# Carrier Frequency Offset Problem Solving in the OFDM/MDPSK System

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*Abstract* – **The main disadvantage of the orthogonal frequency division multiplexing (OFDM) systems is their sensitivity to the carrier frequency offset. This paper presents comparative analysis of two OFDM receivers with good performance in the presence of the carrier frequency offset. The first receiver, denoted as RMSDD-OFDM, uses the MSDD (multiple-symbol differential detection) of MDPSK signal and obtains an improvement by adding ability to reconfigure itself. The second receiver with adaptive remodulation filter (ARF-OFDM), uses a new algorithm for the estimator weights adjustment, which is applied separately in each OFDM channel.**

**In the presence of carrier frequency offset and for the same complexity, ARF-OFDM receiver has better performance, for both modulation formats (BDPSK and 4DPSK). However, if the goal is to have a nearly constant performance in a wide frequency offset range and equal to the error probability for zero frequency offset, we have to use RMSDD-OFDM receiver. In this case, increasing of the system complexity is the main disadvantage.**

*Keywords* – **Detection, estimation, mobile satellite communications, orthogonal frequency division multiplex, synchronization.**

## I. Introduction

Modern telecommunication systems often use OFDM since it provides a broadband communication over fading channels. One of the drawbacks of the OFDM has proven to be its high sensitivity to the frequency offset between the local oscillators at the transmitter and the receiver [1], [2]. The problem with the frequency offset is that it creates the inter-carrier interference since the orthogonality of the OFDM subcarriers is ruined. Since inter-carrier interference may degrade the bit error rate performance severely, the inter-carrier interference suppression has received considerable attention. Depending on the characteristics of the transmitted signal (pilot-based or not), there are different approaches for solving this problem [3]-[8].

In goal to create an OFDM receiver with good performance in the presence of significant frequency offsets, so that it can be used for the $M$-ary differential phase shift keying (MDPSK) signal reception, two different OFDM receivers are proposed in this paper. The first receiver uses the MSDD detection of MDPSK signal and obtains an improvement by adding a ability to reconfigure itself. The second receiver with adaptive remodulation filter (ARF-OFDM), uses a new algorithm for the estimator weights adjustment, which is applied separately in each OFDM channel [9]. This paper presents comparative analyze of these two OFDM receivers and their advantages, disadvantages and complexity.

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## II. System Models

### A. Reconfigurable OFDM/ MSDD receiver

The block diagram of the OFDM/MDPSK signal receiver with reconfigurable MSDD structure (RMSDD-OFDM) is shown in Fig. 1, where $N_{VC}$ is the number of virtual channels. The received signal is down converted, low-pass filtered, and sampled with the period $T_f = T_{GI} + T_S + T_{CP}$, where $T_{GI}$ is the guard interval duration, $T_{CP}$ is the cyclic prefix duration, and $T_S$ is the symbol interval duration. S/P represents serial to parallel converter and it requires timing synchronization. After removing the cyclic prefix, a Fast Fourier transform (FFT) of length $N$ is performed. In this case we use OFDM demodulator with $N$ subcarriers and Fast Fourier transform.

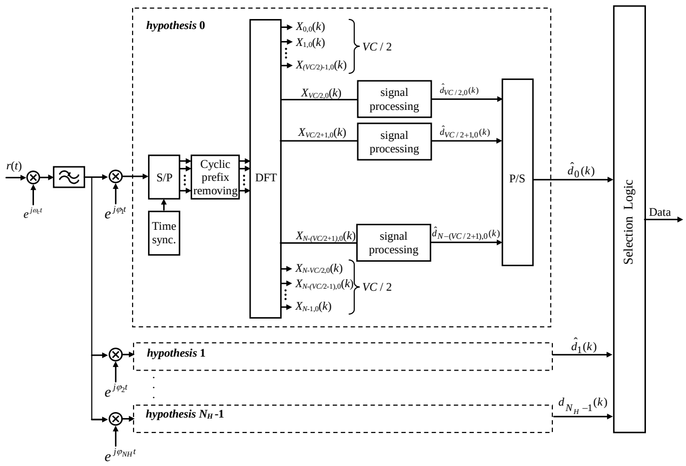


Fig. 1. RMSDD-OFDM receiver model with frequency hypothesis

If we assume that correct frame and timing synchronization is achieved, then the received sequence in $n_c$-th OFDM channel and $k$-th OFDM frame, after stripping the CP, can be expressed as:

