New and advantageous approach for lossless compression of computer tomography image sequences

Peter Ivanov[1], Agata Manolova[2], Roumen Kountchev[3]

Abstract – **In this paper, we introduce a lossless compression method for CT image sequences which is efficient due to compression ability and complexity. It consists of a new decorrelation method called Hierarchical Adaptive Karhunen-Loeve Transform (HAKLT) which has a very good energy compaction and combined with standard AC and RLE coders, it has higher compression ratio than the standard JPEG. The aim is to obtain high decorrelation for each group of consecutive CT images. In result, the main part of the energy of all images in the sequence is concentrated in a relatively small number of eigen images. The computational complexity of the new algorithm is reduced in average 2 times, when compared to that of standard KLT.**

Keywords – **Decorrelation of CT image sequences, lossless compression, Hierarchical Adaptive KLT transform, JPEG**

I. INTRODUCTION

Medical images are widely used for diagnosis purposes and all types of surgery procedures. They include human body pictures and are being present in digital form. Imaging devices improve everyday and generate more data per patient. In the field of profiling patient's data, medical images need long-term storage [1]. Therefore, images need compression and compression ratio is important. However, in real time processes such as telemedicine and online diagnosis systems which need hardware implementation, simplicity of compression algorithm plays an important role. It can accelerate computation process. But for high value images such as medical images where loss of critical information is not acceptable, lossless compression is preferred.

Lossless algorithms are especially important for systems transmitting and archiving medical data, because lossy compression of medical images used for diagnostic purposes is, in many countries, forbidden by law. Furthermore, we have to use lossless image compression when we are unsure whether discarding information contained in the image is acceptable or not. Some systems such as medical CT scanner systems require rapid access to large sets of images that are further processed, analyzed, or just displayed. In such a system, the images or volume slices are stored in the memory since mass storage often turns out to be too slow. In this case a good lossless image compression algorithm could virtually increase the memory capacity allowing the processing of larger sets of data.

Lossless JPEG, JPEG-LS and lossless version of JPEG2000 are lossless methods introduced by JPEG committee and are widely used in the world [2]. We can add also the following methods for CT image compression: inter frame decorrelation based on hierarchical interpolation (HINT) [3], spatial active appearance model [4], and distributed representation of image sets based on Slepian-Wolf coding [5]. Lossless JPEG is about twenty years old standard and due to development and performance enhancement of digital medical imaging systems, no longer performs adequately enough for these kinds of systems.

One of the most efficient methods for decorrelation and compression of groups of images is based on the KLT, also known as transform of Hotelling, or Principal Component Analysis (PCA) [6]. For its implementation the pixels with the same spatial position in a group of N images compose the corresponding N-dimensional vector. The basic difficulty of the KLT implementation is related to the large size of the covariance matrix. For the calculation of its eigenvectors is necessary to calculate the roots of a polynomial of nth degree (characteristic equation) and to solve a linear system of N equations. For large values of N, the computational complexity of the algorithm for calculation of the transform matrix is significantly increased.

In this paper we propose a possible approache for reduction of the computational com-plexity of KLT for N-dimensional group of medical images based on the "Hierarchical Adaptive KLT" (HAKLT). Unlike the known hierarchical KLT (HKLT) presented in [7], this transform is not related to the image sub-blocks, but to the whole image from one group. For this, the HAKLT is implemented through dividing the image sequence into sub-groups of 3 images each, on which is applied Adaptive KLT (AKLT), of size 3×3. This transform is performed using equations, that are not based on iterative calculations, and as a result, they have lower computational complexity. To decorrelate the whole group of medical images is necessary to use AKLT of size 3×3, which to be applied in several consecutive stages (hierarchical levels), with rearranging of the obtained intermediate eigen images after each stage. In result we obtain a decorrelated group of 9 eigen medical images.

