Adaptive Filtering Algorithms Suitable for Real-Time Systems

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Abstract – **In this paper the main characteristics of adaptive systems are introduced. The vital role in the development of adaptive algorithms for the telecommunications is explained. Also, adaptive systems proved to be extremely effective in achieving high efficiency, high quality and high reliability of ubiquitous telecommunication services.**

Keywords – **adaptive filtering, adaptive filtering algorithms, IIR and FIR adaptive filters.**

I. Introduction

Adaptive digital filters can be realized on the basis of different structures. The choice of structure is the main factor influencing the computational complexity (number of arithmetic operations at each iteration), and hence the number of iterations to achieve the desired efficiency. Adaptive digital filters can be divided mainly into two main classes according to the received pulse shape characteristics: finite impulse response characteristic (FIR) and infinite impulse filter characteristics (IIR).

The FIR and IIR filters can be realized with implementation of an adaptive algorithm. The solution to successful adaptive signal processing is understanding the fundamental properties of the adaptive algorithms. The algorithms instead of structure (recursive or not) are also divided into two main classes.

The main characteristics for assessment the performance of adaptive systems are: stability, speed of convergence of the algorithm, missadjustement errors, robustness to both additive noise and signal conditioning (spectral colouration), least mean square error, numerical (computational) complexity, robustness, the order of the filter transfer function and the round-off error analysis of adaptive algorithms.

However, some of these properties are often in direct conflict with each other, since consistent fast converging algorithms tend to be in general more complex and numerically sensitive. Also, the performance of any algorithm with respect to any of these criteria is entirely dependent on the choice of the adaptation update function, that is the cost function used in the minimization process. A compromise must be than reached among these conflicting factors to come up with the appropriate algorithm for the concerned application.

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II. Measurement Parameters

1. *Cost functions*

Before proceeding to discuss any adaptive algorithm, it is necessary to discuss the performance measure (cost function) which is used in adaptive filtering. The adaptive filter has the general form shown in Fig. 1 where the FIR filter of order N is considered here. The filter output $y(n)$ is given by:

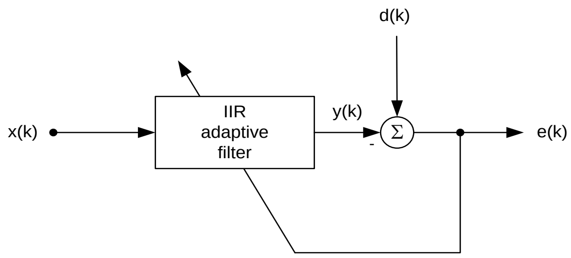

Fig. 1. Figure example

$$y(n) = \sum_{i=0}^{N-1} w_i(n)x(n-i) = \quad (1)$$

$$= w^T(n)x(n) \quad (2)$$

and $x(n) = [x(n), x(n-1), x(n-2), ... , x(n-(N-1))]$, (3)

where $w^T(n) = [w_0(n), w_1(n), w_2(n) ... w_{N-1}(n)]$ (4)

where T denotes transpose. In general, adaptive techniques have been classified under two main categories. In one category, the cost function to be optimised in a running sum of squared errors is given by:

$$J(n) = \sum_{j=0}^{n} e^2(j) \quad (5)$$

where the error $e(n)$ is defined to be the difference value between the desired response $d(n)$ and the output of the adaptive filter $y(n)$, that is, $e(n) = d(n) - y(n)$ The approach, defined by (5), is based on the method of least squares [3-5], which contains the whole class of recursive least squares (RLS) algorithms [6], [7].

In the other category, the cost function to be optimised is a statistical measure of the squared error, known as the mean squared-error (MSE) [7]. This cost function is given by

$$J(n) = E[e^2(n)] \quad (6)$$

where $E[]$ denotes the statistical expectation. This category contains the whole class of gradient algorithms, which includes the least mean-squared (LMS) algorithm [1], [6], [7].

All the functions presented in this section and others not mentioned in this work should be positive and monotonically increasing [35] for their corresponding algorithms to perform correctly.

