Hybrid Evolutionary Algorithm for Integer Multiple-Objective Optimization Problems

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Abstract – **A reference point hybrid evolutionary algorithm is proposed. It combines a heuristic for fast moving the population with fitness function evaluation based on scalarizing approach. A dialog with the decision maker in an interactive manner is included. A test example is presented to illustrate the performance of the new hybrid evolutionary algorithm.**

Keywords **– evolutionary algorithms, multiple-objective optimization, hybrid heuristic techniques.**

I. INTRODUCTION

The evolutionary multi-objective optimization (EMOO) is a popular and useful field for research and development of algorithms which solve many real-life multi-objective problems ([3, 8]). EMOO has been evaluated as a very fast growing field of research and application ([3, 8]).

The Evolutionary Optimization (EO) algorithms use a population-based approach, in which the iterations are performed on a set of solutions (called population) and more than one solution is generated at each iteration. The main reasons for the popularity of EO algorithms are the following:

(i) They do not require any derivative information;

(ii) EO algorithms are relatively simple to implement;

(iii) EO algorithms are flexible and robust, i.e., they perform very well on a wide spectrum of problems ([7]).

To reduce the number of iterations in the new algorithm a combination of EO-approach and a heuristic, designed to move the population to the Pareto-optimal front is used.

II. PROBLEM FORMULATION AND APPROACHES FOR ITS SOLVING

The integer multi-objective convex optimization problem can be stated as follows:

$$\text{Min } f(x) = [\, f_1(x), f_2(x), \ldots, f_k(x)]^T \quad (1)$$

$$\text{subject to: } \boldsymbol{g_j}(x) \le 0, \quad j = 1, 2, ..., m, \quad (2)$$

$$\boldsymbol{x_i}^{(L)} \le x_i \le \boldsymbol{x_i}^{(U)}, \quad i = 1, 2, \ldots, n, \quad (3)$$

$$x \in Z^n, \quad (4)$$

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where $\boldsymbol{g_j}(\boldsymbol{x})$, $j = 1, 2, \ldots, m$ are convex functions;

$\boldsymbol{x_i}^{(L)}$ and $\boldsymbol{x_i}^{(U)}$, $i = 1, 2, \ldots, n$ are the known lower and upper bound of the variable x_i respectively;

$f_j(\boldsymbol{x})$, j = 1, 2,…, k are convex nonlinear functions.

Further on the solution $\boldsymbol{x} \in \boldsymbol{Z^n}$ denotes a vector of n decision variables: $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T$. The constraints (2)-(4) constitute a feasible decision domain $\boldsymbol{V} \subset \boldsymbol{Z^n}$.

$\boldsymbol{S} = \boldsymbol{f}(\boldsymbol{V}) = \{ \boldsymbol{s} = \boldsymbol{f}(\boldsymbol{x}), \boldsymbol{x} \in \boldsymbol{V}\}$ is a k-dimensional objectives' region, $\boldsymbol{S} \subset \boldsymbol{R}^k$.

We shall use the term “solution” as a vector of variables in the decision space and the term “point” as a vector of the criteria values in the objectives' space.

Definition: A solution $x^{(1)}$ is said to dominate the solution $x^{(2)}$, if the following two conditions are true:

1. The solution $x^{(1)}$ is not worse than $x^{(2)}$ in all the objectives. Thus, the solutions are compared based on their objective function values.

2. The solution $x^{(1)}$ is strictly better than $x^{(2)}$ in at least one objective.

All the points which are not dominated by any other point $\boldsymbol{s} \in \boldsymbol{S}$, are called Pareto-optimal points. They constitute together the Pareto-optimal front ([1, 3]) in the objectives' space.

Two basic approaches are applied to solve the above formulated multi objective problem **(1)-(4)**. The first approach is to choose one "best/ final" non-dominated solution among many others. Because the set of non-dominated points consists of equally good points then additional information for the choice is necessary. It comes usually from human factor. In other words the so called Decision Maker (DM) evaluates the solutions obtained during the process of solution. A number of methods realizing this approach exist ([1, 2, 3, 8, 14]).

