Modelling of a Coaxially Loaded Probe-Coupled Cylindrical Cavity using the Cylindrical TLM Method

Tijana Dimitrijević, Jugoslav Joković, Bratislav Milovanović

Abstract – **In this paper the accuracy and usefulness of the cylindrical TLM method enhanced with the compact wire model has been verified on an example of a coaxially loaded probe-coupled cavity. Advantageous of the presented method have been accentuated in regards to the corresponding rectangular model.**

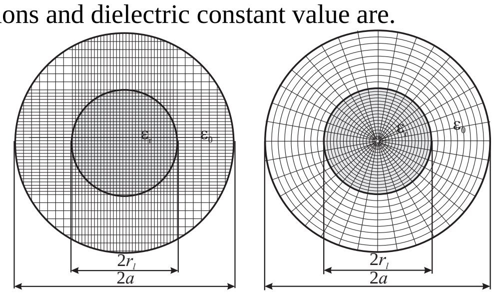
Keywords – **Cavity resonators, Electromagnetic analysis, Coaxial load, Probe antennas, Wire model, Cylindrical grid.**

I. Introduction

Analytical expressions exist describing the fields within simple geometries (i.e., in a waveguide or an unloaded cavity of regular geometric shape). However, as soon as complicated shapes or structures containing lossy loads are encountered, equations become difficult or impossible to solve analytically. Thus, computational electromagnetic techniques emerge as an invaluable tool in the cavity design field. Numerical modelling provides valuable information on such parameters as the electric and magnetic fields, and the power absorbed by the load. Among them, one of the most used in the field is *Transmission-Line Matrix* (TLM) time-domain method [1]. The applicability of this so-called *full-wave* method in the area of modelling of resonant loaded cavities provides better understanding of the mode tuning behavior in a cavity under loading condition, allowing users to make necessary changes in order to optimize the cavity design.

Constant improvements of the TLM method have led to the development of several commercial packages. These packages are convenient as they avoid the tedious task of writing the code. The disadvantage, however, is that they cannot be modified, as would be the case with self-written codes. Furthermore, they are generally based on the rectangular coordinate system and applicable on both rectangular and cylindrical cavity. Opposed to rectangular structures, which can be simply modelled in a rectangular TLM grid, there are problems, such as those with cylindrical or spherical symmetry, where a curved boundary would have to be described approximately (in a step-wise fashion) that, depending on the mesh resolution, might result in a deviation of resonant frequency values as well as in excitation of unwanted modes. This numerical error could be reduced by applying the rectangular TLM mesh of higher resolution around cavity walls. As a result, duration of a simulation is increased, requiring more computer resources. Thus, it is more convenient to use TLM method developed in a cylindrical grid, since it enables precise modeling of boundaries independently of the mesh resolution. Also, due to problem symmetry it is possible to save computer resources and speed up a simulation process.

A problem of choosing an adequate rectangular network becomes more pronounced when a coaxially loaded cylindrical cavity is the subject of modelling. Namely, due to the demand of achieving a time synchronization in a scattering process, the TLM nodes used for describing the load have to be $\sqrt{\varepsilon_r}$ (ε_r is a relative dielectric constant of a load) times less compared to nodes dimension in the rest of the cavity filled with the air. Also, when the rectangular TLM method is applied to model a coaxial load the circular cross section of the load would have to be modelled approximately. This means that the coaxially loaded cavity has to be modelled by non-uniform mesh, including the homogeneous area filled with air, which does not correspond to the real structure (Fig. 1a). As consequence, numerical results might not converge, especially in cases of dielectrics of large dielectric constant as well as loads having large radius, where the rectangular TLM model is not applicable. On the other hand, the cylindrical TLM method allows for changing the resolution only through radial component, providing uniformity of the mesh within the same medium (Fig. 1b). Thus, the cylindrical TLM grid enables correct modelling irrespective whatever load dimensions and dielectric constant value are.



a) b)

