BER Performance of IM/DD FSO System with PIN Photodiode Receiver over Gamma-Gamma Atmospheric Turbulence Channel

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Abstract – **In this paper, we have analyzed the bit error rate (BER) performance of free space optical (FSO) system employing intensity modulation/direct detection (IM/DD) with the on-off keying (OOK). Signal is transmitted over FSO channel under the influence of atmospheric turbulence which is modeled by gamma-gamma distribution since it is convenient fading model in wide range of the turbulence conditions. The new closed-form BER expressions are derived and numerical results are presented. The numerical results are confirmed by Monte Carlo simulation.**

Keywords – **Free space optical (FSO) communications, On-off keying (OOK), Atmospheric turbulence, Gamma-gamma distribution, Bit error rate (BER).**

I. INTRODUCTION

The limitations of radio frequency networks due to spectrum congestion, licensing issues and interference from unlicensed bands are the reason for the increasing interest in free space optical (FSO) communication. Due to its high data rate capacity and wide bandwidth on unregulated spectrum, FSO has been proved to be a good solution to solve the “last mile” problem. Despite a lot of advantages, the FSO system performance is disturbed by the existence of atmospheric turbulence which results in the rapid intensity fluctuations at the received signal, also known as fading or scintillation.

The impact of the atmospheric turbulence has been described with many statistical models. The log-normal distribution has been proved as good solution in weak turbulence conditions. On the other hand, the K distribution has been accepted as an appropriate model for the strong conditions. Because of its excellent agreement between theoretical and experimental data, the gamma-gamma distribution has been received as convenient fading model for a wide range of the turbulence conditions [1]-[4].

The intensity-modulated FSO system with direct detection (IM/DD) with the on-off keying (OOK) with PIN photodiode receiver is practical scheme that has been deployed in commercial FSO systems [5]. In [6] the bit error rate (BER) expression for FSO system with IM/DD using OOK over K-distributed atmospheric turbulence channel has been derived. The BER performance of FSO links over the log-normal atmospheric turbulence channels with spatial diversity has been studied in [7]. In this paper, we observed FSO with IM/DD using OOK signal transmission over channel under the influence of atmospheric turbulence modeled by gamma-gamma distribution. The PIN photodiode is used at the receiver. The new expressions for BER are derived and numerical results are presented.

The rest of the paper is organized as follows. Section II describes the system and channel model. In Section III, new closed-form BER expressions are derived. Section IV shows numerical results and discussion.

II. SYSTEM AND CHANNEL MODEL

The FSO communication system using IM/DD with OOK is presented in Fig. 1. At the transmitter, information bits are modulated by electro-optical modulator IM/OOK. The output of modulator represents the intensity of the laser source which is determined by the transmitting telescope. This telescope forwards the optical signal to the receiver over the turbulence induced channel. At the receiver, the optical signal is converted to an electrical using direct detection and PIN photodetector. The received electrical signal is given by

$$y = x\eta I + n\,, \tag{1}$$

where $x \in \{0,1\}$ represents the information bit, η is the optical-to-electrical conversion coefficient, n is the additive white Gaussian noise with the zero-mean and variance $\sigma_n^2 = N_0/2$ and I is the normalized irradiance which is accounted for the intensity fluctuations due to the atmospheric turbulence.

The channel is under the influence of the atmospheric turbulence which is modeled by the gamma-gamma distribution convenient in wide range of turbulence conditions. The received irradiance I can be considered as a product of two random processes, i.e. $I=I_x I_y$, where I_x and I_y arise from large-scale and small-scale turbulent eddies, respectively, and both of them follow gamma distribution [3]. The probability density function (PDF) of I is defined as

$$p_I(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta)/2-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right), \tag{2}$$

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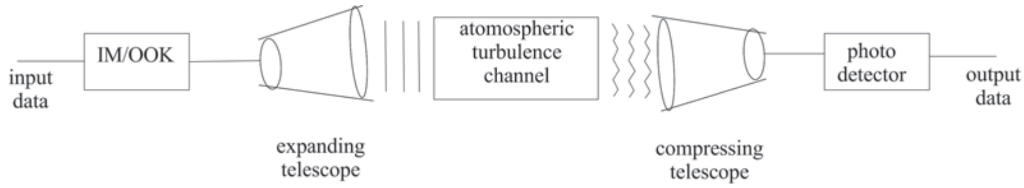

