

Simulation of third-order dispersion in single optical channel

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Abstract – This paper presents a simulation of the effects displayed in a single mode optical fiber. The model was created by means of the MatlabTM environment. The model is based on sequential Fourier and inverse Fourier transformations, as well as on other processing with the aim of a numerical solution of the nonlinear Schrodinger equation with acceptable accuracy.

Keywords – Single Mode, Fiber, Model, Fourier Transformation

I. INTRODUCTION

The existence of contemporary civilization and its image is defined to a great extent by the progress in the methods and means of transmitting great quantities of data at great distances. To make this progress possible, there is a need for constant theoretical and practical research. Some of the research and papers aim to improve the parameters of separate system elements, while others consider the system as a whole [1,2]. Examples of this are: application of pre-transmission dispersion management, amplification and in some circumstances, using different optical carrier sources, etc. [3,4]. Some of the papers aim at wavelength division multiplexing (WDM) where multiple wavelengths on a single SM fiber are typically transmitted. Using WDM optical transmission makes possible the good usage of an optic fiber capacity. In WDM there are different approaches in the analysis depending on the wavelength spacing, the power, the types of sources, etc. However, in general, the studying of more complex systems can be done gradually, which in this case means beginning with a single optical channel system followed by an analysis of the interactions between channels. In all possible cases of studying, it is necessary to have an in-depth knowledge of the functions of the separate parts: modulation, demodulation or generation, detection of the optical signals; impairments in either electronic or photonic domains, dynamics of optical fiber, noise sources by optical amplifiers and receiver electronics; effects of electrical and optical filters.

The consideration of all the parts of an optical system in one paper is not possible. It should also be mentioned that there is variety and alternatives in the separate blocks. For

example the transmission medium may consist of a variety of fiber types such as the standard SMF ITU-G.652 or the non-zero dispersion shifted fiber ITU-G.655. Another example is the receiving sub-system, which can be a single detector with direct detection or a balanced detector with the respective receiving structure.

To make it methodically possible to study and understand optical systems, it is necessary to set up models which describe the actual ones in the best way. The setting up of models also has advantages for the design of much bigger systems because it is a substitute for very expensive experiments. Another advantage is the possibility of an easier quick reconfiguration of the system, as well as for a gradual upgrade.

II. BASIC THEORY

In this paper we consider the modeling of one of the basic parts of the optical communication system, that is the optical fiber.

First, we use the wave equation for light propagation in optical fiber [1]

$$\nabla \times \nabla \times E = -\frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} - \mu_0 \frac{\partial^2 P}{\partial t^2} \quad (1)$$

To be able to use (1) it is necessary to define the relation between P and E . For the wavelength interval $0,5 \div 2 \mu m$, where the self resonance of the medium does not go, it is possible to use the following relation

$$P(r, t) = P_L(r, t) + P_{NL}(r, t) \quad (2)$$

where

$$P_L(r, t) = \epsilon_0 \int_{-\infty}^{\infty} \chi^{(1)}(t-t') \cdot E(r, t') \cdot dt' \quad (3)$$

and

$$P_{NL}(r, t) = \epsilon_0 \iiint \chi^{(3)}(t-t_1, t-t_2, t-t_3) E(r, t_1) \times \\ \times E(r, t_2) E(r, t_3) \cdot dt_1 \cdot dt_2 \cdot dt_3 \quad (4)$$

We will consider pulses with a duration range from 10ns to 10fs. With such impulses there are also dispersion and non-linear effects which affect their form and spectrum. Using (1), (2) and

$$\nabla \times \nabla \times E \equiv -\nabla^2 E \quad (5)$$

we can derive

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$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_L}{\partial t^2} + \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} . \quad (6)$$

Normally it is considered [1], that P_{NL} changes much less compared to P_L . It is also considered that the optical field retains its polarization along the fibers and the pulse envelope (we will mark the pulse envelope by A) changes slowly compared to the optical carrier

$$\begin{aligned} \frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - \frac{i\beta_3}{6} \frac{\partial^3 A}{\partial t^3} = \\ = i\gamma \left(1 + \frac{1}{\omega_0} \frac{\partial}{\partial t} \right) \left(A(z, t) \int_{-\infty}^{\infty} R(t') |A(z, t - t')|^2 dt' \right) \end{aligned} \quad (7)$$

where $\beta(\omega)$ is the wave number (typically it is unknown and the Taylor series approximation is applied)

$$\begin{aligned} \beta(\omega) = \\ = \beta_0 + (\omega - \omega_0) \beta_1 + \frac{1}{2} (\omega - \omega_0)^2 \beta_2 + \frac{1}{6} (\omega - \omega_0)^3 \beta_3 + \dots \end{aligned} \quad (8)$$

In (8) ω_0 is a carrier frequency where $\Delta\omega \ll \omega_0$ is usually fulfilled, γ is a non-linear parameter and $R(t)$ is Response function, which also depicts the Raman and electronic influences [1,5].

