Complex Criterion for Linearity Segment Detection in the Subtraction Procedure for Removing Power-line Interference from ECG

Georgy Mihov[1]

Abstract – **The stage of linear segments detection is the most important stage in the subtraction method for removing powerline interference from ECG. A special criterion is used, corresponding to the second difference for the sampled signal. Its essence is interpreted as a mathematical estimate of the acceleration of the signal and also as a non-recursive digital filter. Transfer functions of simple and complex linearity criteria have been synthesized and analysed. The analysis allows a qualitative estimating of the linearity criterion to be done. Theoretical approaches are supported by numerous practical experiments.**

Keywords – **Digital filters, ECG filtering, Mains interference removing.**

I. Introduction

The subtraction procedure for power-line (PL) interference removal from ECG signals [1-5] has already shown high efficiency. A deep analysis and comparison with other methods for PL interference suppression is done in [6]. Due to its qualities, the subtraction method continues to be subject of complementary investigations [7-9]. Its structure consists of three main stages:

– *Linear segment detection* – every ECG sample is tested whether it belongs to a linear segment;

– *Interference extracting* – if the criterion for linearity is fulfilled the the PL interference is extracted with appropriate digital filter, saved in a temporal FIFO buffer and at the same time is removed from the linear segment;

– *Interference subtracting* – the restored value of the PL interference is subtracted from the original signal in non-linear segments.

The first stage is essential for the accuracy of the subtraction method, i.e. finding linear segments. The error of calculating the interference depends on it. A special criterion for linearity $Cr < M$ is used for this purpose, where M is a practically chosen threshold.

In the practice of the subtraction procedures are used various criteria for linearity [1, 6, 8, 9], but the most proven ones are those using absolute value of ‘second difference’. First differences are taken by samples located at distance of one period of the PL frequency, thus eliminating the interference influence

$$Cr = |D_i| = |(X_{i-n} - X_i) - (X_i - X_{i+n})|. \qquad (1)$$

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The parameter n stands for the ratio between the sampling rate Φ and the PL frequency F, i.e. $n = \Phi/F$ represents number of samples within one period of the PL interference. In the case of multiplicity between the sampling rate and the PL frequency, n is an integer.

The procedure error depends on the threshold M. As less is the threshold, as less is the committed error, but the chance to find a linear segment decreases. The involved absolute error by non-zero value of the threshold can be evaluate on $M/(2n)$.

II. Linearity criterion for the case of non-multiplicity

In cases of non-multiplicity between the sampling rate and the PL frequency, the ratio Φ/F is a real number. The non-multiplicity appears also when the PL frequency deviates around its rated value. The round value n^* is used, which is the greatest integer less than or equal to Φ/F. The introduced in [9] linearity criterion for the case of non-multiplicity is performed by summarizing two criteria, normalized with a factor k_n. The basic linearity criteria is done by the equation

$$D_i = (1-k_n)(X_{i-n^*} + X_{i+n^*}) + k_n(X_{i-n^*-1} + X_{i+n^*+1}) - 2X_i , \qquad (2)$$

having a transfer coefficient for the PL frequency F

$$D_F = -4(1-k_n)\sin^2\frac{n^*\pi F}{\Phi} - 4k_n\sin^2\frac{(n^*+1)\pi F}{\Phi}. \qquad (3)$$

The basic linearity criteria is modified by the auxiliary filter that is summarized by two auxiliary filters, normalized with a factor k_m according the equation

$$A_i = -(X_{i+m^*} + X_{i-m^*})\frac{1-k_m}{4} - (X_{i+m^*+1} + X_{i-m^*-1})\frac{k_m}{4} + \frac{X_i}{2}. \qquad (4)$$

Its transfer coefficient for the PL frequency F is done by

$$A_F = -(1-k_m)\sin^2\frac{m^*\pi F}{\Phi} - k_m\sin^2\frac{(m^*+1)\pi F}{\Phi}. \qquad (5)$$

The round value m^* is used for represent the number of samples in a semi-period of the PL frequency, which is the greatest integer less than or equal to $\Phi/(2F)$. The modified linearity criterion is

$$Cr = |D_i^*| = \left| D_i + A_i\frac{D_F}{A_F} \right|. \qquad (6)$$

Coefficients $k_n = \Phi / F - n^*$ and $k_m = \Phi / F - m^*$ are offered in [9]. More preciously calculation for additional coefficients is done by the formulas

$$k_n = \frac{n^* \cos\frac{\pi n^* F}{\Phi} \sin\frac{\pi n^* F}{\Phi}}{n^* \cos\frac{\pi n^* F}{\Phi} \sin\frac{\pi n^* F}{\Phi} - (n^*+1)\cos\frac{\pi (n^*+1) F}{\Phi} \sin\frac{\pi (n^*+1) F}{\Phi}}, \quad (7)$$

$$k_m = \frac{m^* \cos\frac{\pi m^* F}{\Phi} \sin\frac{\pi m^* F}{\Phi}}{m^* \cos\frac{\pi m^* F}{\Phi} \sin\frac{\pi m^* F}{\Phi} - (m^*+1)\cos\frac{\pi (m^*+1) F}{\Phi} \sin\frac{\pi (m^*+1) F}{\Phi}}. \quad (8)$$

