter and the average signal power, in logarithmic scale, for different number of diversity branches, is shown in Fig 1.

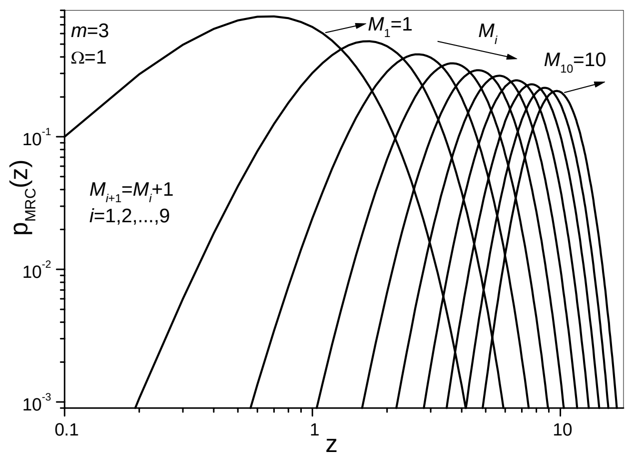

Fig. 1. The received signal pdf versus signal level, in logarithmic scale, for fixed values of m and Ω, with number of diversity branches taking values from 1 to 10.

Analysis of this dependence shows that the increase of number of diversity branches results in smaller values of the pdf maximums, reached for higher values of received signal level, as shown in Fig 1. It can also be concluded that all the maximums lie on a straight line, and that the envelope of maximums is also a straight line, in logarithmic scale. In order to determine the values of received signal level when the maximums are reached, as well as the values of maximums and direction coefficient of envelope, the first derivative of (4) relative to received signal level is determined. Equalization the first derivative with zero obtains:

$$z_{p\max}=\Omega(-1+mM)/m \quad (5)$$

Substituting (5) into (4) yields:

$$p_{MRC}^{z\max}=\frac{1}{\Gamma(mM)}\left(\frac{m}{\Omega}\right)^{mM}\left(\frac{\Omega(mM-1)}{m}\right)^{mM-1}\cdot\exp(1-mM) \quad (6)$$

From (6) we get:

$$\log\left(\max p_{MRC}(\Omega)\right)=k_1\log\Omega+n_1 \quad (7)$$

where the direction coefficient is:

$$k_1 = -1 \qquad (8)$$

and the value on the ordinate axis is:

$$n_1 = \log m - \log\Gamma(mM) + (mM-1)\log(mM-1) + 1 - mM \qquad (9)$$

The direction coefficient of envelope has the value -1, regardless of the value of the fading depth parameter and number of diversity branches.. The envelope determines a certain singular solution of differential equation which can describe the dynamics of this process, while the received signal pdf is its particular solution:

$$p'_{MRC}(z) + p_{MRC}(z)\left(m/\Omega - (mM-1)/z\right) = 0 \qquad (10)$$

The influence of fading parameter m is presented in Fig 2, showing the translation of the pdf curves family maximums envelope. For higher values of m, the value on the ordinate axis n_1 is greater, as shown in Fig 2, while the direction coefficient k_1 has the same value.

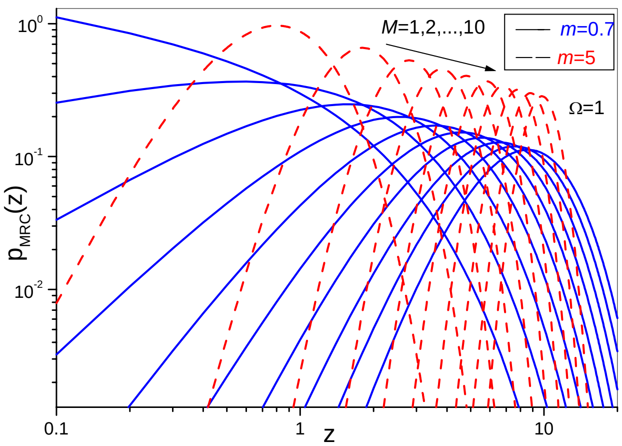

Fig. 2. The received signal pdf versus signal level, in logarithmic scale, for fixed value of Ω, and different values of m, with number of diversity branches taking values from 1 to 10.

