Second order statistics of MRC receiver over α-μ multipath fading channels

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Abstract –**Maximal ratio combining (MRC) receiver in the presence of α-μ is considered. Average level crossing rate of output signal of wireless communication system with MRC receiver operating over multipath fading channel is evaluated. The expression for level crossing rate can be used for calculation of average fade duration of wireless system. Dual and triple MRC receivers are analysed. Numerical results are presented graphically to show the influence of fading parameters on system performances.**

Keywords –**Level crossing rate, maximal ratio combining, α-μ fading**

I. INTRODUCTION

Multipath fading limits system performance and system capacity. Short term fading is result of multipath propagation due to refraction, reflection, diffraction and scattering of radio wave. There are several fading models which can be used for description of signal envelope variation. The most frequently used distribution Rayleigh, Nakagami-m, Weibull and α-μ. α-μ distribution can be used to describe signal envelope variation at output of wireless communication systems operating over multipath fading nonlinear and nonline-of-sight environments. The α-μ distribution has two parameters. Parameter α is related to nonlinearity of environment while the parameter μ is associated with the number of clusters of propagation waves. Some fading distribution for description signal envelope variation can be used assuming a homogeneous diffuse scattering field. In fading environments, the surfaces of the diffuse scattering field can be spatially correlated characterizing a nonlinear medium [1]. The α-μ distribution is proposed to explore the nonlinearity of the propagation channel [2]. This distribution is general distribution and Rayleigh, Nakagami-m, Weibull can be derived from this distribution. By setting α=2, Nakagami-m can be obtained from α-μ distribution. For α=1 α-μ distribution reduces to Weibull distribution. By setting μ=1 and α=2, Rayleigh distribution is derived and for μ=0.5 and α=2, α-μ approximates one sided Gaussian distribution. There are several diversity combining techniques which can be used to reduce fading effects on system performances [3]. Diversity reception using multiple antennas of the receiver is efficient technique to increase quality of service of wireless communication systems. The most popular combining techniques are maximum ratio combining (MRC), equal gain combining (EGC) and selection combining (SC). The MRC technique is optimal solution. The MRC technique requires channel state information available at the receiver. It is why, this combiner is the most complex for implementation. The outage probability, bit error probability, signal mean value, signal square value and channel capacity are the first order performances of the system. The level crossing rate and average fade duration are the second order performance of systems. In this paper the level crossing rate for dual and triple MRC receiver operating over α-μ are calculated [4]. Discussions about second order statistics in the presence of various diversity techniques can be easily found in literature [5-7]. The average fade duration is ratio of outage probability and average level crossing rate. The outage probability is the probability that output signal of MRC receiver falls below the outage threshold. Upgrade transmission reliability without increasing transmission power and bandwidth can be obtained by providing the receiver with multiple fade replica of the same information signal. The MRC output signal-to-noise power ratio is equal to the sum of the signal-to-noise power ratios of MRC receiver inputs. If the equal noise power at MRC inputs is assumed, the signal power of MRC output is equal the sum of signal power at MRC inputs. In this paper second order performances of dual MRC receiver operating over multipath α-μ fading channel are considered.[11-13].

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II. MRC WITH TWO INPUTS

In this paper maximal ratio combiner with two inputs is considered. Multipath α-μ fading at inputs is presented. The short term α-μ fading is independent and identical. In this paper, the average level crossing rate of output signal of MRC receiver in the presence of α-μ fading is calculated. The results, obtained in this paper, can be used for the case, when Rayleigh, Weibull and Nakagami-m are presented. For different parameters values α and μ, α-μ distribution reduces to Rayleigh, Weibull, Nakagami-m distribution.