For pulses with duration below 5ps (7) can be simplified to

$$\begin{aligned} \frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} = \\ = i\gamma \left(|A|^2 A + \frac{i}{\omega_0} \frac{\partial}{\partial T} (|A|^2 A) - T_R A \frac{\partial |A|^2}{\partial T} \right) \end{aligned} \quad (9)$$

where $T = t - \beta_1 z$.

We can make the following associations in (9): β_3 – third order dispersion, which appears in ultra-short pulses, the addend proportional to $1/\omega_0$ is related to the effects of self steering and shock formation; the addend proportional to T_R depicts the effects delayed Raman response and self frequency shift triggered by the intrapulse Raman scattering. Usually T_R is found experimentally. With pulses with duration over 5ps the addends proportional to T_R and $1/\omega_0$ become too small and we ignore them.

III. NUMERICAL METHODS FOR SIMULATION

The equation (9) is non-linear, partial differential and it has no common analytical solution. Because of this, we use many different numerical methods. A short summary depicting the path of the solution chosen for application is shown in (fig.1).

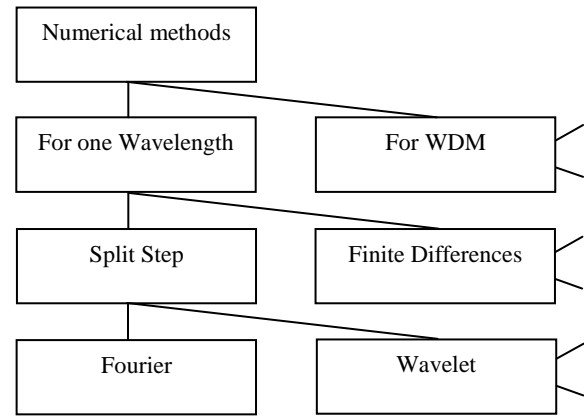


Fig. 1. Classification of the numerical methods used in the solution of equation (10)

We will consider the simulation of a single channel as basic for further simulations of WDM systems. Naturally, with WDM there are further complications resulting from effects such as FWM, XPM etc.

With one single channel the general classification is based on methods with finite differences (perturbation methods) and methods with splitting into parts. The methods with splitting into parts are used because of the quickness and the acceptable accuracy. In particular, for a non-linear dispersive medium is used the Split Step Fourier Method. One of the essential advantages of this method is that FFT is typically used in the calculations, which has been developed and introduced successfully in many systems (for example Matlab™) [5,6,7]. The main idea of this method is that (9) is represented as

$$\frac{\partial A}{\partial z} = (\hat{D} + \hat{N}) A , \quad (10)$$

where

\hat{D} and \hat{N} are operators respectively:

$$\hat{D} = -\frac{i\beta_2}{2} \frac{\partial}{\partial T^2} + \frac{\beta_3}{6} \frac{\partial^3}{\partial T^3} - \frac{\alpha}{2} , \quad (11)$$

$$\hat{N} = i\gamma \left(|A|^2 + \frac{i}{\omega_0} \frac{1}{A} \frac{\partial}{\partial T} (|A|^2 A) - T_R \frac{\partial |A|^2}{\partial T} \right) . \quad (12)$$

The optical fiber is split into small lengths h . For its part, h is split into two parts: in the first part functions \hat{N} , and $\hat{D} = 0$, while in the second part functions \hat{D} , and $\hat{N} = 0$.

We can derive that

$$A(z+h, T) \approx \exp\left(h\hat{D}\right) \exp\left(h\hat{N}\right) A(z, T) \quad (13)$$

$\exp\left(h\hat{D}\right)$ is calculated in the frequency domain

$$\exp\left(h\hat{D}\right) B(z, T) = F_T^{-1} \exp\left[h\hat{D}(i\omega)\right] F_T B(z, T) \quad .$$

The using of FFT has the advantage that the differential operators in (11) are substituted by powers of $i\omega$.

The accuracy of the calculations improves if we use a modified version of (13)

$$A(z+h, T) \approx \exp\left(\frac{h}{2}\hat{D}\right) \exp\left(\int_z^{z+h} \hat{N}(z') dz'\right) \exp\left(\frac{h}{2}\hat{D}\right) A(z, T) \quad (14)$$

(14) is also known as a symmetrical method with splitting (h is split into three parts). The integral from (14) can be calculated using the method of the trapeziums

$$\int_z^{z+h} \hat{N}(z') dz' \approx \frac{h}{2} \left[\hat{N}(z) + \hat{N}(z+h) \right] \quad (15)$$

In (15) the problem is that, $\hat{N}(z+h)$ is not known yet, when we have reached $h/2$. This is solved by additional iterations or additional simplification to $\hat{N}(z)$.

IV. RESULTS

For the possibility of the future research, on the basis of the expressions (11)-(15) and what is published in [6] we have developed a simulation in the programming environment of MatlabTM. We have created suitable vectors for time and angular frequency. We have considered the case for transmitting of rectangular pulses with different relative durations and the presence of third order dispersion. The choice of the rectangular pulse is not accidental. It is made to show what is the fiber effect using OOK modulation.

From the obtained results it is clear that at a high relative pulse duration dispersion effects occur, but they can easily be avoided. When reducing the length in full agreement with our expectations it is evident that the impulse is significantly influenced both in the time and the frequency domains.