The paper is organized as follows. In Sections 2 and 3 we will introduce the principle for decorrelation of CT images group through HAKLT, the calculation of eigen images through AKLT with 3×3 matrix. Section 3 is devoted to the experimental results and comparison of the new method with JPEG using compression ratio and PSNR calculated on sequences of CT images. Finally, Section 4 gives our conclusion remarks.

[1]Peter Ivanov, [2]Agata Manolova, [3]Roumen Kountchev are with the Faculty of Telecommunications at Technical University of Sofia, 8 Kl. Ohridski Blvd, Sofia 1000, Bulgaria, E-mail: peter.n.ivanov@gmail.com, amanolova@tu-sofia.bg, rkountch@tu-sofia.bg.

II. Principle of CT Image Sequence Decorrelation Using Hierarchical Adaptive KLT

The sequence of CT images is divided into Groups of 9 images (GMI), for which we suppose that they are highly correlated. On the other hand, each GMI is further divided into 3 sub-groups. The algorithm for 2-levels HAKLT for one GMI is shown on Fig. 1. As it is easily seen there, on each sub-group of 3 images from the first hierarchical level of HAKLT we apply AKLT with matrix of size 3×3. In result are obtained 3 eigen images (colored in yellow, blue and green correspondingly). After that, the eigen images are rearranged so that the first sub-group of 3 eigen images comprises the first images from each group, the second group of 3 eigen images – the second images from each group, etc. For each GMI of 9 intermediate eigen images of the first hierarchical level we apply in similar way the next AKLT, with a 3×3 matrix, on each sub-group of 3 eigen values. In result are obtained 3 new eigen images (colored in yellow, blue, and green correspondingly) for the second hierarchical level. Then the eigen images are rearranged again so, that the first group of 3 eigen images contains the first images from each group before the rearrangement; the second group of 3 eigen images - the second image before the rearrangement, etc. At the end of the process we obtain a decorrelated sequence of eigen images. Through inverse HAKLT the original sequence could be restored.

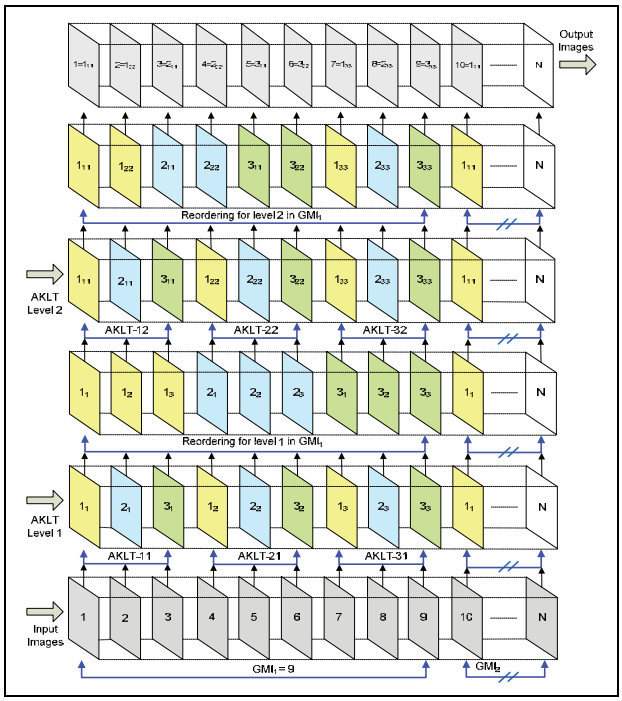


Fig. 1. Algorithm for 2-levels Hierarchical Adaptive KLT for Group of 9 Medical Images

III. Calculation of Eigen Images through AKLT with 3×3 Matrix

For the calculation of eigen images through AKLT with 3×3 matrix for GMI sub-group we use the approach for the representation of the 3D colour vector in the KLT space, given in [9]. From each sub-group of 3 medical images with S pixels each, shown on Fig. 2, are calculated the vectors $\vec{C}_s = [C_{1s}, C_{2s}, C_{3s}]^t$ with s = 1, 2, . . . , S (on the figure are shown the vectors for the first 4 pixels only).