Fig. 1. Block diagram of the FSO system using IM/OOK

where $\Gamma(.)$ is the gamma function [8, Eq. (8.310.1)] and $K_\nu(.)$ is the νth-order modified Bessel function of the second kind [8, Eq. (8.432)]. The parameters α and β represent the effective of small-scale and large scale cells and can be related to the atmospheric conditions. If the plane wave propagation and zero inner scale are assumed, the parameters α and β can be expressed as [1], [4]

$$\alpha=\left(\exp\left[\frac{0.49\sigma_R^2}{\left(1+1.11\sigma_R^{12/5}\right)^{7/6}}\right]-1\right)^{-1},$$

$$\beta=\left(\exp\left[\frac{0.51\sigma_R^2}{\left(1+0.69\sigma_R^{12/5}\right)^{5/6}}\right]-1\right)^{-1}, \qquad (3)$$

where σ_R^2 is the Rytov variance here used as a metric of the strength of turbulence. It is given by [1], [4]

$$\sigma_R^2=1.23C_n^2k^{7/6}L^{11/6}, \qquad (4)$$

where $k=2\pi/\lambda$ is the wave-number, λ is the wavelength, L is the propagation distance, and C_n^2 denotes the index of refraction structure parameter which is assumed to be constant for horizontal paths. It should be noted that the Rytov variance is used only as a metric because it brings together all physical operating conditions. For simplification, turbulence strength is counted by σ_R.

Fig. 2 shows the PDF of irradiance I modeled by gamma-gamma distribution given by Eq. (2). The parameters α and β are calculated using Eq. (3). The result is obtained by analytical calculation and confirmed by simulation.

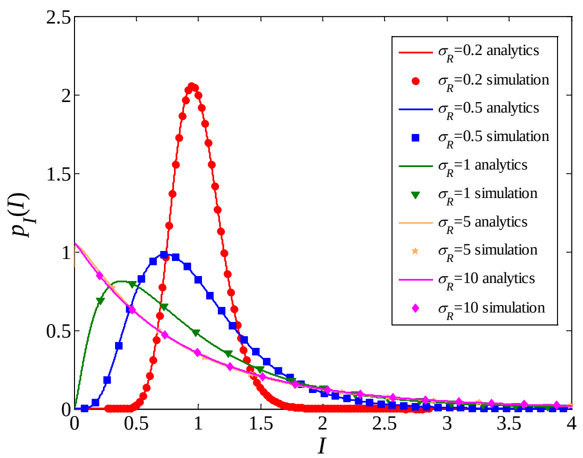

Fig. 2. Figure example Gamma-gamma distribution of irradiance I for some values of turbulence strength σ_R

The instantaneous electrical signal-to-noise ratio (SNR) is expressed as

$$\gamma=(\eta I)^2/N_0 \qquad (5)$$

and the average electrical SNR is

$$\mu=(\eta E[I])^2/N_0, \qquad (6)$$

where $E[.]$ denotes the statistical expectation. It should be noted that $E[I]=1$ since I is normalized. Also, μ is different from $\bar{\gamma}=E[\gamma]$ since the latter quantity is defined as $\bar{\gamma}=\eta^2E[I^2]/N_0$. The PDF of γ can be found using simple power transformation of the random variable I:

$$p_\gamma(\gamma)=\frac{(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)\mu^{(\alpha+\beta)/4}}\gamma^{(\alpha+\beta)/4-1}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\sqrt{\frac{\gamma}{\mu}}}\right). \qquad (7)$$

III. BER ANALYSES

If IM/DD with OOK is employed, BER expressions for FSO links can be calculated as [6], [7]

$$P_e=P(1)P(e\,|\,1)+P(0)P(e\,|\,0), \qquad (8)$$

where $P(1)$ and $P(0)$ represent the probabilities of transmitting „*on*“ and „*off*“ bits, respectively, and $P(e|1)$ and $P(e|0)$ are the conditional bit error probabilities when the transmitted bit is „*on*“ or „*off*“. It is considered that $P(1)=P(0)=0.5$ and $P(e|1)=P(e|0)$ which are conditioned on I. It is shown that [6], [7]

$$P(e\,|\,I)=P(e\,|\,1,I)=P(e\,|\,0,I)=Q\left(\frac{\eta I}{\sqrt{2N_0}}\right), \qquad (9)$$

where $Q(.)$ is the Gaussian Q-function defined as $Q(x)=\left(1/\sqrt{2\pi}\right)\int_0^\infty \exp\left(-t^2/2\right)dt$ and related to the complementary error function erfc (.) [8, Eq. (8.250.4)] by $\mathrm{erfc}(x)=2Q\left(\sqrt{2}x\right)$.