The second approach is to find the whole set of non-dominated points (efficient frontier). This problem is solved completely only for linear case ([4, 6, 11, 20]). There exist also approximate algorithms applying this approach.

Evolutionary algorithms seems to be very suitable to apply the second approach, namely to find an approximation for the whole non-dominated set. Indeed there are a number of EO methods ([1, 5, 10]).

Here an evolutionary algorithm is proposed, which applies the first approach. It performs with limited population, but large enough to approximate locally the efficient frontier directed by the DM's preferences. The iterative procedure is repeated until a final solution is found. Thus we exploit the advantages of EO approach to generate a good approximation of efficient frontier. Using traditional heuristic scalarizing methods there arises the question – how to support the DM in setting new preferences. Some of those methods use trade-off, other use search in a reference direction, or generate a reference points ([12, 15, 16, 18]). In this paper DM can set his local preferences at each iteration in terms of desired improvements or relaxations of the criteria (as a reference point). On this basis a discrete optimization scalarizing problem is constructed. A small ranked set of relatively close alternatives is defined with the help of this scalarizing problem. The ranked set is presented to the DM for selection of the most preferred alternative or for entering his/her new local preferences. In short the characteristics of the new proposed algorithm are the following:

- A heuristic procedure is used to accelerate the moving the whole population towards the Pareto front. It is similar to those, described in [9]. In this way we avoid the slow speed performance of the evolutionary algorithms.
- An interaction step is included, where the Decision Maker sets a reference point f^r in the objectives' space (see [13, 17, 19]). The DM has the possibility to change his/her preferences periodically and to replace the former reference point by a new one. This step ensures the convergence of the proposed algorithm to a desired non-dominated solution.

III. The Proposed Evolutionary Algorithm

We use a population P of N solutions found during the search process.

We propose a *heuristic procedure* to move quickly the initial generated population to the Pareto-optimal front. For this purpose we calculate the direction $\boldsymbol{y} = Cref - Ci$, where $Cref$ is a reference point given by DM and Ci is the weight centre of all solutions in P. Then we move the population as close as possible to the Pareto-optimal front (reaching eventually the boundaries of the system (2)-(3)). We perform consecutive steps calculating solution $x' = x + \alpha.\boldsymbol{y}$, where α is the step length. In case x' violates any constraint in the system (2) or in case the current step in $\boldsymbol{y}$ – direction leads to deteriorating the criteria values, the corresponding feasible solution is calculated using the Golden section method for line search along the segment xx' and by rounding it to an integer solution. The Pareto-optimal front may be located:

1) on the boundary of the feasible domain.
2) inside the feasible domain.

We present below the scheme of this heuristic, reaching the Pareto-optimal front in both cases:

A. Scheme of the proposed heuristic procedure

We use the function $\varphi(x) = \sum_{i=1}^{k} f_i(x)$, where $f_i(x)$ is current value for the i-th objective, $i = 1, \ldots, k$.

Step: Find the minimal value of the function $\boldsymbol{\varphi(x)}$ over the rays defined by each population solution belonging to the initial population P_0 and the vector $\boldsymbol{y}$. The Golden section method is used for this calculation.

The above heuristic is based on the following theoretical properties:

1) The direction y is an improving direction by construction. This means that between every two different solutions x_1 and x_2 lying on a ray $\boldsymbol{y}^{\rightarrow}$ with starting solution Ci the following relations are satisfied: $f(x_1) \leq f(x_2)$ or $f(x_1) \geq f(x_2)$, but the solutions x_1 and x_2 are not incomparable.

2) The function $\boldsymbol{\varphi(x)}$ obtains its minimum at a point which is located on the Pareto optimal front.