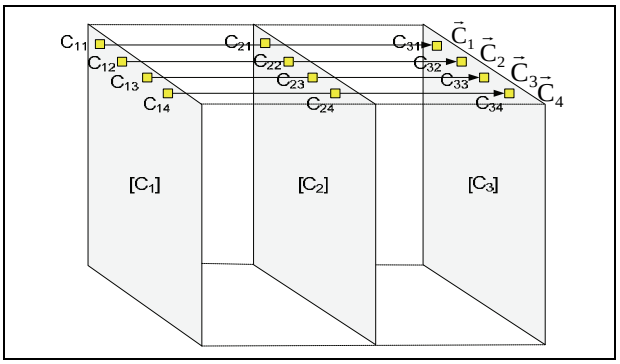


Fig. 2. Sub-group of 3 images from the GMI

Each vector is then transformed into corresponding vectors $\vec{L}_s = [L_{1s}, L_{2s}, L_{3s}]^t$ through APCA with the matrix [Φ] of size 3×3.

The covariance matrix $[K_C]$ of size 3×3 for vectors $\vec{C}_s$ is calculated:

$$[K_C] = \left[\frac{1}{S}\sum_{s=1}^{S}\vec{C}_s\vec{C}_s^t\right] - \vec{m}_c\vec{m}_c^t = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}, \quad (1)$$

where $\vec{m}_C = [\overline{C}_1, \overline{C}_2, \overline{C}_3]^t$ is the mean vector. Here, $\bar{x} = E(x_s) = \frac{1}{S}\sum_{s=1}^{S} x_s$; $E(.)$ – operator for calculation of the mean value of x_s.

The elements of the mean vector $\vec{m}_c$ and of the matrix $[K_C]$ are defined in accordance with the relations:

$$\overline{C}_1 = E(C_{1s}),\ \ \overline{C}_2 = E(C_{2s}),\ \ \overline{C}_3 = E(C_{3s}), \quad (2)$$

$$k_{11} = k_1 = E(C_{1s}^2) - (\overline{C}_1)^2,$$
$$k_{22} = k_2 = E(C_{2s}^2) - (\overline{C}_2)^2, \quad (3)$$
$$k_{33} = k_3 = E(C_{3s}^2) - (\overline{C}_3)^2,$$

$$k_{12} = k_{21} = k_4 = E(C_{1s}C_{2s}) - (\overline{C}_1)(\overline{C}_2),$$
$$k_{23} = k_{32} = k_6 = E(C_{2s}C_{3s}) - (\overline{C}_2)(\overline{C}_3), \quad (4)$$

$$k_{13} = k_{31} = k_5 = E(C_{1s}C_{3s}) - (\overline{C}_1)(\overline{C}_3). \quad (5)$$

The eigen values $\lambda_1, \lambda_2, \lambda_3$ of the matrix $[K_C]$ are defined in accordance to the solution of the characteristic equation:

$$\det|k_{ij} - \lambda\delta_{ij}| = \lambda^3 + a\lambda^2 + b\lambda + c = 0, \quad (6)$$

$$a = -(k_1 + k_2 + k_3),$$
$$\text{where: } b = k_1k_2 + k_1k_3 + k_2k_3 - (k_4^2 + k_5^2 + k_6^2),$$
$$c = k_1k_6^2 + k_2k_5^2 + k_3k_4^2 - (k_1k_2k_3 + 2k_4k_5k_6).$$

Since the matrix $[K_C]$ is symmetric, its eigen values are real numbers. For their calculation the equations of Cardano could be used:

$$\lambda_1 = 2\sqrt{\frac{|p|}{3}}\cos\left(\frac{\varphi}{3}\right) - \frac{a}{3};$$
$$\lambda_2 = -2\sqrt{\frac{|p|}{3}}\cos\left(\frac{\varphi+\pi}{3}\right) - \frac{a}{3}; \quad (7)$$
$$\lambda_3 = -2\sqrt{\frac{|p|}{3}}\cos\left(\frac{\varphi-\pi}{3}\right) - \frac{a}{3} \quad \text{for } \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0,$$

$$q = 2(a/3)^3 - (ab)/3 + c, \quad p = -(a^2/3) + b < 0,$$
$$\varphi = \arccos\left[-q/2\Big/\sqrt{(|p|/3)^3}\right], \quad (8)$$