The α-μ random variables x_1 and x_2 of inputs of MRC are [8]

$$x_1 = y_1^{\frac{2}{\alpha}} \tag{1}$$

$$x_2 = y_2^{\frac{2}{\alpha}} \tag{2}$$

where y_1 and y_2 are Nakagami-m random variables with probability density functions

$$P_{y_1}(y_1) = \left(\frac{m}{\Omega_1}\right)^m \frac{2}{\Gamma(m)} {y_1}^{2m-1} e^{-\frac{m}{\Omega_1}{y_1}^2} \qquad (3)$$

$$P_{y_2}(y_2) = \left(\frac{m}{\Omega_2}\right)^m \frac{2}{\Gamma(m)} {y_2}^{2m-1} e^{-\frac{m}{\Omega_2}{y_2}^2} \qquad (4)$$

The squared random variable z is equal to the sum of squared random variables x_1 and x_2

$$z^2 = {x_1}^2 + {x_2}^2 = ({y_1}^2)^{\frac{2}{\alpha}} + ({y_2}^2)^{\frac{2}{\alpha}}$$

$$= (y_{11}^2 + y_{12}^2 + \cdots + y_{12m}^2)^{\frac{2}{\alpha}} + (y_{21}^2 + y_{22}^2 + \cdots + y_{22m}^2)^{\frac{2}{\alpha}} \quad (5)$$

where are $y_{11}, y_{12}, \ldots y_{12m}, y_{21}, y_{22}, \ldots y_{22m}$ Gaussian random variables. The first derivative of previous expression is

$$\dot{z} = \frac{2}{\alpha z}(y_{11}^2 + y_{12}^2 + \cdots + y_{12m}^2)^{\frac{2}{\alpha}-1}$$

$$* (y_{11}\dot{y_{11}}, + y_{12}\dot{y_{12}} + \cdots + y_{12m}\dot{y_{12m}})$$

$$+ \frac{2}{\alpha z}(y_{21}^2 + y_{22}^2 + \cdots + y_{22m}^2)^{\frac{2}{\alpha}-1}$$

$$* (y_{21}\dot{y_{21}}, + y_{22}\dot{y_{22}} + \cdots + y_{22m}\dot{y_{22m}}) \qquad (6)$$

The first derivatives $\dot{y_{11}}, \dot{y_{12}}, \ldots \dot{y_{12m}}, \dot{y_{21}}, \dot{y_{22}}, \ldots \dot{y_{22m}}$ have Gaussian distributions. The linear transformationof Gaussian random variables is Gaussian variable. Therefore, random variable $\dot{z}$ has conditional Gaussian distribution. The means values of $\dot{y_{11}}, \dot{y_{12}}, \ldots \dot{y_{12m}}$ and $\dot{y_{21}}, \dot{y_{22}}, \ldots \dot{y_{22m}}$ are zero. Therefore the mean value of $\dot{z}$ is zero. The variance of $\dot{z}$ is

$${\delta_{\dot{z}}}^2 = \frac{4}{\alpha^2 z^2}(y_{11}^2 + y_{12}^2 + \cdots + y_{12m}^2)^{\frac{4}{\alpha}-2}$$

$$* \left({y_{11}}^2 {\delta_{\dot{y_{11}}}}^2 + {y_{12}}^2 {\delta_{\dot{y_{12}}}}^2 + \cdots + {y_{12m}}^2 {\delta_{\dot{y_{12m}}}}^2\right)$$

$$+ \frac{4}{\alpha^2 z^2}(y_{21}^2 + y_{22}^2 + \cdots + y_{22m}^2)^{\frac{4}{\alpha}-2}$$

$$* \left({y_{21}}^2 {\delta_{\dot{y_{21}}}}^2 + {y_{22}}^2 {\delta_{\dot{y_{22}}}}^2 + \cdots + {y_{22m}}^2 {\delta_{\dot{y_{22m}}}}^2\right) \qquad (7)$$

The variances, ${\delta_{\dot{y_{11}}}}^2, {\delta_{\dot{y_{12}}}}^2, \ldots {\delta_{\dot{y_{12m}}}}^2$ are equal

$${\delta_{\dot{y_{11}}}}^2 = {\delta_{\dot{y_{12}}}}^2 = \cdots = {\delta_{\dot{y_{12m}}}}^2 = \pi^2 \Omega_1 {f_m}^2 = f_1^2 \qquad (8)$$

where f_m is maximal Doppler frequency.