2. *Convergence Rate*

The speed of convergence determines the rate at which the filter converges to the optimal solution. The main objective in the design of adaptive systems is to achieve fast convergence. The speed of convergence depends on all the other characteristics of the filter.

If some of the parameters are changed to receive good convergence of the adaptive digital filter, then this will lead to increase or decrease of the other characteristics. Very often when the speed of convergence increases, the stability characteristics will get worst, this make the system to diverge to a solution instead of converge to the optimal solution. This proves that the speed of convergence to the optimal solution can be considered and evaluated only in the context of assessment of the other key features.

3. *Minimum Mean Square Error*

The least mean square error (MSE) is a parameter indicating how exact the system adapts to the particular solution. If the MSE has a very small value, this means that the adaptive system exactly converges to the optimal solution of the system. If the size of the MSE is very large, it means that the adaptive filter can not properly model the system or initial conditions that are set are wrong starting point is not correct and the filter converges, but not to the right/optimal solution. There are many parameters by which can be determinated the minimum of the MSE. Some of those parameters are the noise due to quantization effects, order of the adaptive system, measurement noise and the gradient error due to the final step size of adaptation.

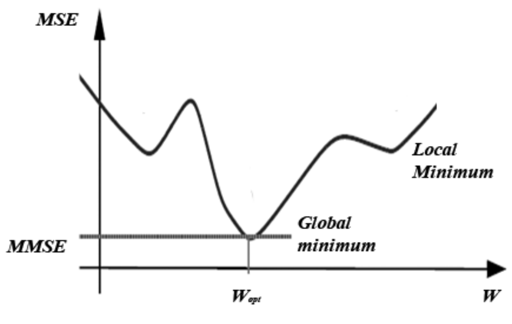

Fig. 2. Figure example

If an adaptive FIR filter has two weights then the performance surface has the form of a paraboloid in 3 dimensions. If the filter has more than three weights then it cannot be drawn the performance surface in three dimensions. In mathematical point of view there is only one minimum point which occurs when the gradient vector becomes equal to zero. When the performance surface is quadratic with more than three dimensions is called a hyperparaboloid.

If a FIR filter is used, then the MSE is determined, leading to a point situated in the lowest part of a hyperboloid. The mathematical representation of this is the process of finding the zero of the gradient vector.

If the implementation is based on an IIR filter structure, then the MSE will have local minimum as well the desired global one. The graphical representation of this is:

4. *Computational Complexity of adaptive algorithms*

The computational complexity is very important parameter in real time applications of adaptive digital filters. If the applications are in real-time systems there are limitations which are introduced by the hardware, and they can affect the behavior of the whole system. If is used a very complicated algorithm it will require more complex hardware than it gets when a simpler ones are used.

The efficiency of the algorithm is in close relation with the computational complexity. The number of additions and multiplications per iteration are the limit of the adaptive system, because they take time on the processor to process the signals. The tendency is to develop more and more complex algorithms. One of the most interesting areas for researchers is the development of algorithms with lower computational requirements, due to the limitations of the hardware realizations.

Important feature for an algorithm is the time necessary for processing. This time is based on the number of operations in a single operation. Usually adaptive algorithms are iterative.

5. *Stability*

The stability is the next important feature which is important to be investigated during the process of design of adaptive filters. This is may be the most important characteristic. Because of their nature adaptive systems have very few completely asymptotically stable systems can be realized. In most cases, the systems used are marginally stable, which is predetermined by the initial conditions, the system transfer function and step at the entrance.

6. *Robustness*

The robustness of a system is directly related to the stability of a system. Robustness can be defined as the ability of the system to tolerate changes. The redundant, concurrent system models allow for a quick context switching on occurrences of abrupt changes and also for concurrent simulation and testing to continuously adapt to the environment or to the requirement.

Robustness is a parameter by which is measured how the system will work when are introduced the effects of the input noise, the noise due to the quantization and the insensitivity to external errors. Those analyses the behavior of a system against internal errors due to the effects listed above.