Applying Eq. (5) in Eq. (9), the conditional bit error probabilities are now conditioned over the instantaneous SNR:

$$P_e = \frac{(\alpha\beta)^{(\alpha+\beta)/2}\, 2^{\frac{\alpha+\beta}{4}-3}}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)\mu^{(\alpha+\beta)/4}} G^{4,2}_{2,5}\left(\frac{(\alpha\beta)^2}{4\mu}\,\middle|\, \begin{matrix} 1-(\alpha+\beta)/4 \quad 1/2-(\alpha+\beta)/4 \\ (\alpha-\beta)/4 \quad (\alpha-\beta)/4+1/2 \quad (\beta-\alpha)/4 \quad (\beta-\alpha)/4+1/2 \quad -(\alpha+\beta)/4 \end{matrix}\right) \tag{12}$$

$$P_e = \frac{2^{\alpha+\beta-3}}{\pi^{3/2}\Gamma(\alpha)\Gamma(\beta)} G^{4,2}_{2,5}\left(\frac{(\alpha\beta)^2}{4\mu}\,\middle|\, \begin{matrix} 1 \quad 1/2 \\ \alpha/2 \quad (\alpha+1)/2 \quad \beta/2 \quad (\beta+1)/2 \quad 0 \end{matrix}\right) \tag{13}$$

$$\begin{aligned} P_e = \frac{2^{\alpha+\beta-2}}{\pi\Gamma(\alpha)\Gamma(\beta)} \Bigg\{ & \frac{1}{\alpha}\Gamma\left(\frac{\beta-\alpha}{2}\right)\Gamma\left(\frac{\beta-\alpha+1}{2}\right)\Gamma\left(\frac{\alpha+1}{2}\right)\left(\frac{(\alpha\beta)^2}{4\mu}\right)^{\frac{\alpha}{2}} {}_2F_4\left[\frac{\alpha}{2},\frac{1+\alpha}{2};\frac{1}{2},\frac{2+\alpha-\beta}{2},\frac{1+\alpha-\beta}{2},\frac{2+\alpha}{2};\left(\frac{(\alpha\beta)^2}{4\mu}\right)\right] \\ & -\frac{2}{1+\alpha}\Gamma\left(\frac{\beta-\alpha}{2}\right)\Gamma\left(\frac{\beta-\alpha-1}{2}\right)\Gamma\left(\frac{\alpha+2}{2}\right)\left(\frac{(\alpha\beta)^2}{4\mu}\right)^{\frac{\alpha+1}{2}} {}_2F_4\left[\frac{1+\alpha}{2},\frac{2+\alpha}{2};\frac{3}{2},\frac{3+\alpha-\beta}{2},\frac{2+\alpha-\beta}{2},\frac{3+\alpha}{2};\left(\frac{(\alpha\beta)^2}{4\mu}\right)\right] \\ & +\frac{1}{\beta}\Gamma\left(\frac{\alpha-\beta}{2}\right)\Gamma\left(\frac{\alpha-\beta+1}{2}\right)\Gamma\left(\frac{\beta+1}{2}\right)\left(\frac{(\alpha\beta)^2}{4\mu}\right)^{\frac{\beta}{2}} {}_2F_4\left[\frac{\beta}{2},\frac{1+\beta}{2};\frac{2-\alpha+\beta}{2},\frac{1-\alpha+\beta}{2},\frac{1}{2},\frac{2+\beta}{2};\left(\frac{(\alpha\beta)^2}{4\mu}\right)\right] \\ & -\frac{2}{1+\beta}\Gamma\left(\frac{\alpha-\beta}{2}\right)\Gamma\left(\frac{\alpha-\beta-1}{2}\right)\Gamma\left(\frac{\beta+2}{2}\right)\left(\frac{(\alpha\beta)^2}{4\mu}\right)^{\frac{\beta}{2}} {}_2F_4\left[\frac{1+\beta}{2},\frac{2+\beta}{2};\frac{3-\alpha+\beta}{2},\frac{2-\alpha+\beta}{2},\frac{3}{2},\frac{3+\beta}{2};\left(\frac{(\alpha\beta)^2}{4\mu}\right)\right]\Bigg\} \end{aligned} \tag{14}$$

$$P(e\,|\,\gamma) = P(e\,|\,1,\gamma) = P(e\,|\,0,\gamma) =$$

$$= Q\left(\sqrt{\frac{\gamma}{2}}\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{\sqrt{\gamma}}{2}\right). \tag{10}$$

Finally, the average BER over gamma-gamma fading channel can be found by averaging (10) over γ:

$$P_e = \int_0^\infty p_\gamma(\gamma)\left(\frac{1}{2}\mathrm{erfc}\left(\frac{\sqrt{\gamma}}{2}\right)\right)d\gamma. \tag{11}$$