When testing the model with Gaussian pulse are derived results very close to those shown in fig.4.14 in [1].

$u_0(nt/2:nt/2+50)=1;$

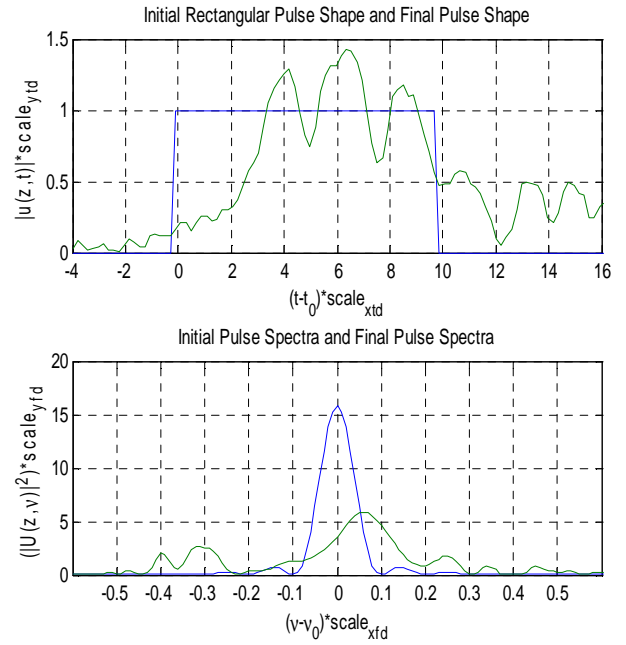


Fig. 2. Testing model with "long" pulse ($u_0=\text{zeros}(nt,1);$ $u_0(nt/2:nt/2+50)=1;$);

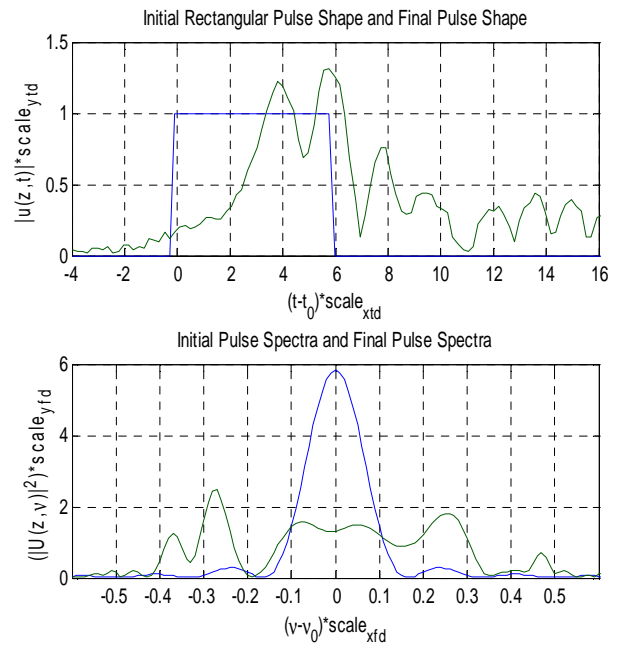


Fig. 3. Testing model with "medium" pulse ($u_0=\text{zeros}(nt,1);$ $u_0(nt/2:nt/2+30)=1;$);

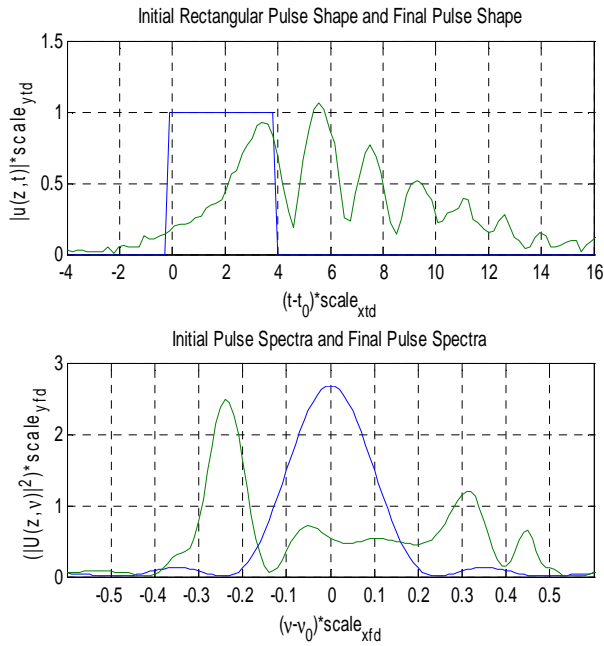


Fig. 4. Testing model with "short" pulse ($u_0=\text{zeros}(nt,1)$; $u_0(nt/2:nt/2+20)=1$);

V. CONCLUSION

The developed model is the first part of a larger system of models. Models of source, receiver, amplifier, etc. are yet to be developed. In the future we also plan to adapt them from the Matlab environment to the environment of Matlab/Simulink.

Except for the study of distortions in the optical channel of

cable communications systems, this model is very suitable for use in training students of a master's course. The model is also suitable for doctoral students.

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