Fig. 1 shows responses of the linearity criterion by Eq. (6), regarding as a digital filter (called D-filter).

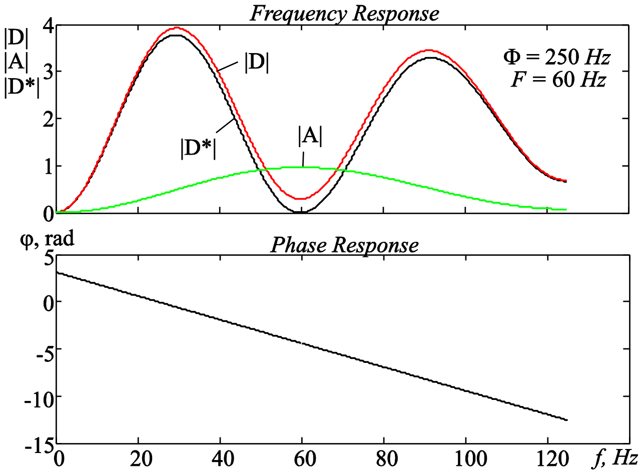

Fig. 1. Linearity criterion for Φ = 250 *Hz*, *F* = 60 *Hz*.

III. COMPLEX LINEARITY CRITERION FOR THE CASE OF NON-MULTIPLICITY

The complex criterion for linearity in cases of multiplicity [1] uses $n+1$ successive first subtractions and the complex second difference is formed as a subtraction of the maximal and minimal first difference

$$Cr = \left|\max(FD_0, FD_1, \ldots, FD_n) - \min(FD_0, FD_1, \ldots, FD_n)\right|. \quad (9)$$

For cases of non-multiplicity two first differences

$$FD0_i = X_i - X_{i+n^*} \text{ and } FD1_i = X_{i-1} - X_{i+n^*+1}, \quad (10)$$

with frequency responses respectively

$$FD0(f) = \sin\frac{n^* \pi f}{\Phi} \text{ and } FD1(f) = \sin\frac{(n^*+2)\pi f}{\Phi}, \quad (11)$$

are summarised with a normalising factor k_d, i.e.

$$FD^*_i = (X_i - X_{i+n^*})(1-k_d) + (X_{i-1} - X_{i+n^*+1})k_d. \quad (12)$$

The factor k_d may be determined by equalising the transfer function of the modified first difference to zero for $f = F$, i.e.

$$FD^*_F = \sin\frac{n^* \pi F}{\Phi}(1-k_d) + \sin\frac{(n^*+2)\pi F}{\Phi} k_d = 0,$$

$$k_d = \frac{\sin\frac{n^* \pi F}{\Phi}}{\sin\frac{n^* \pi F}{\Phi} - \sin\frac{(n^*+2)\pi F}{\Phi}}. \quad (13)$$

Fig. 2 shows responses of first differences represented as digital filters.

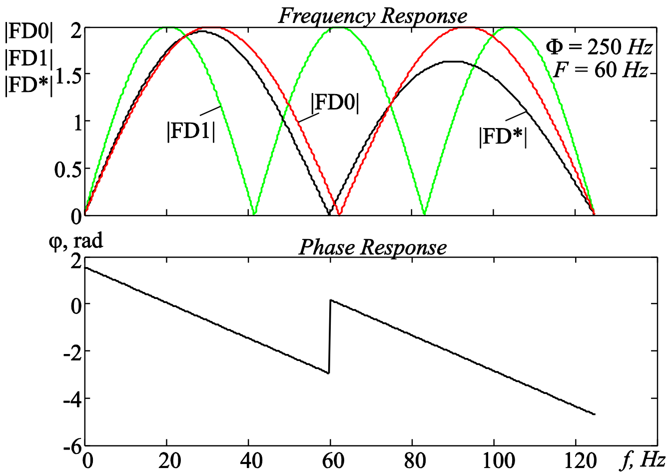

Fig. 2. Linearity criterion for Φ = 250 *Hz*, *F* = 60 *Hz* (n^* = 4).