$$X_{n_c}(k) = \sqrt{\frac{1}{N}} \sum_{i=kN}^{k(N+1)-1} r(i) e^{-j2\pi \frac{i n_c}{N}}, 0 \le n_c \le N-1 \qquad (1)$$

and $r(i) = s(i) + n(i)$, where $n(i)$ represents an Additive White Gaussian Noise (AWGN) term, with power spectrum density $N_0 / 2$.

Based on [10], there are $N_H$ blocks in Fig. 1 marked with dashed lines. These blocks represent different phase hypothesis. The input signal is multiplied by the signal with the corresponding frequency offset $\theta_{n_h}$

$$\theta_{n_h} = n_h \Delta\varphi - \frac{(N_H - 1)\Delta\varphi}{2},\ n_h = 0,...N_H - 1 \qquad (2)$$

where $n_h$ represents number of the hypotheses, $\Delta\varphi$ is the algorithm parameter that represents a phase step.

Thus, equation (1) for $n_h$-th hypothesis block, can be expressed as:

$$X_{n_c,n_h}(k) = \sqrt{\frac{1}{N}} \sum_{i=kN}^{k(N+1)-1} r(i) e^{-j2\pi \frac{i n_c}{N}} e^{ji\theta_{n_h}} \qquad (3)$$

The signal processing block choose the phase vector $\mathbf{\Phi}_{n_c,n_h} = \{\phi_{n_c,n_h}(k), \phi_{n_c,n_h}(k-1),...,\phi_{n_c,n_h}(k-N_B+1)\}$ which maximizes $S_{n_c,n_h}(k)$, for each OFDM channel and for each hypothesis, where:

$$S_{n_c,n_h}(k) = \left| \sum_{n_b=0}^{N_B-1} X_{n_c,n_h}(k-n_b) e^{-j\phi(k-n_b)} \right|^2 \qquad (4)$$

$X_{n_c,n_h}(k-n_b)$ is $n_b$-th transmitted MDPSK symbol in $n_c$-th OFDM channel, and for $n_h$-th hypothesis, and $n_b = 0,..., N_B - 1$, $n_c = 0,..., N - VC - 1$, $n_h = 0,...N_H - 1$, $k$ is the discrete time. $N_B$ represents number of symbol in multiple symbol differential detection used for detection, and $N_H$ represents the number of hypothesis.

One hypothesis algorithm is described as follows. To find $\mathbf{\Phi}_{n_c,n_h}$, first we need to remodulate the $X_{n_c,n_h}(k-n_b)$ in MDPSK sector $\left[0, \frac{2\pi}{M}\right)$, for each $n_b = 0,..., N_B - 1$. Let $\tilde{\mathbf{\Phi}}_{n_c,n_h}$ be the unique $\mathbf{\Phi}_{n_c,n_h}$ for which

$$\arg\left[X_{n_c,n_h}(k-n_b) e^{-j\tilde{\phi}(k-n_b)}\right] \in \left[0, \frac{2\pi}{M}\right) \qquad (5)$$

for $n_b = 0,..., N_B - 1$. Define $Z_{n_c}(k-n_b)$ as

$$Z_{n_c,n_h}(k-n_b) = X_{n_c,n_h}(k-n_b) e^{-j\tilde{\phi}(k-n_b)} e^{jn_b\theta_{n_h}} \qquad (6)$$