Fig. 1. a) Rectangular TLM network, b) Cylindrical TLM network

One of the aims that should be reached during designing process of these structures is to enable modelling conditions that are in large amount similar to the real experimental procedure. For instance, knowing in what way wire elements used for excitation and detection of resonant modes influences a model behavior is of great significance. To enable this, the compact wire model has been developed and implemented in the TLM method based on the rectangular grid [2]. When this method is used to model a cylindrical probe-coupled cavity, two opponent demands have to be fulfilled. One is regarding precise modelling of boundaries which might be achieved by

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mesh of higher resolution, and the other is related to the wire model implementation that assumes an optimal ratio between wire radius dimension and dimension of nodes through which wire propagates. As consequence, a maximum probe radius that could be modelled is limited.

This limitation of a rectangular TLM model for describing a probe-fed cylindrical cavity can be overcome by the compact wire model implemented in the cylindrical TLM model that enables precise curved boundaries modelling irrespective of the mesh resolution applied. Due to the cylindrical grid structure and empirical nature of the compact model, this implementation has to take into account a change of wire parameters with a variable cross-section of the TLM nodes through which a radially placed wire conductor passes. This has been solved by the development and implementation of an additional algorithm for connecting procedure for wire segments belonging to TLM nodes with different cross-sections.

Based on the compact TLM wire model embedded in the cylindrical model the noncommercial code *3DTLMcyl_cw* has been written. It was verified on an example of a cylindrical cavity containing wire elements for resonant modes excitation and detection [3]. Further, this model has been succesfully applied for resonant frequencies determination in case of a probe-coupled cylindrical cavity loaded with planparallel layers of dielectrics. [4].

The subject of this paper is to explore usefulness and efficiency of the cylindrical TLM model enhanced with the compact wire model for accurate modelling of a cylindrical cavity loaded with a concentric dielectric cylinder (Fig. 2). The transmission and reflection coefficient based on the coupling of two wire probes inserted into the cavity have been considered numerically and experimentally. Advantageous of the presented cylindrical method have been considered with regard to the corresponding rectangular method.

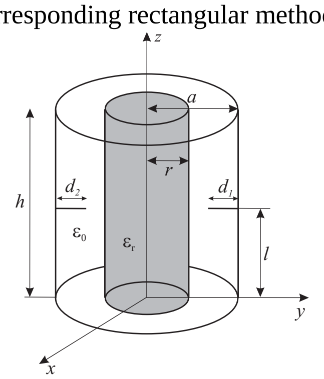

Fig. 2. A probe-coupled coaxially loaded cylindrical cavity

II. MODELLING PROCEDURE

The TLM method is a time-domain numerical method for solving field problems using circuit equivalents. The method is based on the equivalence between Maxwell's equations and the equations for voltages and current on a mesh of transmission lines, the latter representing the propagation space.

In the conventional TLM time-domain method, the EM field strength in three dimensions, for a specified mode of oscillation in a metallic cavity, is modelled by filling the field space with a network of link lines and exciting a particular field component through incident voltage pulses on appropriate lines. An efficient computational algorithm of scattering properties, based on enforcing continuity of the electric and magnetic fields and conservation of charge and magnetic flux, is implemented to speed up the simulation process. EM properties of different mediums in the cavity are modelled by using a network of interconnected nodes, a typical structure known as the symmetrical condensed node – SCN. To operate at a higher time-step, a hybrid symmetrical condensed node (HSCN) is used. Each node describes a portion of the medium shaped like a cubic (Cartesian rectangular mesh) or a slice (Non-Cartesian cylindrical mesh) depending on the coordinate system applied. Additional stubs may be incorporated into the TLM network to account for inhomogeneous materials and/or electric and magnetic losses [1].

When cylindrical structures are concerned, a non-Cartesian cylindrical mesh in the coordinate system (φ, r, z) can be used for the modelling purpose. The coordinate system used and the port designations are shown in Fig. 3. Simulation proceeds exactly as for a SCN with stubs in a Cartesian grid. The only modification involves the calculation of stub parameters where account must be taken of the details of the new geometry.