7. Filter order

The order of the adaptive filter system is inherently related to many of the other parameters involved in the assessment of the system. The order determines how exact a system can be modeled by an adaptive filter. It also affects the speed of convergence by increasing or decreasing the time necessary for calculations, the stability of the system at a fixed step size of adaptation and the minimum of the MSE. If the order of the filter increase then will increase the number of calculations, thus reducing the maximum speed of convergence. In order to achieve stability because of increasing the order of the filter can be added poles and zeros, which is less than it already has. In such cases, the maximum step size or the maximum rate of convergence will have to be reduced to ensure stability. Finally, if the system is under specified, meaning there are not enough pole and/or zeroes to model the system, the least mean square error will converge to a constant different from zero. Usually when the system is over specified, meaning that it has to many poles and/or zeros it will be possible to converge to zero, but the increasing number of calculations will affect the maximum of possible speed of convergence.

This division according to different types of algorithms is depicted on the next graph.

III. ADAPTIVE ALGORITHMS

Types of adaptive algorithms

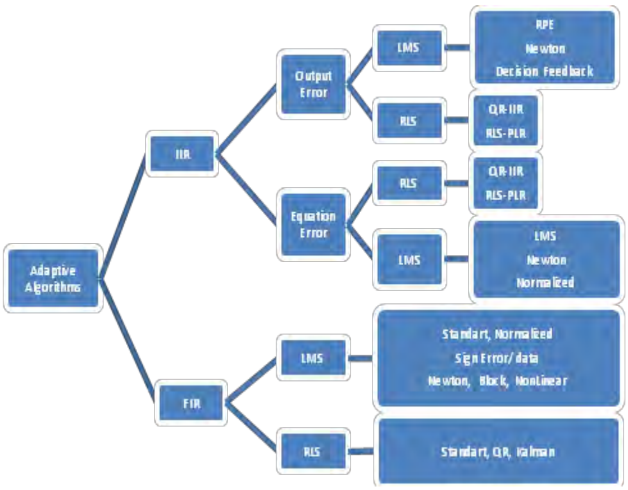

Fig. 3. Adaptive algorithms tree

The adaptation of the filter parameters is based on minimizing the mean squared error between the filter output and a desired signal. The most common adaptation algorithms are the Recursive Least Square (RLS) and the Least Mean Square (LMS). The RLS algorithm has higher convergence speed compared to the LMS algorithm. If the main characteristic is the computation complexity, then the LMS algorithm is much faster than the RLS. Due to the computational simplicity, the LMS algorithm is most commonly used in the design and implementation of integrated adaptive filters. The LMS digital algorithm is based on the gradient search according to the equation (1).

3. 1. Finite Impulse Response (FIR) Algorithms

The adaptive algorithms can be divided in two main types according to the mathematical formulation used. As it was previously mentioned – RLS and LMS and the second point of division – IIR or FIR. The implementation depends on the specific requirements in each realization. One of the most widely used is:

3.1.1 Least Mean Squares Gradient Approximation Method

If is given an adaptive filter with an input signal *x(n)*, an impulse response *w(n)* and an output signal *y(n)* can be derived a mathematical relation for the transfer function of the system:

$$w(n+1) = w(n) - \mu \Delta_{E^2}(n) \tag{7}$$

$$y(n) = w^T(n)x(n) \tag{8}$$

and $$x(n) = [x(n), x(n-1), x(n-2), \ldots, x(n-(N-1))] , \tag{9}$$

where $$w^T(n) = [w_0(n), w_1(n), w_2(n) \ldots w_{N-1}(n)] \tag{10}$$

are the time domain coefficients for an N-th order FIR filter.

In the above equation *w(n+1)* represents the new coefficient values for the next time interval, μ is called scaling factor, and $\Delta_{E^2}(n)$ is the ideal cost function with respect to the tap weight *w(n)*. From the above formula can be derived the estimate for the ideal cost function

$$w(n+1) = w(n) - \mu e(n)x(n) \tag{11}$$

where $e(n) = d(n) - y(n)$ and $y(n) = x^t(n)w(n)$.

