Noise- Resistance Performance Estimation of a Chaos Shift Keying Signals

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Abstract – **In this paper the performance evaluation of error probability for a chaos shift keying coherent system over an Additive White Gaussian Noise (AWGN) channel is presented.**

Keywords – **Chaos communication, Chaos-shift keying, Error probability**

I. Introduction

The chaotic signal is a relatively new field in communication systems. Motivation derives from the advantages offered by chaotic signals, such as spread spectrum, robustness in multipath environments and resistance to jamming. Chaotic signals are non-periodic, broadband, and difficult to predict and to reconstruct; they may be generated by mathematical map functions, electronic circuits or laser optics. Their properties coincide with requirements for signals used in communication systems, in particular for secure communication systems.

Chaos Shift Keying (CSK) [1,2] is one of the encoding/modulation methods, proposed in the literature to send digital messages. The CSK method employs the advantages offered by the inherent phase synchronization [2] given by the drive response principle and well understood correlation methods [3].

The receiver in CSK communication system can use coherent or non-coherent detection techniques. In coherent detection, the receiver is required to reproduce the same chaotic signals sent by the transmitter, often through a chaos synchronization process which is unfortunately a fragile process [3]. In non-coherent detection of CSK, however, the receiver does not have to reproduce the chaotic signals. Rather, it makes use of some distinguishable property of the chaotic signals to determine the identity of the digital symbol being transmitted. Different attractors may differ in variance, meaning of the absolute value, dynamic range, and many other statistical properties [3]. The optimal decision level of the threshold detector, however, will depend on the signal-to-noise ratio of the received signal.

One of the main characteristics determining the effectiveness of a radio communication system is the stability against disturbances [3]. It is characterized with the dependency of the fidelity of received communications on the line energy parameters, algorithms used to transmit information and statistical characteristics of disturbances [3]. With discrete systems of connections, the error probability of distinguishing signals is used for fidelity assessment. [3].

The purpose in this paper, is to present the error probability of a coherent CSK digital system under the influence of additive white Gaussian noise (AWGN), assuming ideal synchronization at the receiver. The solution for the error probability has been derived, in terms of the signal-to-noise power ratio.

II. Review of CSK Approach

The idea of CSK Approach is to encode digital symbols with chaotic basis signals. A block diagram of the communication system with CSK is shown in Fig. 1. The transmitter dynamics is dissipative and chaotic and the transmitter state trajectory converges to a strange attractor. A message is transmitted by changing one or more parameters of the transmitter dynamics which results in a change of the attractor dynamics. At the receiver the message is decoded by estimating to which message the received chaotic attractor corresponds.

The transmitter consists of two chaos generators 1 and 2, producing signals $x_1(t)$ and $x_2(t)$, respectively.

Assume that a chaotic signal is generated by the map

$$x[n+1]=f(x[n]), \tag{1}$$

where

$$x[n]=(x_1[n], x_2[n], \ldots\ldots, x_m[n]) \tag{2}$$

is the state, $\boldsymbol{f} = (f_1, f_2, \ldots\ldots\ldots f_m)$ is the discrete functional transformation (maps) the state $\mathbf{x}[n]$ to the next state $\mathbf{x}[n+1]$.

With two different initial conditions, we can generate two sets of chaotic sequences which can be used to represent two binary symbols. Let $\{x_{n1}\}$ and $\{x_{n2}\}$ be the two chaotic sequences representing ”0” and ”1” respectively.

The outputs of the chaotic signal generators, denoted by $x_1(t)$ and $x_2(t)$, are given by

$$x_1(t) = \sum_{n=0}^{\infty} x_{n1} r(t - nT_r) \tag{3}$$

and

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$$x_2(t)=\sum_{n=0}^{\infty} x_{n2} r(t-nT_r) \ , \qquad (4)$$

where $r(t)$ is a rectangular pulse of unit amplitude and width T_r , i.e.,

$$r(t)=\begin{cases}1, & 0\le t<T_r\\ 0, & \text{elsewhere}\end{cases} \ . \qquad (5)$$

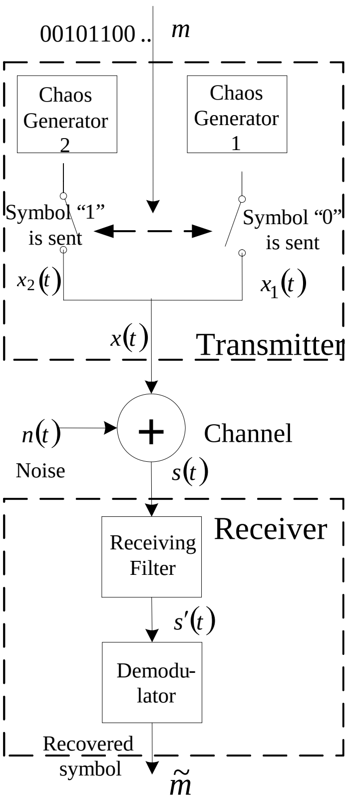

Fig. 1 Binary CSK digital communication system

Assume that the system starts at t=0 and the binary data to be transmitted has a period T_b .