Integral in (11) can be solved by representing the complementary error function and modified Bessel function in terms of the Meijer's G functions [8, Eq. (9.301)] using [9, Eqs. (06.27.26.0006.01) and (03.04.26.0009.01)], and afterwards using [9, Eq. (07.34.21.0013.01)]. The expression for the average BER is given by Eq. (12). Using [9, Eq. (07.34.16.0001.01)], Eq. (12) can be formulated in convenient form given by Eq. (13). The Meijer G-function in Eq. (13) can be represented in terms of well-known hypergeometric function [8, Eq. (9.41.1)] using [9, Eqs. (07.34.26.0004.01) and (06.05.16.0002.01)], under the condition $|\alpha-\beta| \neq n$, (n is integer). So, Eq. (13) can be equivalent expressed as Eq. (14).

IV. Numerical results

Using previously derived expressions, the numerical results are presented and confirmed by simulation.

Fig. 3 shows the BER dependence on the average SNR for different values of turbulence strength σ_R. When the values of the turbulence strength is higher, the influence of atmospheric turbulence is more expressed. When the turbulence strength is low (σ_R<2), the system is under the influence of weak turbulence conditions. The curves are at the larger distance for two low values of σ_R than for two high values (compare σ_R=1 and 2 and σ_R=8 and 10). It means that increasing of turbulence strength has greater impact on BER in weak turbulence conditions.

Fig. 4 shows the BER dependence on turbulence strength for some values of average electrical SNR. If the certain BER is wanted, greater value of SNR is required with increasing of the parameter σ_R, (i.e. when the fading is more expressed).

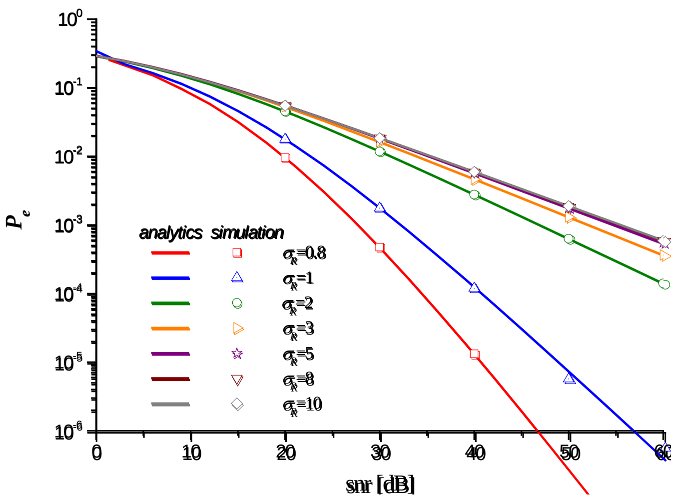

Fig. 3. BER dependence on average electrical SNR with different values of parameter σ_R

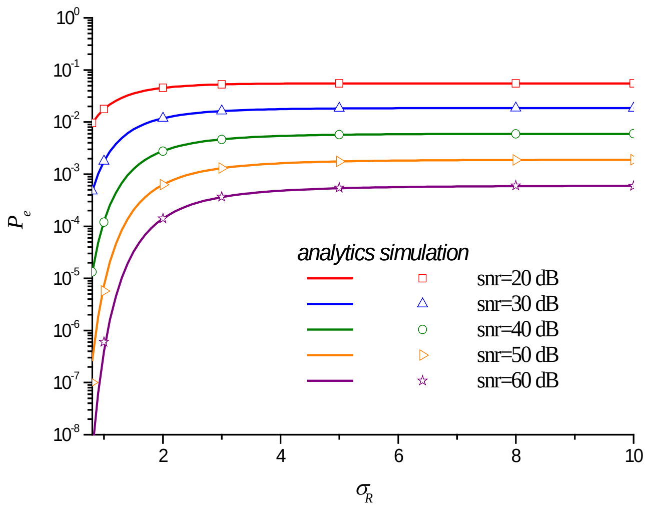

Fig.4. BER dependence on parameter σ_R with different values of average electrical SNR

For lower values of σ_R, we have rapid increase of BER. In strong turbulence conditions, BER is slightly increased. It means that BER is almost constant for different values of σ_R in the case of strong fading, especially at lower values of SNR.

V. Conclusion

In this paper the new closed-form BER expressions of FSO system employing IM/DD with OOK have been derived. System is under the influence of atmospheric turbulence modeled by gamma-gamma distribution. The effects of turbulence strength have been analyzed and numerical results have been presented. The obtained results have been confirmed by Monte Carlo simulation.

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