For each $n_b$, $n_b = 0,..., N_B - 1$, calculate $\arg\left[Z_{n_c,n_h}(k-n_b)\right]$, and list the values $\arg\left[Z_{n_c}(k-n_b)\right]$ in order, from largest to smallest. Define the function $l_{n_c,n_h}(i)$ as giving the values of $n_b$ of $Z_{n_c,n_h}(k-n_b)$ for the $i$-th list position, $i$ =0,…$N_B$-1.

$$\begin{aligned} 0 &\le \arg\left[Z_{n_c,n_h}(k - l_{n_c,n_h}(N_B - 1))\right] \le \\ &\le \arg\left[Z_{n_c,n_h}(k - l_{n_c,n_h}(N_B - 2))\right] \le \\ &\le \cdots \le \arg\left[Z_{n_c,n_h}(k - l_{n_c,n_h}(0))\right] < \frac{2\pi}{M} \end{aligned} \qquad (7)$$

Below, we make the following sums:

$$\left| \sum_{i=q}^{q+NB-1} Z_{n_c,n_h}(k - l_{n_c,n_h}(i)) \right|^2,\ q = 0,\dots N_B\text{-}1 \qquad (8)$$

and select the largest. The points $Z_{n_c,n_h}(k - l_{n_c,n_h}(i))$, $i$=0,…$N_B$ – 1, are the remodulations of $X_{n_c}(k-n_b)$, $n_b = 0,..., N_B$, in $\left[0, \frac{2\pi}{M}\right)$ ordered by the value of their angle.

Suppose the largest magnitude in (8), occurs for $q = q'$. We now find the phase vector $\hat{\mathbf{\Phi}}_{n_c,n_h}$ corresponding to $q = q'$. Based on previous consideration, with $i$ in the range $q' \le i \le q' + N_B - 1$, we have

$$\phi_{n_c,n_h}(k - l_{n_c,n_h}(i)) = \tilde{\phi}_{n_c,n_h}(k - l_{n_c,n_h}(i)), q' \le i \le N_B - 1 \quad (9)$$

$$\begin{aligned} &\phi_{n_c,n_h}(k - l_{n_c,n_h}(i) + 1) = \tilde{\phi}_{n_c,n_h}(k - l_{n_c,n_h}(i) + 1) + \frac{2\pi}{M}, \\ &N_B - 1 < i \le q' + N - 2 \end{aligned} \qquad (10)$$

The evaluation of (9) and (10) gives elements $\phi_{n_c,n_h}(k - l_{n_c,n_h}(l))$, $l$=0,…$N_B$ – 1. By arranging the elements $\phi_{n_c,n_h}(k - l_{n_c,n_h}(l))$, $l$=0,…$N_B$ – 1, in order of value $l_{n_c,n_h}(l)$, we form the sequence $\phi_{n_c,n_h}(k), \phi_{n_c,n_h}(k-1),...\phi_{n_c,n_h}(k-N_B+1)$, which is the vector $\mathbf{\Phi}_{n_c,n_h}$ for one hypothesis.

After calculating of vector $\mathbf{\Phi}_{n_c,n_h}$ for each hypothesis, we calculate the following

$$\hat{S}_{n_h}(k) = (1-A)\hat{S}_{n_h}(k-1) + A \cdot \sum_{n_c=0}^{N-VC+1} S_{n_c,n_h}(k) \qquad (11)$$

$$n_h \Leftarrow \max_{n_h} \hat{S}_{n_h}(k) \qquad (12)$$

Because the data is differentially encoded, to find $\Delta\hat{\Phi}$, we need to use the following transformation:

$$\begin{aligned} &\Delta\phi_{n_c}(k) = \phi_{n_c,n_h}(k) - \phi_{n_c,n_h}(k-1), \\ &k = 1,...N_B - 1, \quad n_c = \frac{VC}{2},..., N - \left(\frac{VC}{2} + 1\right) \end{aligned} \qquad (13)$$

From (13) we can determine the received symbol for MDPSK constellation, and based on (12) selection logic block switches to the corresponding hypothesis block.