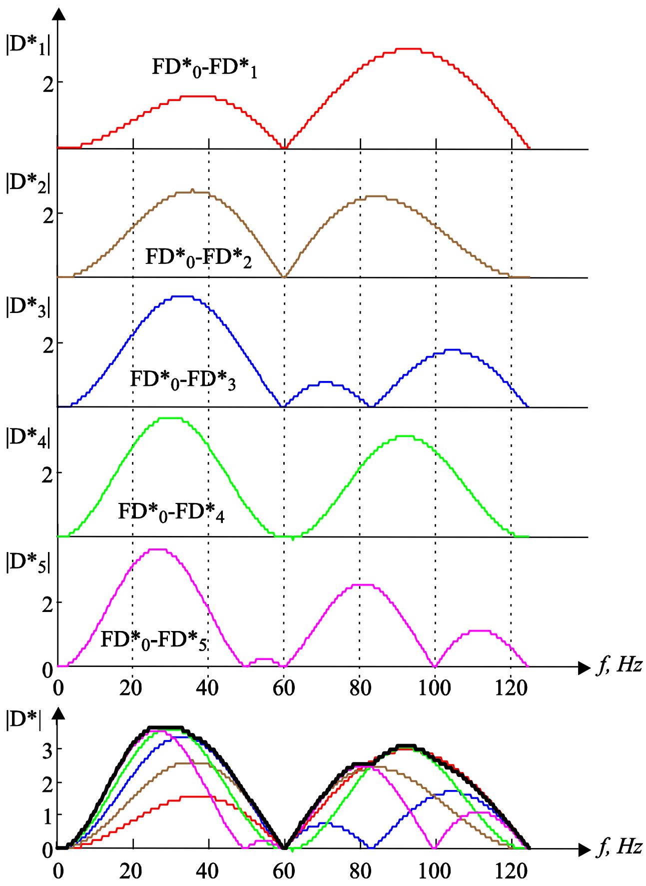

Fig. 3. Linearity criterion for Φ = 250 *Hz*, *F* = 60 *Hz* (n^* = 4).

In Fig. 4 are shown frequency responces of the family D-filters in case of Φ = 250 *Hz* and F = 60 *Hz* (n^* = 4). The last graphic shows the total (simultaneous) operation of all linearity criteria. The wrapper of the family of curves forms the transfer function of the filter according to formula

$$Cr = \left|\max\left(FD^*_0, \ldots FD^*_{n^*+1}\right) - \min\left(FD^*_0, \ldots FD^*_{n^*+1}\right)\right|. \quad (13)$$

IV. Experiments

The experimental investigation is performed in Matlab environment in the following sequence:

1. An episode from a signal of AHA database AHA_1001d1, which is considered as a conditionally clean from PL frequency (*Original conditionally clean signal*) The testing episode have got a duration of 16 *s* and sampling rate Φ = 250 *Hz*.
2. A synthesized PL interference with amplitude p = 0,5 *mV* is added to the original signal (*Original signal + interference*).
3. The contaminated signal is treated by the subtraction procedure and the filtered signal is shown on the third subplot (*Processed signal*).
4. The error is calculated as an absolute difference between the processed signal and the originally conditionally clean signal. It is shown on the forth subplot, together the linearity criterion (*Zoomed absolute error & linearity criterion course*).

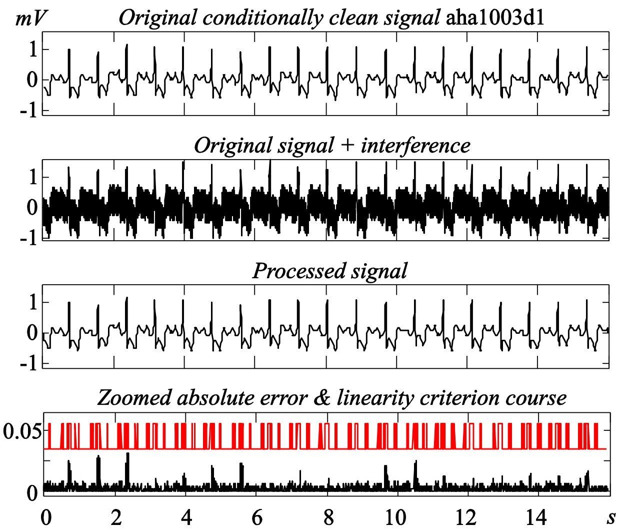

Fig. 4. Experiment with F = 60 *Hz* (n^* = 4) and a linearity criterion by Eq. (6).