The variances, ${\delta_{\dot{y_{21}}}}^2, {\delta_{\dot{y_{22}}}}^2, \ldots {\delta_{\dot{y_{22m}}}}^2$ are equal

$${\delta_{\dot{y_{21}}}}^2 = {\delta_{\dot{y_{22}}}}^2 = \cdots = {\delta_{\dot{y_{22m}}}}^2 = \pi^2 \Omega_2 {f_m}^2 = f_2^2 \qquad (9)$$

After substituting (8) and (9) in (7), variance of $\dot{z}$ becomes

$${\delta_{\dot{z}}}^2 = \frac{4}{\alpha^2 z^2}\left(f_1^2 {y_1}^{\frac{8}{\alpha}-2} + f_2^2 {y_2}^{\frac{8}{\alpha}-2}\right) \qquad (10)$$

From (5)

$$y_1 = \left(z^2 - {y_2}^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}} \qquad (11)$$

The variance of $\dot{z}$ in terms of z and $y_2$is

$${\delta_{\dot{z}}}^2 = \frac{4}{\alpha^2 z^2}\left(f_1^2 \left(z^2 - {y_2}^{\frac{4}{\alpha}}\right)^{2-\frac{\alpha}{2}} + f_2^2 {y_2}^{\frac{8}{\alpha}-2}\right) \qquad (12)$$

The conditional probability density function of the first derivative of MRC output signal envelope is

$$p_{\dot{z}}(\dot{z}/zy_2) = \frac{\alpha z}{2\sqrt{2\pi\left(f_1^2\left(z^2 - {y_2}^{\frac{4}{\alpha}}\right)^{2-\frac{\alpha}{2}} + f_2^2 {y_2}^{\frac{8}{\alpha}-2}\right)}}$$

$$* e^{-\frac{\dot{z}^2 \alpha^2 z^2}{\left(f_1^2\left(z^2 - {y_2}^{\frac{4}{\alpha}}\right)^{2-\frac{\alpha}{2}} + f_2^2 {y_2}^{\frac{8}{\alpha}-2}\right)} y^2} \qquad (13)$$

The joint probability density function of $\dot{z}$, z and y_2 is equal to the product of the conditional probability density function of $\dot{z}$ and joint probability density function of z and y_2

$$p_{z\dot{z}y_2}(z\dot{z}y_2) = p_{\dot{z}}\left(\frac{\dot{z}}{zy_2}\right) p_{zy_2}(zy_2) \qquad (14)$$

The joint probability density function of z and y_2 is equal to the product of the conditional probability density function of z and probability density function of y_2

$$p_{zy_2}(zy_2) = p_z(z/y_2) p_{y_2}(y_2) \qquad (15)$$

The conditional probabilitydensityfunction of z can be obtained using transformation method

$$p_z(z/y_2) = \left|\frac{dy_1}{dz}\right| p_{y_1}\left(\left(z^2 - {y_2}^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}}\right) \qquad (16)$$

where

$$\left|\frac{dy_1}{dz}\right| = \frac{\alpha}{4}\left(z^2 - {y_2}^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}-1} 2z \qquad (17)$$

By substituting (15), (16) and (17) in (14), the joint PDF of ż, z and y_2 becomes

$$p_{zz\dot{y}_2}(zz\dot{y}_2) = p_{\dot{z}}\left(\frac{\dot{z}}{yz}\right)\frac{1}{y} p_x\left(\frac{z}{y}\right) p_y(y)$$

The joint probability density function of z and $\dot{z}$ is

$$p_{z\dot{z}}(z\dot{z}) = \int_0^{z^{\frac{\alpha}{2}}} p_{zz\dot{y}_2}(zz\dot{y}_2) dy_2$$

$$= \frac{2z\alpha}{4}\int_0^{z^{\frac{\alpha}{2}}} p_{\dot{z}}\left(\frac{\dot{z}}{zy_2}\right)\left(z^2 - {y_2}^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}-1} p_{y_1}\left(\left(z^2 - {y_2}^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}}\right) p_{y_2}(y_2)$$