The eigen vectors $\vec{\Phi}_1, \vec{\Phi}_2, \vec{\Phi}_3$ of the covariance matrix $[K_C]$ are the solution of the system of equations:

$$\left| \begin{array}{l} [K_C]\vec{\Phi}_m = \lambda_m \vec{\Phi}_m \\ |\vec{\Phi}_m|^2 = \sum_{i=1}^{3} \Phi_{mi}^2 = 1 \end{array} \right., \text{ for } m = 1,2,3. \quad (9)$$

The solution of the system of Eq. (9) is used to calculate components of mth eigenvector $\vec{\Phi}_m = [\Phi_{1m}, \Phi_{2m}, \Phi_{3m}]^t$, which corresponds to the eigen value λ_m. The direct AKLT for vectors $\vec{C}_s = [C_{1s}, C_{2s}, C_{3s}]^t$, from which are obtained vectors $\vec{L}_s = [L_{1s}, L_{2s}, L_{3s}]^t$, is:

$$\begin{bmatrix} L_{1s} \\ L_{2s} \\ L_{3s} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{21} & \Phi_{31} \\ \Phi_{12} & \Phi_{22} & \Phi_{32} \\ \Phi_{13} & \Phi_{23} & \Phi_{33} \end{bmatrix} \begin{bmatrix} (C_{1s} - \overline{C}_1) \\ (C_{2s} - \overline{C}_2) \\ (C_{3s} - \overline{C}_3) \end{bmatrix} \quad (10)$$

The components of vectors $\vec{L}_s = [L_{1s}, L_{2s}, L_{3s}]^t$ could be processed in various ways (such as orthogonal transforms, quantization, decimation and interpolation, etc.).

In result are obtained the corresponding vectors $\vec{L}_s^q = \psi(\vec{L}_s) = [\psi_1(L_{1s}), \psi_2(L_{2s}), \psi_3(L_{3s})]^t$ with components $L_{1s}^q = \psi_1(L_{1s})$, $L_{2s}^q = \psi_2(L_{2s})$, $L_{3s}^q = \psi_3(L_{3s})$, where $\psi_1(.), \psi_2(.), \psi_3(.)$ are the functions of the used transform. For the restoration of vectors $\hat{\vec{C}}_s = [\hat{C}_{1s}, \hat{C}_{2s}, \hat{C}_{3s}]^t$ through inverse AKLT we needed not only the vectors $\hat{\vec{L}}_s = [\hat{L}_{1s}, \hat{L}_{2s}, \hat{L}_{3s}]^t$, but also the elements Φ_{ij} of the matrix $[\Phi]$, and the values of $\overline{C}_1, \overline{C}_2, \overline{C}_3$ as well. The total number of these elements could be reduced representing the matrix $[\Phi]$ as the product of matrices $[\Phi_1(\alpha)], [\Phi_1(\beta)], [\Phi_1(\gamma)]$ with rotation around coordinate axes for each transformed vector in Euler angles α, β and γ correspondingly:

$$[\Phi] = \begin{bmatrix} \Phi_{11} & \Phi_{21} & \Phi_{31} \\ \Phi_{12} & \Phi_{22} & \Phi_{32} \\ \Phi_{13} & \Phi_{23} & \Phi_{33} \end{bmatrix} = \quad (11)$$
$$[\Phi_1(\alpha)][\Phi_2(\beta)][\Phi_3(\gamma)] = [\Phi(\alpha,\beta,\gamma)].$$

Then, for the calculation of the elements of the inverse matrix $[\Phi]^{-1}$ is enough to know the values of the 3 rotation angles α, β and γ. In result, the number of the needed values for the calculation of the matrix $[\Phi]^{-1}$ is reduced from 9 down to 3, i.e. 3 times reduction.