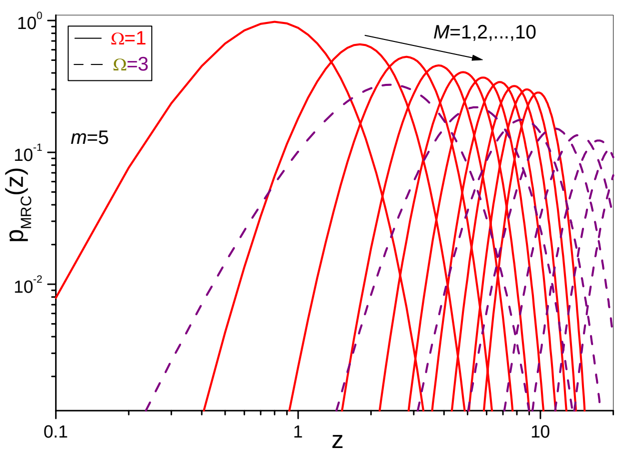

Fig. 3. The received signal pdf versus signal level, in logarithmic scale, for fixed value of m, and different values of Ω, with number of diversity branches taking values from 1 to 10.

The influence of average signal power is presented in Fig 3, also showing the translation of the pdf curves family maximums envelope. For higher values of Ω, the value on the ordinate axis n_1 is smaller, as shown in Fig 3, while the direction coefficient k_1 has the same value.

Graphic presentation of dependence of Nakagami-m pdfs after MRC, versus average signal power, for a fixed value of the fading depth parameter and received signal level, in logarithmic scale, for different number of diversity branches is shown in Fig 4.

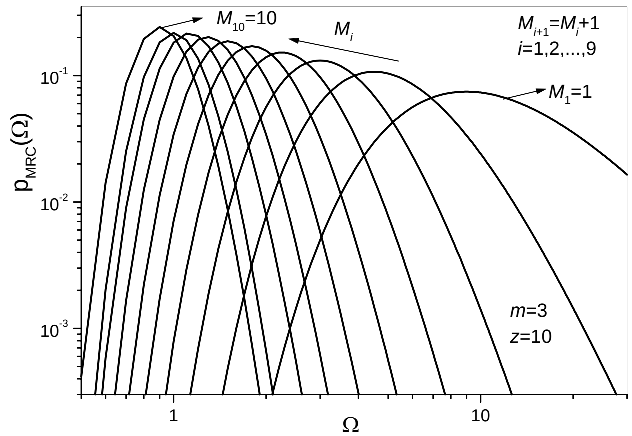

Fig. 4. The received signal pdf versus average signal power, in logarithmic scale, for fixed value of m and z, with number of diversity branches taking values from 1 to 10.

The analysis of this dependence shows that the increase of number of diversity branches results in higher values of the pdf maximums, reached for smaller values of average signal power, as shown in Fig 4. It can also be concluded that all the maximums lie on a straight line, and that the envelope of maximums is also a straight line, in logarithmic scale. In order to determine the values of average signal level when the maximums are reached, as well as the values of maximums and direction coefficient of envelope, the first derivative of (4) relative to average signal power is determined. Equalization the first derivative with zero obtains:

$$\Omega_{p\max} = z/M \qquad (11)$$

Substituting (11) into (4) yields:

$$p_{MRC}^{\Omega\max} = \frac{(mM)^{mM}}{\Gamma(mM)} \cdot \frac{1}{z} \cdot \exp(-mM) \qquad (12)$$