*B. OFDM receiver with adaptive remodulation filter*

OFDM receiver with adaptive remodulation filter is shown in Fig. 2. Signal processing structure is shown in Fig. 3 and the receiver is denoted as ARF-OFDM. The proposed algorithm with DPSK input signal is equivalent to the LMS algorithm with the CW (Continuous Wave - a signal with constant amplitude and frequency, and a random phase) input signal in terms of adaptation rate and convergence, in case of correctly determined remodulation weights $R_{n,l}(k)$. The remodulation weights are sufficiently correctly determined in the range of error probabilities of practical importance. Transversal filter with remodulation lowers the noise level in the input signal with as little degradation of the useful signal as possible. Therefore, if we put the transversal filter in front of the detector, the detector will work with the estimated input signal which has a smaller noise variance, and it will make better decisions.

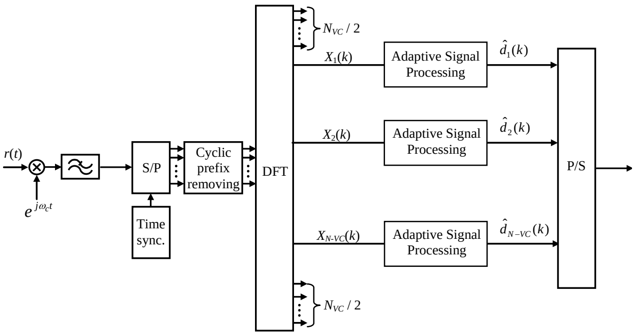


Fig. 2. ARF-OFDM receiver

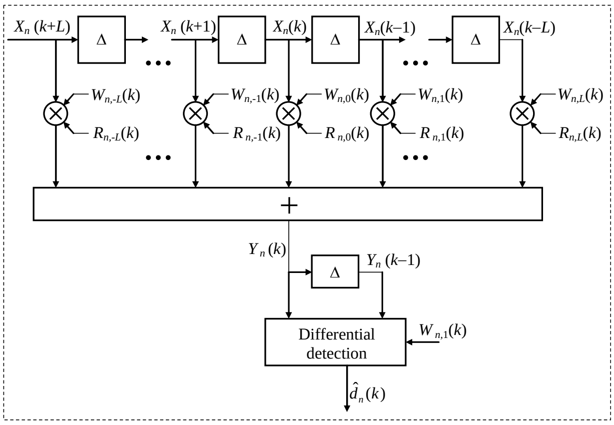


Fig. 3. Adaptive signal processing structure in n-th ARF-OFDM channel

The signal processing is described with the following equations:

$$Y_n(k) = \frac{1}{2L+1}\sum_{l=-L}^{L} R_{n,l}(k)\cdot X_n(k-l)\cdot W_{n,l}(k) \tag{14}$$

where $n = 1,2,...,(N - N_{VC})$ denotes the $n$-th OFDM channel, $2L$ is the length of the proposed structure, and $R_{n,l}(k)$ are remodulation weights. The remodulation weights are determined independently for each branch in order to avoid the error propagation. $R_{n,l}(k)$ are determined as:

$$\begin{gathered} R_{n,l}(k) = \arg\min_{w\in S}\left\{| X_n(k) - w\cdot X_n(k-l)\cdot W_{n,l}(k)|^2\right\} \\ R_{n,l}(k)\in S \end{gathered} \tag{15}$$

where $S = \left\{ e^{j\frac{\pi}{2}m}, m \in (0,1,2,3)\right\}$, $n = 1,2,...,(N - N_{VC})$,

$l = -L,...L,\quad l \neq 0$, and $R_{n,0}(k) = 1$.

Weights $R_{n,l}(k)$ are used for the modulation removal, and $W_{n,l}(k)$ are complex weights trying to compensate the phase rotation due to frequency offset. The initial value of the weights is equal to 1 (phase is equal to 0). The convergence and the cost function of the algorithm are discussed in more details in the next subsection.