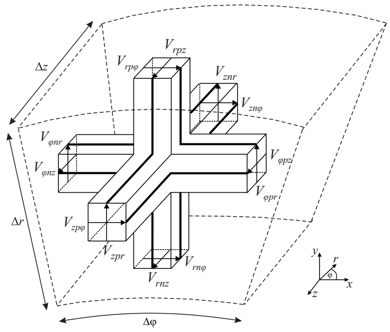

Fig. 3. A cylindrical SCN

Since the considered cylindrical model is based on the HSCN node type I, characteristic admittances of link lines are calculated as:

$$Y_{ij} = Y_{ji} = Y_0 \frac{2c\Delta t\Delta k}{\mu_{rk}\Delta i\Delta j}, \quad (1)$$

and characteristic admittances of open stubs are as follows:

$$Y_{ok} = Y_0\left[\frac{2\varepsilon_{rk}\Delta i\Delta j}{c\Delta t\Delta k} - \frac{4c\Delta t}{\Delta k}\left(\frac{\Delta i}{\mu_{ri}\Delta j} + \frac{\Delta j}{\mu_{rj}\Delta i}\right)\right], \quad (2)$$

where (Δi, Δj, Δk) = ($r\Delta\varphi$, Δr, Δz).

When modelling of cavities containing lossy loads is concerned, implementation of losses in the TLM model is carried out by introduction of stubs with losses in the nodes where scattering is going on. Stubs with losses may be considered as infinitely long transmission lines, or equivalently, as lines terminated with its characteristic impedance. They can be used to model either electric or magnetic losses. In case of the symmetrical condensed node, stubs with losses are directly implemented in the scattering procedure, including coupling with the corresponding EM field component.

After defining a loss tangent at the appropriate frequency, corresponding equations for reflected total voltages and currents in corresponding direction have to be modified in case of modelling of mediums with losses [1].

For the purpose of modelling of real microwave devices involving presence and coupling of wire elements, the compact wire TLM model has been developed. Signal propagation along the wire and interaction with EM field is simulated through the wire network formed of additional link and stub lines interposed over the existing network to account for increase of capacitance and inductance of the medium caused by wire presence. The single column of TLM nodes, through which a wire conductor passes, can be used to approximately form the fictitious cylinder which represents capacitance and inductance of a wire per unit length.

An equivalent radius of the fictive cylindre in a cylindrical grid for calculating the capacitance and inductance, r_{Cr} and r_{Lr}, respectively, for a wire segment running along radial direction are $r_{Cr} = k_{Cr}\Delta r_c$ and $r_{Lr} = k_{Lr}\Delta r_c$, where Δr_c represents a mean dimension of the node cross-section in r direction ($\Delta r_c = \left(\frac{(r_i + r_{i+1})}{2}\Delta\varphi + \Delta z\right)/2$, (where r_i and r_{i+1} are lower and upper limits of the TLM wire node in radial direction (Fig.3)), while r_{Cr} and r_{Lr} are factors empirically obtained by using known characteristics of the TLM network.

Distributed capacitance and inductance per unit length, needed for modelling of wire segments, may be expressed as:

$$C_{wr} = \frac{2\pi\varepsilon}{\ln(r_{Cr}/r_w)},\ L_{wr} = \frac{\mu}{2\pi}\ln(r_{Lr}/r_w) \tag{3}$$

where r_w is a real probe radius.

An equivalent radius of the fictitious cylinder can be easily kept constant along nodes column in a rectangular grid. However, for a radial wire conductor in a cylindrical grid, as it is shown in Fig. 3, mean cross-section dimensions of TLM nodes, through which a wire passes, are variable making difficult to preserve distributed capacitance and inductance of a wire per unit length. As a result, admittance of the wire network link line, interposed over the existing network to account for wire presence, varies from one TLM node to another (Fig. 4). Therefore, an additional connecting procedure for wire segments with different link-lines admitances has been implemented into the existing TLM-based software [3].