B. Scalarizing problem formulation

At each iteration *iter* a ranked set $M = \{i_1, i_2, \ldots\ldots, i_l\}$ of alternatives is generated. The first alternative is the current preferred alternative and l is the number of generated alternatives, which the DM is willing or is able to evaluate at this iteration. The DM has to estimate the relatively close alternatives and to choose one of them either as a current preferred or as the most preferred alternative. In the second case the discrete multicriteria choice problem is solved. In the first case the DM sets the desired changes of the criteria (desired values for improving (relaxing)) in order to search for a new better alternative in the reference neighborhood of the current preferred alternative.

Let h denotes the index of the current preferred alternative. Let us introduce the following notations in relation to the current preferred alternative:

$K_k^{\geq}$ - the set of indices of the criteria $j \in J$ that the DM wants to improve by desired (aspiration) values Δ_{hj};

$K_h^{\leq}$ - is the set of indices of the criteria $j \in J$ that the DM agrees the values of the criteria to be deteriorated by no more than δ_{hj};

K_h^0 - the set of indices of the criteria $j \in J$ in which the DM is not interested concerning alteration at the moment and these criteria can be freely altered;

$\bar{a}_{hj}$ - the desired (aspiration) value of the criterion with an index $j \in K_h^{\geq}$;

$$\bar{a}_{hj} = a_{hj} + \Delta_{hj}, \quad j \in K_h^{\geq}$$

a_{hj} - the value of a criterion with an index $j \in K_h^{\geq}$ in the current preferred alternative;

Λ_j - the difference between the maximal and minimal value for the criterion with an index j;

$$\Lambda_j = \max_{i \in I} a_{ij} - \min_{i \in I} a_{ij}$$

The set $M = (i_1, \ldots . i_l)$ is computed solving the following discrete scalarizing problem:

$$\min_{i\in I} S(i,h) = \min_{i\in I}\{\max[\max_{j\in K_h^{\geq}}(\bar{a}_{hj} - a_{ij})/\Lambda_j, \max_{j\in K_h^{\leq}}((a_{hj} - a_{ij})/\Lambda_j]\} \quad (5)$$

subject to:

$$a_{ij} \geq a_{hj} - \delta_{hj}, \; j \in K_h^{\leq}. \quad (6)$$

When solving a discrete optimization problem S the value of *S(i,h)* is computed for all alternatives for which the conditions (6) are satisfied. Function *S(i,h)* denotes the distance between alternatives *i* and *h* with respect to the “modified” Chebyshev norm. The alternatives correspond to the fitness function values of solutions in the current population in the proposed new hybrid algorithm.

C. Scheme of the new algorithm

Step 1. Set the iteration counter *iter* = 0. Generate *N* uniform distributed solutions’ vectors around the Chebyshev centre *Ch* of the feasible domain by using a deviation of $\pm\delta$, where δ is a % of the corresponding component variation (for example, δmax = ±5%). Use them to create the initial population P_{iter}.

Step 2. Find the solutions in P_{iter} whish are the best according to the value of each objective. Show them to DM and ask him/her to choose one of them as a initial reference solution x^r, or a group of them, which weight centre will be the initial reference solution x^r. The corresponding reference point is denoted by $f^{\,r}$. Compute the weight centre *Ci* of all solutions P_{iter}. Form the moving direction $d = x^r - Ci$.

Step 3. Move the population P_{iter} in direction *d*: $\{P_{new}\} = \{P_{iter} \rightarrow d\}$. Each solution is moved as close as possible to the Pareto-optimal front (reaching the boundaries of the system (2)-(3) if necessary) along this direction by using the Golden section method for line search as in the heuristic procedure.

Step 4. Calculate fitness values for each solution in P_{iter} using the scalarization function S(*i*,*h*) of (5)-(6), where *i*=1,..*N*; and *h* denotes the index of the current preferred point.

Step 5. Arrange all the points corresponding to the solutions in P_{iter} in ascending order according their S-values. Show the first ten points to DM. DM evaluates visually them and if he/she is satisfied by one of them, go to *Step 8*, otherwise DM chooses the current best solution x^{best}. Set *iter=iter*+1, $P_{iter}=P_{new}$, and go to *Step 6*.