In the above equation the coefficient μ is very often multiplied by 2, but here we will assume it is in the μ factor.

The Least Mean Squares Gradient method, usually presented as the Method of Steepest Descent, an investigation based on the current filter coefficients is made, and the gradient vector, the derivative of the MSE with respect to the filter coefficients, is calculated from the investigated. Secondly is made tap-weight vector estimation by making a change in the present guess in a direction opposite to the gradient vector. This process is repeated until the derivative of the MSE is zero.

3.1.2 Quasi-Newton Adaptive Algorithms

The quasi-Newton adaptive algorithms are based on the implementation of second order statistic in order to reduce the speed of convergence of the adaptive filter, according to the Gauss-Newton method. The fames one quasi-Newton algorithm is the RLS algorithm. It is important to be notet that even the speed of convergence is increased the RLS requires a great amount of processor power, which will lead to difficulties in their implementation in real-time systems.

In the family of quasi-Newton algorithms are some having good convergence properties and are alternative to process information signals in real-time. In the paper [7] and [9] it is well described.

3.1.3 Adaptive Lattice Algorithms

The main reason for the use of lattice structures is to reduce quantization noise introduced by the filter coefficients in systems with limited word length. The purpose of developed lattice adaptive algorithms is to reduce the effects of quantization noise and thus to try to reduce the length of the register maintaining good behavior. In [7] and [9] this is well described.

3.2. Infinite Impulse Response (IIR) Adaptive Filters

The most important advantage of IIR filters is that they are the basis to receive an equivalent amplitude frequency response of a FIR filter, but with a lower number of coefficients. This theoretically decreases also the number of adders, multipliers and other mathematical operations to perform filtering. This is the main reason for implementing them. The lower number of coefficients leads to less number of undesired sources of noise (for example due to the limited length of the digital registers-finite word length). The recursive filters, however, lead to many problems in their use, due to their instability issues.

The main problem with the use of adaptive recursive filters is the possible instability of the poles position of the transfer function. In some cases they could get out of the unit circle, during the process of training the system which means that the system will become unstable. Even if the system is stable at the beginning and in the end, there still is a possibility the system to be destabilized during the process of convergence. One good solution to this is to introduce restrictions on the position of the poles (in order to limit them within the unit circle) but this method requires a small step size, which will significantly reduce the speed of convergence.

Due to the interplay between the movement of the poles and zeros, the convergence of IIR systems tends to be slow [3]. The result is that even though IIR filters have fewer numbers of coefficients, therefore fewer calculations per iteration, the number of iterations may increase and this will cause a change and loss of time in processing time to reach the convergence. This, however, is not a problem when implementing all pole filters.

In IIR system the MSE surface can have a local minimum, which can lead to convergence of this system to a local minimum and not to a global one. It must be considered also the initial conditions for adaptive IIR filters.

The IIR filters are more susceptible to quantization errors of the coefficients of the FIR, which is due to the presence of a feedback.

There have been a number of studies done on the use of IIR adaptive filters, but due to the problems stated above, they are still not widely used in industry today.

IV. CONCLUSION

In conclusion in this contribution was made an outline of the main adaptive algorithms and characteristics important in investigations of an adaptive system. The choice of the mathematical algorithm for IIR and FIR filters depends on the concrete realization and performance desired. In some cases adaptive RLS algorithms are preferred, in other LMS as the most widely used for both types of filters because of its very good properties: fast convergence and good performance stability.

Investigations on different structure realizations are also important in order to avoid finite word length effects, possible instability when IIR filters are implemented. A crucial point for researchers is to develop structures with canonical number of multipliers and with low sensitivity to all those undesired effects.

The role of adaptive systems is wide spread covering almost all aspects of telecommunication engineering, but perhaps most notable in the context [3] of ensuring high-quality signal transmission over unknown and time varying channels.

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