Reflected voltages on both directions of the interface between nodes with different cross-section, which at the same time represent incident voltages respect to the node center for the next time step, can be expressed as follows:

$$V_e^w = 2V_{i+1}^{w,inc}\frac{Y_{i+1}^w}{Y_i^w + Y_{i+1}^w} + 2V_i^{w,inc}\frac{Y_i^w}{Y_i^w + Y_{i+1}^w} \tag{4}$$

$$V_i^{w,ref} = \frac{Y_i^w - Y_{i+1}^w}{Y_i^w + Y_{i+1}^w}\left(V_i^{w,inc} - V_{i+1}^{w,inc}\right) + V_{i+1}^{w,inc} \tag{5}$$

$$V_{i+1}^{w,ref} = \frac{Y_i^w - Y_{i+1}^w}{Y_i^w + Y_{i+1}^w}\left(V_i^{w,inc} - V_{i+1}^{w,inc}\right) + V_i^{w,inc} \tag{6}$$

where V_e^w is an equivalent voltage at the interface, $V_i^{w,inc}$ and $V_{i+1}^{w,inc}$ are the incident voltages.

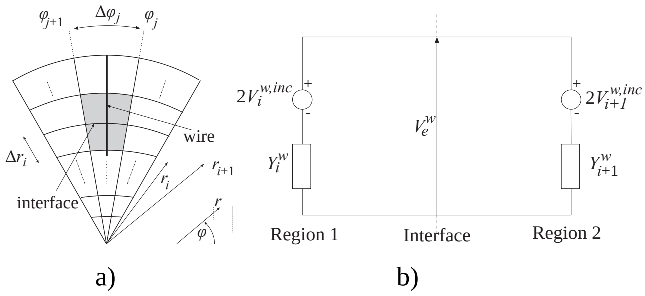

Fig. 4. a) TLM nodes in $r\varphi$ plane through which wire runs and b) an interface between two nodes

Such compact wire model allows for simple incorporation of voltage/current sources and lumped loads and takes into account the physical dimensions of wire probes, determined only by TLM mesh resolution.

III. Numerical Results

In order to investigate possibilities and effectiveness of the compact wire TLM model of a cylindrical cavity loaded with a dielectric cylinder, *3DTLMcyl_cw* has been used for modelling of a coaxially loaded probe-coupled cylindrical cavity with dimensions a = 7 cm and h = 14.24 cm, chosen to follow the experimental ones [3]. Wire probes, representing a feed and receiving probe, were placed at the height l = 7.4 cm from the bottom of the cavity, along the radial direction and opposite to each other (Fig. 2). In order to model real coaxial cable characteristics, the probes were connected, through the TLM wire port, to the real voltage source and resistances of 50Ω. A probe radius and length were chosen to be r = 0.5mm and d_1=d_2=d=2.5cm, respectively, whereas it was asumed that the cavity walls and wire probes were made out of the perfect conducted material. The water of relative dielectric constant $\varepsilon_r = 77 - j6$ has been used as a coaxial load. In order to provide convergence of the results in the frequency range of interest, whilst taking care of time simulation and computer resources, coaxialy loaded cavities for two different dielectrics have been modelled by properly chosen cylindrical network. A radius of the load was set to be r_{diel} = 3cm, whereas the load has been modelled with nodes which dimension was $\sqrt{\varepsilon_r}$ times less than nodes in the air-filled area.

Simulated reflection characteristic in the frequency range of interest is shown in Fig. 5, whereas numerical values of resonant frequencies are compared with measured ones in Table I. Fig. 6. shows both numerical and experimental results representing transmission characteristic in the frequency range of interest. As can be seen numerical results follow the experimental ones in a great extent.

However, when a rectangular TLM mesh would be applied to model the cilindrical cavity loaded with water it would not be possible to obtain reflection or transmission characteristic in the consedered frequency range. The reason for that can be found in the fact that maximum probe radius, which can be modelled, is limited due to extra fine nonunifrom mesh (Fig. 1a) used in the homogeneous air-filled area where wire probes are placed. This means that in case of loads having large permitivitty, such as the water, simulation based on the rectangular grid can be proceed only if probes of a very small radius are used, which does not correspond to the real case.