Step 6. Compute the weight centre *C* of the population P_{iter}. Form the moving direction $p = x^{best} - C$. Compute a series of solutions $t^{\,j} = C + j.p$, $j = 1,\ldots$; and present the corresponding points $f(t^{\,j})$ to the DM as possible reference points. The DM chooses one of them as a new reference point $f^{\,r}$. The corresponding solution is denoted by x^r.

Step 7. Calculate direction *d* in which will be moved the whole population P_{ite}: $d = x^r - C$. Go to *Step 3*.

Step 8. End.

IV. Illustrative example

We consider the following problem:

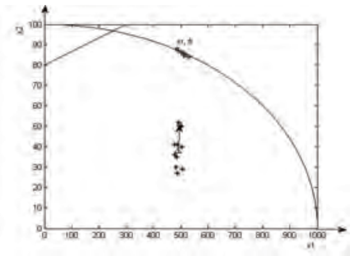
$$\text{Min} \quad f_1 = 1/(x_1+1),$$
$$\text{Min} \quad f_2 = 1/(x_2+1),$$
$$\text{Min} \quad f_3 = (x_1.x_2)/(x_1+1).(x_2+1)^2,$$

subject to: $x_1^{\,2} + 100x_2^{\,2} \leq 10^6$; $-x_1 + 15x_2 - 1200 \leq 0$;

$0 \leq x_1 \leq 1000$; $\quad 0 \leq x_2 \leq 100$;

$x_1, x_2 \in \mathbf{Z}$.

The search process of one iteration is presented on Fig.1



Legend:
* – initial population P_0
Δ – weight centres *Ci* and *Ce*
□ – members of Pe_0
+ – P_h at the end of *Step* 2
x – solutions $t^{\,j}$ at *Step 3*
o – members of final population
✡ – ref. solution x^r*

Fig. 1. One iteration

We denote values $f^i_{\,r} = f^i.10^3$. The initial population P_0 at Step 1 is: x^1 = (500; 50), $f^1_{\,r}$ = (1.996; 19.608; 19.185), x^2 = (506; 29), $f^2_{\,r}$ = (1.972; 33.333; 32.158), x^3 = (482; 30), $f^3_{\,r}$ = (2.070; 32.258; 31.163), x^4 = (493; 52), $f^4_{\,r}$ = (2.024; 18.868; 18.474), x^5 = (477; 41), $f^5_{\,r}$ = (2.092; 23.810; 23.194), x^6 = (485; 35), $f^6_{\,r}$ = (2.058; 27.778; 26.950), x^7 = (504; 40), $f^7_{\,r}$ = (1.980; 24.390; 23.748), x^8 = (487; 41), $f^8_{\,r}$ = (2.049; 23.810; 23.195), x^9 = (488; 27), $f^9_{\,r}$ = (2.045; 35.714; 34.368), x^{10} =(479; 36), $f^{10}_{\,r}$ =(2.083; 27.027; 26.242);. At Step 2 the best solutions chosen by the DM are x^1 and x^4. Their weight centre is x^r = (497; 51). The weight centre of all solutions in the population P_0 is *Ci* = (490; 38). So d = x^r – *Ci* = [7; 13]. At Step 3 the whole population is moved to the Pareto-optimal front. The following solutions are obtained: x^1 = (519; 85), $f^1_{\,r}$ = (1.923; 11.628; 11.471), x^2 = (534; 84), $f^2_{\,r}$ = (1.869; 11.765; 11.605), x^3 = (510; 86), $f^3_{\,r}$ = (1.957; 11.494; 11.340), x^4 = (511; 85), $f^4_{\,r}$ = (1.953; 11.628; 11.470), x^5 = (502; 86), $f^5_{\,r}$ = (1.988; 11.494; 11.340), x^6 = (512; 85), $f^6_{\,r}$ = (1.949; 11.628; 11.470), x^7 = (527; 84), $f^7_{\,r}$ = (1.894; 11.765; 11.604), x^8 = (510; 86), $f^8_{\,r}$ = (1.957; 11.494; 11.340), x^9 = (519; 85), $f^9_{\,r}$ = (1.923; 11.628; 11.471), x^{10} =(506; 86), $f^{10}_{\,r}$ =(1.972; 11.494; 11.340);. At Step 5 are calculated the S-values of all solutions in *P*. At Stem 6 as x^{best} is chosen the solution x^{10} =(506; 86). The weight centre C of all solutions in *P* is C = (515; 85). The vector p = x^{best} – C = [-9; 1]. For t^2 the DM makes a choice of reference solution x^r = (488; 88) and the corresponding reference point is: $f^{\,r}$ = (2.045; 11.236; 11.087). At Step 7 the vector d = x^r – C = [-27; 3]. At Step 3 are obtained the solutions: x^1 = (492; 87), $f^1_{\,r}$ = (2.028; 11.364; 11.212), x^2 = (507; 86), $f^2_{\,r}$ = (1.969; 11.494; 11.340), x^3 = (474; 88), $f^3_{\,r}$ = (2.105; 11.236; 11.086), x^4 = (484; 87), $f^4_{\,r}$ = (2.062; 11.364; 11.211), x^5 = (474; 88), $f^5_{\,r}$ = (2.105; 11.236; 11.086), x^6 = (485; 87), $f^6_{\,r}$ = (2.058; 11.364; 11.211), x^7 = (500; 86), $f^7_{\,r}$ = (1.996; 11.494; 11.339), x^8 = (483; 87), $f^8_{\,r}$ = (2.066; 11.364; 11.211), x^9 = (492; 87), $f^9_{\,r}$ = (2.028; 11.364; 11.212), x^{10} =(474; 88), $f^{10}_{\,r}$ =(2.105; 11.236; 11.086);. At Step 4 DM is satisfied by solution x^3 = (474; 88), $f^3_{\,r}$ = (2.105; 11.236; 11.086) and the calculations are terminated.