The sequence of the testing is applied in two referent experiments. The first is shown in Fig. 5 with a signal aha1003d1, that is contaminated by a powerline interference with a frequency F = 60 *Hz*. The used linearity criterion is performed by Eq. (6) with a threshold M = 80 μ*V*. One may observe that the absolute error do not exceed 30 μ*V*.

At the same condition is performed the second referent experiment. The original signal is contaminated by PL interference with a frequency F = 16,7 *Hz*. The absolute error is higher than in the previous experiment and reaches 60 μ*V*.

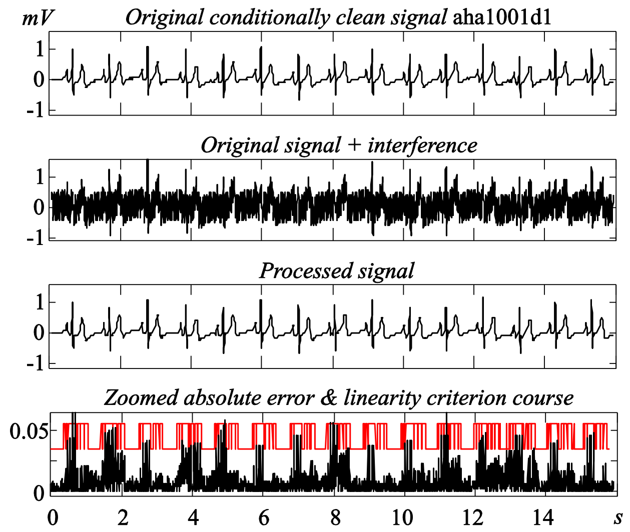

Fig. 5. Experiment with F = 16,7 *Hz* and linearity criterion Eq. (6).

Next two experiments are performed at the same conditions as previous, but using the offered complex criterion Eq. (9).

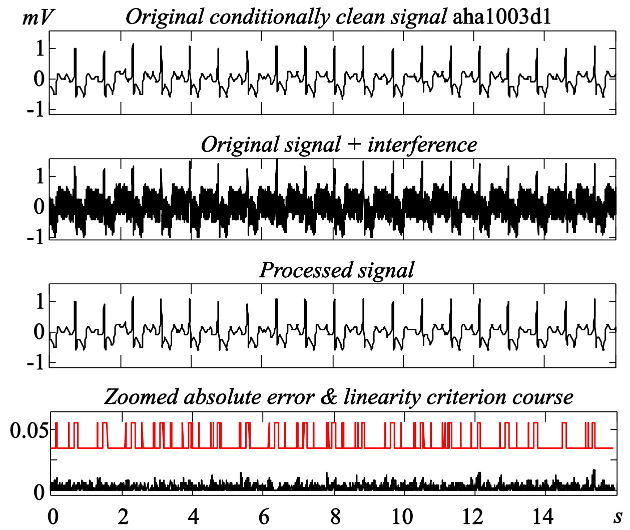

Fig. 6. Experiment with F = 60 *Hz* and linearity criterion Eq. (9).

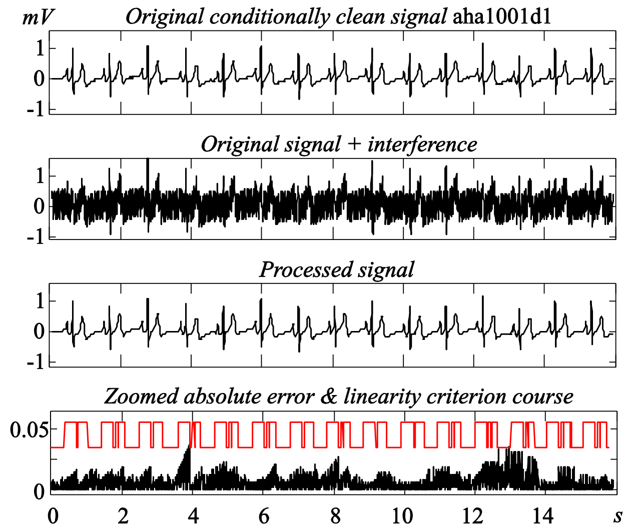

Fig. 7. Experiment with F = 16,7 *Hz* and linearity criterion Eq. (9).

Next two experiments are performed in condition on PL frequency deviation. An abrupt change in PL frequency F from $+dF$ to $-dF$ is simulated in the middle of the epoch. The fifth subplot (*Frequency deviation & linearity criterion course*) shows the PL frequency deviation (curve a – green), the estimated diversion of the PL frequency (curve b – black) and the criterion for linearity (curve c – red). It is obviously that the error committed, after reaching the stationary value of the PL frequency, is the same as in Figs. 6 and 7.