The level crossing rate of MRC output signal envelope is

$$N_z = \int_0^\infty \dot{z} p_{z\dot{z}}(z\dot{z})\, d\dot{z} \tag{20}$$

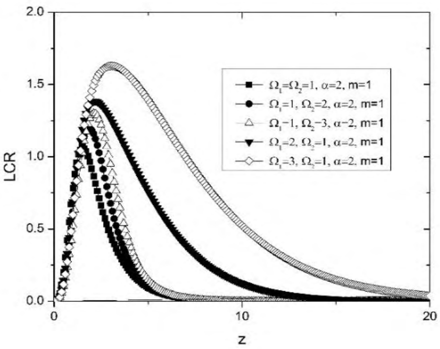

Fig. 1. LCR for dif. Ω_1, Ω_2

In Figure 1. is shown the average level crossing rate versus envelope of signal for different values of signal envelope input power. For lower values of signal envelope level crossing rate increases and for higher values of signal envelope level crossing rate decreases. In Figure 1. the influence of parameters Ω_1 and Ω_2 on average level crossing rate is also shown. As power of inputs signal envelope increases the level crossing rate increases.

$$N_z = \frac{2z\alpha}{4\sqrt{2\pi}} \int_0^{z^{\frac{\alpha}{2}}} dy_2 \sqrt{\left(f_1^2\left(z^2 - y_2^{\frac{4}{\alpha}}\right)^{2-\frac{\alpha}{2}} + f_2^2 y_2^{\frac{8}{\alpha}-2}\right)}$$

$$* \left(z^2 - y_2^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}-1}$$

$$* p_{y_1}\left(\left(z^2 - y_2^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}}\right) p_{y_2}(y_2) \tag{21}$$

The expression for level crossing rate can be used for calculation of the average fade duration of wireless communication system using dual MRC space diversity technique operating over α-μ multipath fading channel in nonlinear and non-line-of-sight environment.

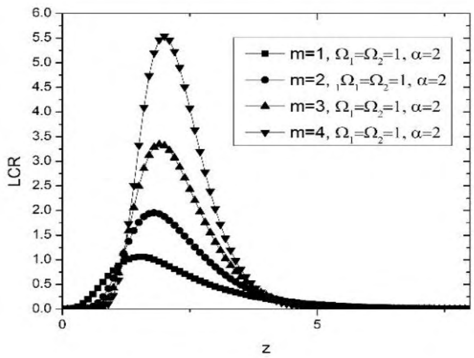

Fig. 2. LCR for dif. parameters *m*

In Figure 2. the influence of parameter m on average level crossing rate is shown. The parameter *m* is associated to the number of clusters of propagation environment. As parameter m increases then level crossing rate increases. For lower values of signal envelope, when parameter *m* has higher values, average level crossing rate decreases. These values of signal envelope are important in performance analysis of wireless communication systems.

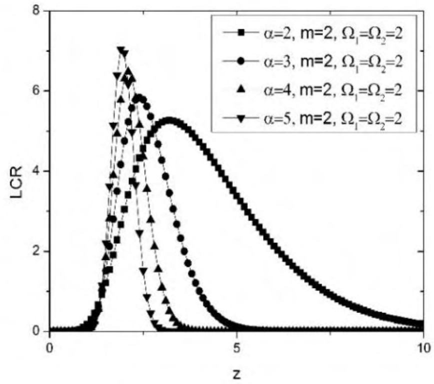

Fig. 3. LCR for dif. parameters α

In Figure 3. the influence of parameter α on the average level crossing rate is presented. The parameter α is related with nonlinear environment. As the parameter α increases, the level crossing rate, also, increases.

III. MRC WITH THREE INPUTS

In this section, the average level crossing rate of output signal envelope of wireless communication system with space diversity MRC receiver with three inputs operating over α-μ multipath fading environment is determined. The inputs signal envelopes are x_1 , x_2 and x_3. Signal envelope at the output of MRC receiver is z. The random α-μ variables x_1 , x_2 and x_3 are

$$x_1 = y_1^{\frac{2}{\alpha}} \tag{22}$$

$$x_2 = y_2^{\frac{2}{\alpha}} \tag{23}$$

$$x_3 = y_3^{\frac{2}{\alpha}} \tag{24}$$

where y_1, y_2 and y_3 follows Nakagami-m distribution

The first derivative of *z* now is

$$\dot{z} = \frac{2}{\alpha z}(y_{11}^2 + y_{12}^2 + \cdots + y_{12m}^2)^{\frac{2}{\alpha}-1}$$