Denote the transmitted data by:

$$\boldsymbol{m}=(m_1, m_2, \ldots\ldots) \ , \quad m_m \in \{0,1\}$$

If a binary "0" is to be sent during the interval $[(m-1)T_b, mT_b]$, ; $x(t)=x_1$ is transmitted by the communication channel. If the binary symbol "1" is to be sent, $x(t)=x_2$ is transmitted. Here, m is a number of the transmitted symbol.

Let

$$\beta=\frac{T_b}{T_r} \qquad (6)$$

be the spreading factor, which is an integer.

Thus

$$u^{(m)}(t)=\sum_{n=0}^{\beta-1} y^{(m)}_{n+(m-1)\beta}\, r[t-(nT_r+(m-1)T_b)] \qquad (7)$$

is the transmitted waveform for the m-th bit, where

$$y^{(m)}_{n+(m-1)\beta}=\begin{cases}x_{[n+(m-1)\beta]1} \ , & \text{if } \mathrm{m_m}=0\\ x_{[n+(m-1)\beta]2} \ , & \text{if } \mathrm{m_m}=1\end{cases} . \qquad (8)$$

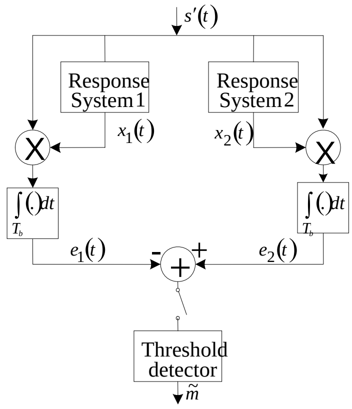

Fig. 2. Block diagram of a coherent CSK receiver

The overall transmitted waveform, $x(t)$, is

$$x(t)=\sum_{m=1}^{\infty} p^{(m)}(t) \ . \qquad (9)$$

III. Functional Kind of the Noise-Resistant Performance of a Coherent Receiver with Additive White Noise

For ideal communication system, the channel is anticipated only with additive white Gaussian noise $n(t)$ where it is free from intersymbol interference.This is usually a good starting point for understanding basic performance relationships. In the following analysis, $n(t)$ is replaced by an equivalent noise source $n'(t)$ given by

$$n'(t)=\sum_{n=0}^{\infty}\zeta_n r(t-nT_r), \qquad (10)$$

where the coefficients $\{\zeta_n\}$ are independent Gaussian random variables with zero mean and variance

$$\sigma^2_{n'}=\frac{N_0}{2T_r} \quad , \qquad (11)$$

where N_0 is the white Gaussian noise spectral power density.

The block diagram of a coherent CSK receiver as shown in Fig. 2. Assume that the transmitted signal by the receiving filter is

$$s'(t)=x(t)+n'(t)=\sum_{m=1}^{\infty}u^{(m)}(t)+\sum_{n=0}^{\infty}\zeta_n r(t-nT_r)=$$

$$=\sum_{m=1}^{\infty}\sum_{n=0}^{\beta-1}y^{(m)}_{n+(m-1)\beta}r[t-(nT_r+(m-1)T_b)]+\sum_{n=0}^{\infty}\zeta_n r(t-nT_r). \quad (12)$$

For the m-th received symbol, the output of correlator$_{(“0”)}$ at the end of the bit duration equals to

$$e_1=\int_{(m-1)T_b}^{mT_b}s'(t)x_1(t)dt=T_r\sum_{n=(m-1)\beta}^{m\beta-1}y^{(m)}_n x_{n1}+\zeta_n x_{n1}. \quad (13)$$

The output of correlator$_{(“1”)}$ can be shown equal to

$$e_2=\int_{(m-1)T_b}^{mT_b}s'(t)x_2(t)dt=T_r\sum_{n=(m-1)\beta}^{m\beta-1}y^{(m)}_n x_{n2}+\zeta_n x_{n2}. \quad (14)$$

The input to the threshold detector equals to

$$e=e_2-e_1=$$

$$=T_r\sum_{n=(m-1)\beta}^{m\beta-1}y^{(m)}_n x_{n1}+\zeta_n x_{n1}-y^{(m)}_n x_{n2}-\zeta_n x_{n2}. \quad (15)$$

If a “ 1” has been transmitted for the m-th symbol, i.e., $m_m=1$ and $y^{(m)}_n=x_{n2}$ for $(m-1)\beta\le n\le m\beta-1$, the input of the detector will be given by

$$e_{“1”}(mT_b)=T_r\sum_{n=(m-1)\beta}^{m\beta-1}x^2_{n2}+\zeta_n x_{n2}-x_{n2}x_{n1}-\zeta_n x_{n1}. \quad (16)$$

The error probability given a “1” has been transmitted is given by [3]:

$$p(e_{“1”})=F\left(\frac{\overline{e_{“1”}(mT_b)}}{\sqrt{\mathrm{var}[e_{“1”}(mT_b)]}}\right), \quad (17)$$

where

$$F(z)=\frac{2}{\sqrt{2\pi}}\int_0^z\exp\left(\frac{-t^2}{2}\right)dt \quad (18)$$

is the integral function of Cramp’s distribution [3]; $\overline{e_{“1”}(mT_b)}$ is the mean value and $\mathrm{var}[e_{“1”}(mT_b)]$ is the variance of $e_{“1”}(mT_b)$.