IV. Experimental results

On the basis of the 2-level HAKLT algorithm, we have experimented with sequences of CT images with size 512×512 pixels, 8 bpp. The sequence was divided into sets, each containing 9 consecutive CT images. Set 3 is illustrated on Fig. 3.

The results represented on Fig. 4 a) and b) show that the main part of the energy of all 9 images is concentrated in the first eigen image, and the energy of each next eigen image decreases quickly. On these figures the power distribution of pixels of eigen images from Set 3 is given after first level of HAKLT, before and after their rearrangement in correspondence to Fig. 1.

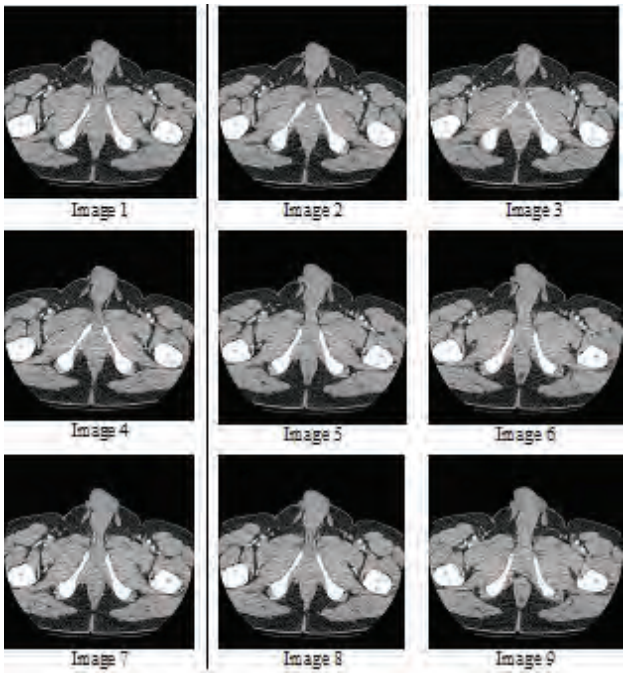
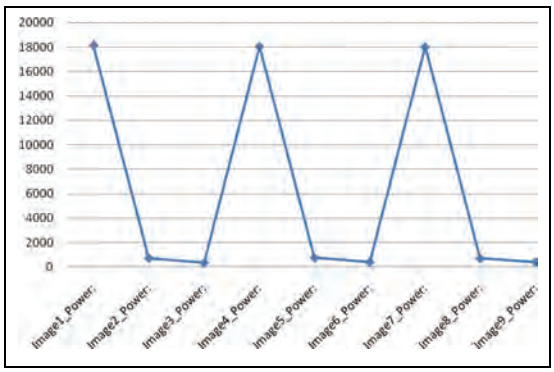
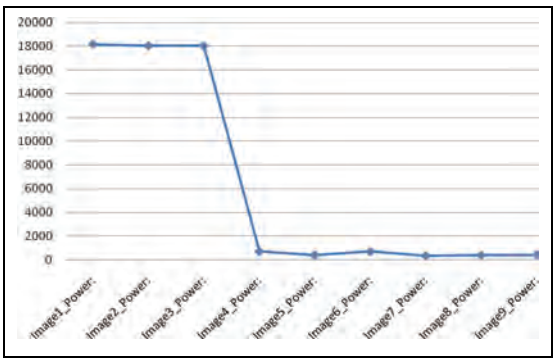


Fig. 3. Group of 9 consecutive CT images in Set 3.