From (12) we get:

$$\log\left(\max p_{MRC}(z)\right) = k_2 \log z + n_2 \qquad (13)$$

where the direction coefficient is:

$$k_2 = -1 \qquad (14)$$

and the value on the ordinate axis is:

$$n_2 = mM \log(mM) - \log\Gamma(mM) - mM \quad (15)$$

The direction coefficient of envelope has the value -1, regardless of the value of the fading depth parameter and number of diversity branches. The envelope determines a certain singular solution of differential equation which can describe the dynamics of this process, while the received signal pdf is its particular solution:

$$p'_{MRC}(\Omega) + p_{MRC}(\Omega)(M - z/\Omega)\cdot m/\Omega = 0 \quad (16)$$

The influence of fading depth parameter *m* is presented in Fig 5, showing the translation the translation of the pdf curves family maximums envelope. For higher values of *m*, the value on the ordinate axis n_2 is greater, as shown in Fig 5, while the direction coefficient k_2 has the same value.

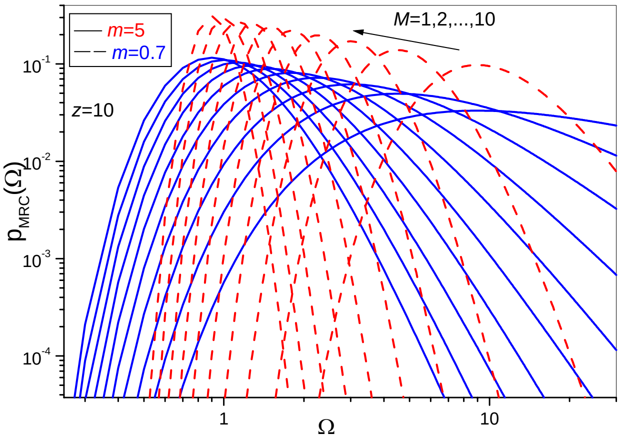

Fig. 5. The received signal pdf versus average signal power, in logarithmic scale, for fixed value of *z*, and different values of *m*, with number of diversity branches taking values from 1 to 10.

The influence of received signal level is presented in Fig 6, also showing the translation of the pdf curves family maximums envelope. For higher values of *z*, the value on the ordinate axis n_2 is smaller, as shown in Fig 6, while the direction coefficient k_2 has the same value.

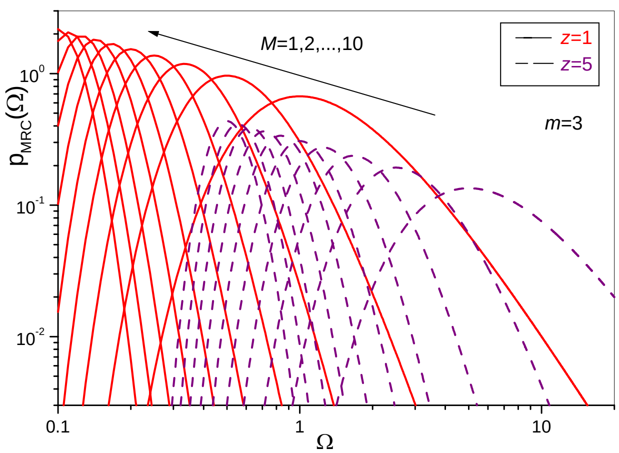

Fig.6. The received signal pdf versus average signal power, in logarithmic scale, for fixed value of *m*, and different values of *z*, with number of diversity branches taking values from 1 to 10.

IV. Conclusion

This paper presents some properties of the envelope of the pdf curves family in Nakagami-*m* environment when MRC receiver is applied. The obtained results show that the position of the maximums of these pdfs is uniquely determined by the maximums envelope's equation, which presents the singular solution of corresponding differential equation describing the complete dynamics of the signal transmission process.

In such a way, the boundary conditions for radio transmission, with given propagation conditions, could be defined, while some system performance measures could be evaluated, since they are all related to the received signal pdf.

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