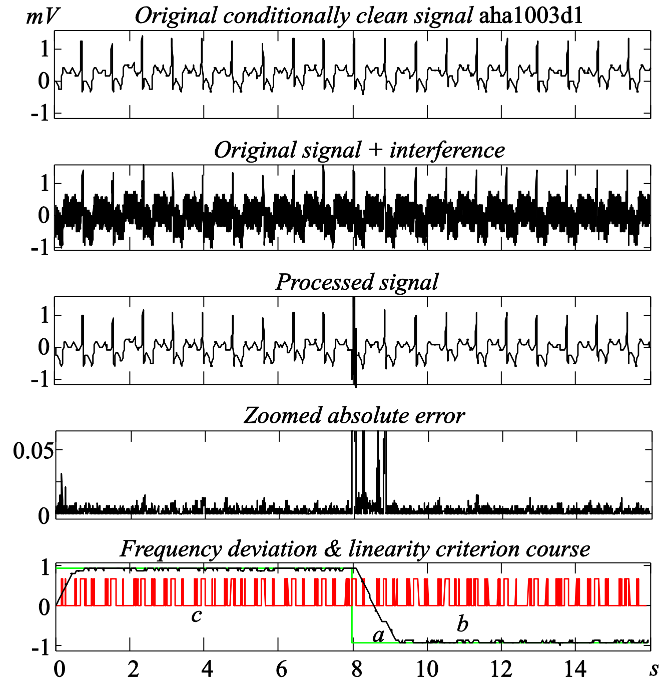

Fig. 8. Experiment with F = 60 Hz (n^* = 4) and a linearity criterion by Eq. (9).

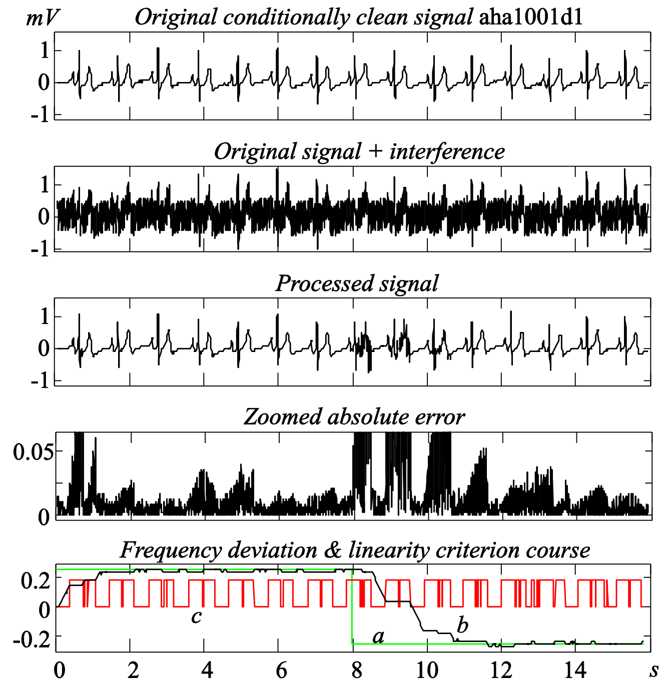

Fig. 9. Experiment with F = 16,7 Hz (n^* = 14) and a linearity criterion by Eq. (9).

IV. Conclusion

The article develops the subtraction procedure for removing PL interference from ECG in general case of non-multiple sampling and power line frequency deviation.

The stage of linear segments detection is the most important stage in the subtraction method for removing power-line interference from ECG. A new complex modification of the linear criterion is introduced that retains all needed features for non-multiplied sampling. The offered complex linearity criterion approximately twice reduces the absolute error of PL interference rejection – see experiment shown in Fig. 6 in comparison with Fig. 4 and Fig. 7 in comparison with Fig. 5.

Analogously results are obtained using the offered complex linearity criterion with signals contaminated by PL interference with frequency deviations – see Figs. 8 and 9. Despite good results obtained, one disadvantage has to be pointed: the range of PL frequency deviation, using complex criterion by Eq. (9), is shorten to ±0.5% in comparison to ±3%, using the criterion by Eq. (6). The computational complexity is also higher using complex criterion by Eq. (9), than using the criterion by Eq. (6), despite the Eq. (13) results in a constant that is calculated ones at procedure starting.

The involved absolute error caused by non-zero threshold $M = 120 \mu V$, is less than 20 μV for the case of Φ = 250 Hz and F = 50 Hz. The additional error is due to the own frequency components F in the ‘conditionally clean’ signal.

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