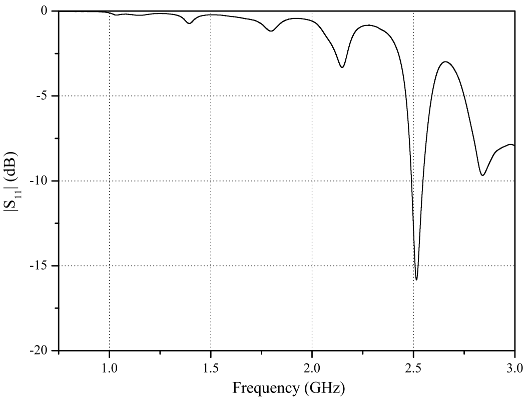

Fig. 5. Reflection characteristic of a probe-coupled cavity coaxially loaded with water

TABLE I
RESONANT FREQUENCY VALUES OF A COAXIALLY LOADED CAVITY ACCORDING TO THE REFLECTION COEFFICIENT

Rezonant frequency (GHz)	*TLMcyl_cw*	Measured values
water	1.394	1.391
	1.797	-
	2.148	2.122
	2.515	2.472
	2.840	2.816

On the other hand, when a cylindrical TLM mesh is applied, a mesh resolution used to model the coaxial load is chosen in order to achieve time synchronization and have no influence on the mesh uniformity within the air-filled area. This yields possibility of modelling of probes of greater radius.

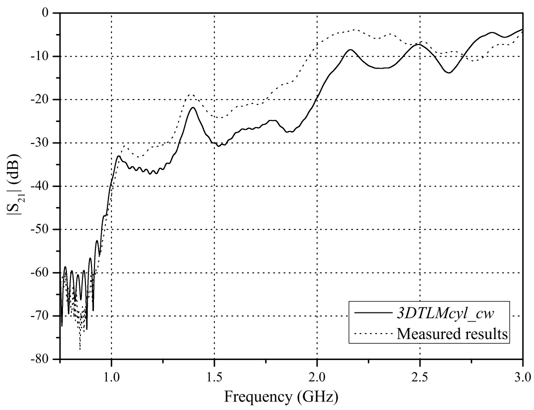

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Fig. 6. Transmission characteristic of a probe-coupled cavity coaxially loaded with water

IV. CONCLUSION

This paper presents an efficiency of the compact wire model implemented into the 3-D TLM cylindrical mesh for the purpose of the analysis of a probe-coupled coaxially loaded cylindrical microwave cavity. The model accuracy has been experimentally verified on an example of a probe-coupled cylindrical cavity containing water as a dielectric cylinder. Possibilities and usefulness of the presented model have been considered in regards to the rectangular grid based TLM method.

The main advantage of cylindrical grid based TLM method is precise modelling of cylindrical cavity external amd internal boundaries which allows for mesh resolution to be chosen only in regards to the wire probe radius. Another advantage is seen in case of a coaxially loaded cylindrical cavity, where it is possible to accuratelly model a nonhomogeneous medium independently of a load dimension and permitivitty. Moreover, an implementation of the compact wire model in the cylindrical TLM grid has enabled modelling of probes of greater radius than a rectangular grid allows for since there is no need for increasing the mesh resolution to provide correct modelling of boundaries. Finally, in a presence of a coaxial load of large permittivity it is possible to model probes of much greater radius compared to the rectangular TLM grid, since the mesh resolution used to model a load, which must be changed in order to achieve time synchronization in the scattering process, has no effect on the mesh resolution in the air-filled area, as it is the case in the rectangular TLM grid.

The considered configuration of a probe-coupled cavity loaded with coaxially placed dielectric is of a great importance in a realization of microwave resonant applicators, widely used for thermal processing of materials.

ACKNOWLEDGEMENT

This paper is funded by means of the Ministry of Education and Science of Republic of Serbia within the project numbered III 32052.

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