We can conclude that for all the 7 sets of 9 images in GMI, 95,7 % of the total mean power is concentrated in the first 3 eigen images of each set. The mean power of the first eigen image for all sets is more than 250 times larger than that of each of the next 8 eigen images. The values for pixels of the eigen images, obtained in result of the direct 2-level HAKLT, were calculated with full accuracy, and after corresponding rounding could be transformed into 8-bit numbers.



a)



b)

Fig. 4. Power distribution for Set 3, level 1: a) - not arranged, b) - arranged.

Then, if on the 8 bpp eigen images is applied the inverse 2-level HAKLT, the quality of corresponding restored images in GMI, evaluated by their peak signal-to-noise ratio (PSNR), is ≥ 45 dB. Hence, the sequence of 9 images could be restored with retained visual quality. This result illustrates the ability for efficient compression of a sequence of CT images, when HAKLT is used.

The experimental results were obtained with the software implementation of HAKLT, in Visual C.

To be able to compare the represented approach with the standard JPEG we combine the HAKLT with Burrows-Wheeler transformation (BWT) [9] and entropy encoding. The results represented in fig. 5 and 6 show that the proposed algorithm for lossless data compression demonstrates higher accuracy than the JPEG and retain fully quality of the restored images. The mean compression ratio for all 7 sets of CT images for the HAKLT BWT+RLE+AC Coder is 1.74 compared to the compression ratio of 1.69 for JPEG, the mean Quality PSNR for HAKLT is 46.21 dB and for JPEG is 54.67 dB.

V. Conclusion

The computational complexity of the 2-level HAKLT algorithm compared with that of the KLT algorithm, is at least 1,7 times smaller than KLT algorithm for each value of *P* pixels (in average, about 2 times). The basic qualities of the offered HAKLT for processing a group of sequential medical images are:

- Lower computational complexity than KLT for the whole GMI, due to the lower complexity of AKLT;
- Ability for efficient lossless compression of GMI with retained visual quality of the restored images;

There is also a possibility for further development of the HAKLT algorithm, through compression of video sequences from stationary TV camera; sequences of multispectral and multi-view images, etc.

Acknowledgement

This work is supported by the Technical University of Sofia Science Fund – Grant No. 122pd0061-07/2012 „Decorrelation of multispectral images, based on hierarchical adaptice principal component analysis”.

References

[1] F. Sepehrband, M. Mortazavi, S. Ghorshi and J, Choupan, Simple lossless medical image compression based on a new method of transformation, *The 3rd International Conference on Machine Vision*, 2010.

[2] D. Santa-Cruz; T. Ebrahimi; J. Askelof, M. Larsson; C. A. Christopoulos, JPEG 2000 still image coding versus other standards, *Applications of Digital Image Processing* XXIII, Proceedings Vol. 4115, 2000, pp.446-454.

[3] P. Roos, M. Viergever, Reversible interframe compression of medical images: a comparison of decorrelation methods, *IEEE Trans. Medical Imaging*, 1991, vol. 10(4), pp. 538-547.

[4] S. Szilágyi, L. Szilágyi, Z. Benyó, Echocardiographic Image Sequence Compression Based on Spatial Active Appearance Model, *Progress in Pattern Recognition, Image Analysis and Applications*, LN in Computer Science, 2007, vol. 4756, pp. 841-850.

[5] V. Thirumalai, Distributed Compressed Representation of Correlated Image Sets, Thesis No 5264, 2012, Lausanne, EPFL.

[6] R. Dony, *Karhunen-Loeve Transform*, Ch. 1 in The transform and data compression handbook (Eds. K. Rao, P. Yip), 2001, Boca Raton, CRC Press.

[7] M. Hanafi, A. Kohler, E. Qannari. Shedding new light on Hierarchical Principal Component Analysis, *Journal of Chemometrics*, 2010, Vol. 24 (11-12), pp. 703-709.

[8] R. Kountchev, R. Kountcheva. Image Color Space Transform with Enhanced KLT, *New Advances in Intelligent Decision Technologies*, Springer, 2009, pp. 171-182.

[9] P. Ferragina, G, *Manzini: Burrows-Wheeler Transform*, Encyclopedia of Algorithms 2008.