ARF-OFDM receiver for DPSK modulation contains a new algorithm, corresponding to the nature of OFDM signals. The adjustment of the adaptive filter weights, $W_{n,l}(k)$, used in all OFDM channels, is performed by the following algorithm:

$$W_{n,l}(k+1) = W_{n,l}(k) + \frac{\mu}{2\cdot L}\sum_{n=1}^{N-N_{VC}} \frac{E_n(k)[X_n(k-l)R_{n,l}(k)]^*}{\overline{|X_n(k)|^2}}, \tag{16}$$

where $l = -L,...L,\quad l \neq 0$, $W_{n,0}(k) = 1$ and $E_n(k)$ is a partial (for each OFDM channel) LMS algorithm error signal, given by

$$E_n(k) = X_n(k) - Y_n(k) \tag{17}$$

$\mu$ is the adaptation factor, $(\cdot)^*$ represents complex conjugate, and $\overline{|X_n(k)|^2}$ is the average power of the input signal. So, weights $W_{n,l}(k)$ are being used in all OFDM channels, and are being adjusted in each channel, one after another.

In the case of continuous wave ($CW$) input signal and if the thermal noise is neglected, the filter weight $W_{n,1}(k)$ contains the estimated frequency offset $\Delta f$ as in [11]

$$W_{n,1}(k) \sim e^{j2\pi\Delta f T_f}. \tag{18}$$

Having the above in mind, we propose a correction for the $k$-th symbol detection using weight $W_{n,1}(k)$ and the differential detection of $k$-th symbol is performed in the following way:

$$\hat{d}_n(k) = \arg\min_m\left\{\left|Y_n(k) - Y_n(k-1)W_{n,1}(k)\exp\left(j\frac{m\pi}{2}\right)\right|^2\right\}, \tag{19}$$

$m = 0,1,2,3$

The proposed structure does not minimize the frequency offset, but has the ability to operate in a wide range of frequency offset. The algorithm has been applied due to its good overall properties, which include satisfactory speed, stability, and not so high complexity.

## III. NUMERICAL RESULTS

The performance of the proposed receiver is analyzed using Monte-Carlo simulation with one million simulation steps. The carrier frequency is 2.4 GHz, the sampling period before DFT block is $T_c$ = 100 ns. OFDM simulation parameters are $N$ = 64, number of virtual channel $N_{VC}$ = 8, $T_{CP}$ = $T_{GI}$ =8$T_c$, which does not limit the generality of the results. In most OFDM literature, the cyclic prefix occupies the same interval as the guard interval, but in our simulation the general format of the OFDM signal is considered, and $T_{CP}$ may differ from $T_{GI}$. From the above, frame time duration is $T_f = T_{GI} + T_S + T_{CP} = 80T_c = 8\mu s$, and OFDM date rate is 7 MSym/s.

Since OFDM schemes are primarily intended for the mobile and wireless systems where a fading channel is assumed, in the following figures the performance analysis is performed in Rician fading channel with Rician factor $K$ = 10 dB. Figs. 4, 5 show the symbol error rate versus the normalized frequency offset of the receiver if the signal processing is performed by: fast multi-symbol differential detection ($N_H$ = 1) and proposed reconfigurable MSDD algorithm (RMSDD-OFDM) and adaptive remodulation filter (ARF-OFDM).