$$* (y_{11}\dot{y_{11}}, + y_{12}\dot{y_{12}} + \cdots + y_{12m}\dot{y_{12m}})$$

$$+ \frac{2}{\alpha z}(y_{21}^2 + y_{22}^2 + \cdots + y_{22m}^2)^{\frac{2}{\alpha}-1}$$

$$* (y_{21}\dot{y_{21}}, + y_{22}\dot{y_{22}} + \cdots + y_{22m}\dot{y_{22m}})$$

$$+\frac{2}{\alpha z}(y_{31}^2+y_{32}^2+\cdots+y_{32m}^2)^{\frac{2}{\alpha}-1}$$

$$*(y_{31}\dot{y_{31}},+y_{32}\dot{y_{32}}+\cdots+y_{32m}\dot{y_{32m}}) \quad (25)$$

The variance of $\dot{z}$ is now

$$\delta_{\dot{z}}{}^2=\frac{4}{\alpha^2 z^2}\left(f_1^2 y_1{}^{\frac{8}{\alpha}-2}+f_2^2 y_2{}^{\frac{8}{\alpha}-2}+f_3^2 y_3{}^{\frac{8}{\alpha}-2}\right) \quad (26)$$

The variances, $\delta_{\dot{y_{31}}}{}^2, \delta_{\dot{y_{32}}}{}^2, \ldots\ \delta_{\dot{y_{32m}}}{}^2$ are equal

$$\delta_{\dot{y_{31}}}{}^2=\ \delta_{\dot{y_{32}}}{}^2=\cdots=\ \delta_{\dot{y_{32m}}}{}^2=\pi^2\Omega_3 f_m \quad (27)$$

From

$$z^2=(y_1{}^2)^{\frac{\alpha}{2}}+(y_2{}^2)^{\frac{\alpha}{2}}+(y_3{}^2)^{\frac{\alpha}{2}}$$

Nakagami-m random variable y_1 is

$$y_1=\left(z^2-y_2{}^{\frac{4}{\alpha}}-y_3{}^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}} \quad (28)$$

The variance of $\dot{z}$ in terms of z, y_2 and y_3 is now

$$\delta_{\dot{z}}{}^2=\frac{4}{\alpha^2 z^2}$$

$$*\left(f_1^2\left(z^2-y_2{}^{\frac{4}{\alpha}}-y_3{}^{\frac{4}{\alpha}}\right)^{2-\frac{\alpha}{2}}+f_2^2 y_2{}^{\frac{8}{\alpha}-2}+f_3^2 y_3{}^{\frac{8}{\alpha}-2}\right)$$

Similar mathematical apparatus is used like in (12)-(18) considering three inputs of MRC.

The level crossing rate of MRC output signal envelope in this case is [10]

$$N_z=\frac{2z\alpha}{4\sqrt{2\pi}}\int_0^{z^{\frac{\alpha}{2}}}dy_2\int_0^{\left(z^2-y_2{}^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}}}dy_3$$

$$*\left(z^2-y_2{}^{\frac{4}{\alpha}}-y_3{}^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}-1}$$

$$*\,p_{y_1}\left(\left(z^2-y_2{}^{\frac{4}{\alpha}}-y_3{}^{\frac{4}{\alpha}}\right)^{\frac{\alpha}{4}}\right)p_{y_2}(y_2)p_{y_3}(y_3)\,*$$

$$\sqrt{\left(f_1^2\left(z^2-y_2{}^{\frac{4}{\alpha}}-y_3{}^{\frac{4}{\alpha}}\right)^{2-\frac{\alpha}{2}}+f_2^2 y_2{}^{\frac{8}{\alpha}-2}+f_3^2 y_3{}^{\frac{8}{\alpha}-2}\right)} \quad (29)$$

The expression for level crossing rate can be used for calculation of the average fade duration of wireless communication system using triple MRC space diversity technique operating over α-μ multipath fading channel in nonlinear and non-line-of-sight environment.

IV. Conclusion

In this paper maximal ratio combining technique is considered. Analyzed MRC has two and three inputs.Statistics of second order of MRC output signal envelope such as level crossing rate of wireless communication systems is evaluated. The considered wireless communication system operates over α-μ multipath fading channel. α-μ distribution describes small scale signal envelope variation in nonlinear and non-line-of-sight environment. Results are shown graphically for different parameters.

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