Likewise, it can be shown that when a “0” has been transmitted, the conditional error probability is given by

$$p(e_{“0”})=F\left(\frac{\overline{e_{“0”}(mT_b)}}{\sqrt{\mathrm{var}[e_{“0”}(mT_b)]}}\right), \quad (19)$$

where

$$e_{“0”}(mT_b)=T_r\sum_{n=(m-1)\beta}^{m\beta-1}x^2_{n1}+\zeta_n x_{n1}-x_{n2}x_{n1}-\zeta_n x_{n2}. \quad (20)$$

The average bit energy of the system is given by [3]:

$$\overline{E}_b=T_b\lim_{T\to\infty}\frac{1}{T}\int_0^T x^2(t)dt=$$

$$=T_r\left[\beta\overline{\gamma}(x_{n2},\beta)p(e_{“1”})+\beta\overline{\gamma}(x_{n1},\beta)p(e_{“0”})\right], \quad (21)$$

where $\overline{\gamma}(x_{n2},\beta)$ and $\overline{\gamma}(x_{n1},\beta)$ denote the mean-squared values of chaotic sequence samples of length β taken from the chaotic series $\{x_{n1}\}$ and $\{x_{n2}\}$ respectively.

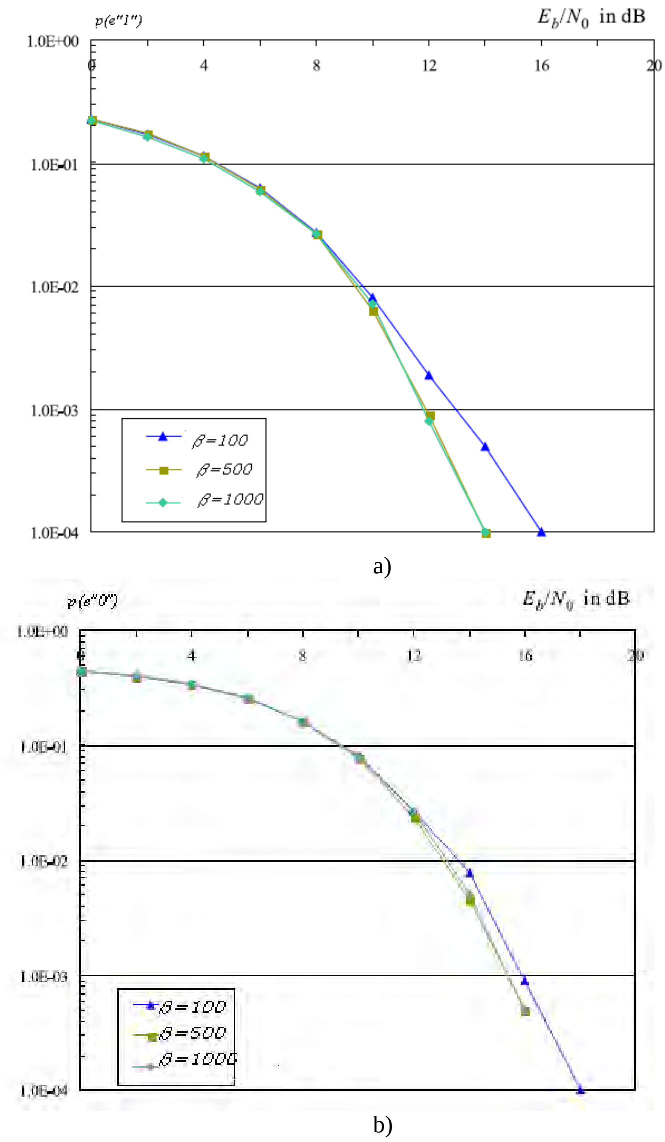

Fig.3. Error probability vs. $\overline{E_b}/N_0$

Since both series $\{x_{n1}\}$ and $\{x_{n2}\}$ are generated from the same map with different initial conditions, we have $\bar{\gamma}(x_{n2}, \beta)$ = $\bar{\gamma}(x_{n1}, \beta)$ and (21) can be simplified to

$$\overline{E}_b = T_r \beta \bar{\gamma}(x_{n2}, \beta)[p(e_{"1"}) + p(e_{"0"})] = T_r \beta \bar{\gamma}(x_{n2}, \beta) \ . \quad (22)$$

The dependence of errors probability $e_{"1"}$ and $e_{"0"}$ for various average-bit-energy-to-noise-power-spectral-density ratios ($\overline{E_b}/N_0$) is shown in Fig. 3.

IV. CONCLUSION

In this paper we present the the error probability for various average-bit-energy-to-noise-power-spectral-density ratios ($\overline{E_b}/N_0$).The obtained results show that the error probability decreases with increasing $\overline{E_b}/N_0$. Moreover, it can be reduced for the same by using a higher spreading factor.

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