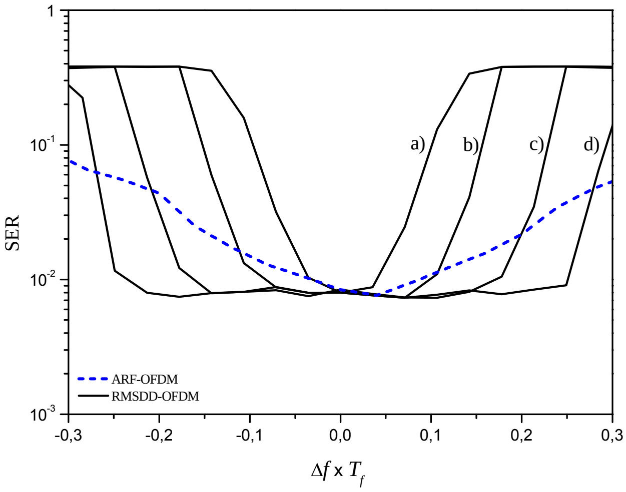


Fig. 4. Symbol error probability versus normalized frequency offset $N$ = 64, $N_{VC}$ = 8 and $T_{GI} = T_{CP} = 8T_c$, BDPSK modulation, $E_s / N_0$ = 7dB, $NB$ = 4, $\Delta\varphi / B_C$ = 3% with $N_H$ as a parameter
a) $N_H$ = 1, b) $N_H$ = 5, c) $N_H$ = 10, d) $N_H$ = 15

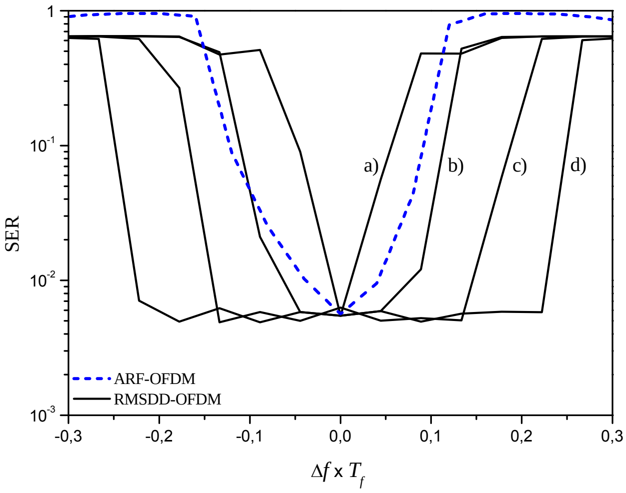


Fig. 5. Symbol error probability versus normalized frequency offset $N$=64, $N_{VC}$=8 and $T_{GI}$=$T_{CP}$=8$T_c$, 4DPSK modulation, $E_s / N_0$ = 12dB, $NB$ = 4, $\Delta\varphi / B_C$ = 3% with $N_H$ as a parameter
a) $N_H$ = 1, b) $N_H$ = 5, c) $N_H$ = 10, d) $N_H$ = 15

Figs. 4. and 5. represent systems performance with parameter which represents number of hypothesies $N_H$. The curves for RMSDD-OFDM receiver show that range where is satisfying transfer quality becomes wider with the increase of the number of hypotheses. This method increases the complexity of the system, and this is the main disadvantage.

Comparing the corresponding curves in Fig. 4 an improvement may be noticed in system performance in the presence of frequency offset using ARF-OFDM receiver. ARF-OFDM receiver works in a wider frequency offset range compared with DD-OFDM and FMSDD-OFDM systems. Therefore, the range where there is a satisfying transmission quality is significantly wider for ARF-OFDM receiver and we don`t increase the system complexity, as it is for RMSDD-OFDM.

## IV. Conclusion

In a goal to create an OFDM receiver with good performance in the presence of significant frequency offsets, so that it can be used for the *M*-ary differential phase shift keying (MDPSK) signal reception, two different OFDM receivers (ARF-OFDM and RMSDD-OFDM) are proposed in this paper. For the same complexity ($N_H$ = 1), in the presence of carrier frequency offset, ARF-OFDM receiver has the better performance, independent of the modulation. However, if we want a smaller receiver sensitivity at significant frequency offsets ($\Delta f \times T_f > 0,2$), RMSDD-OFDM receiver can be implemented with parameter $N_H$ = 15. Performance is nearly constant in a wide frequency offset range and equal to the error probability for zero frequency offset. The receiver has excellent performnce, but at the price of high complexity. This conclusion is valid for both modulation formats (BDPSK and 4DPSK).

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