V. CONCLUSION

The basic characteristics of the proposed interactive evolutionary method solving convex integer multi-criteria problems may be summarized as follows:

- it is an interactive algorithm;
- the new algorithm is population – based and combines a heuristic for fast moving the population to the Pareto-optimal front with scalarizing approach to arrange the solutions in the current population.
- the algorithm proposed explores only a desired part of the Pareto optimal front in contrast to other algorithms, where the purpose is to obtain a representative sample of the whole Pareto-optimal front;
- an accelerated moving the whole population to the Pareto-optimal front is achieved, leading to better efficiency in comparison to the usual evolutionary algorithms;
- The increasing the number of objectives in the optimization problem does not have essential influence on the performance of the search procedure.
- The Decision Maker is supported in the choice of a suitable reference point, so that he/she can easily direct the search process to the desired region.

The approach demonstrated by the new algorithm may be used for solving linear and nonlinear multiple objective optimization problems, having continuous and/or integer variables.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the support of Bulgarian National Science Fund, Grant No DTK02/71 "Web-Based Interactive System, Supporting the Building Models and Solving Optimization and Decision Making Problems".

REFERENCES

[1] Branke J., K. Deb, K. Miettinen, R. Slowiń ski, *Multiobjective Optimization. Interactive and Evolutionary Approaches.* (Berlin Heidelberg, Springer-Verlag, 2008).

[2] Coello C. A. C. & Lamont G. B., *Applications of Multi-Objective Evolutionary Algorithms*, (World Scientific, Singapore, 2004).

[3] Deb K., *Multi-Objective Optimization Using Evolutionary Algorithms*. Wiley-Interscience Series in Systems and Optimization. Chichester, (John Wiley & Sons, 2001).

[4] Ecker. J.G., N. S. Hegner and I.A. Kouada, „Generating All Maximal Efficient Faces for Multiple Objective Linear Programs", *Journal of Optimization Theory and Applications*, Vol. 30, No 3, (1980) pp. 353-381.

[5] Fonseca, C. M., P. J. Fleming, Genetic Algorithms for Muitiobjective Optimization: Formulation, Discussion and Generalization. – In: *Proc. of the Fifth Int. Conf. on Genetic Algorithms*, San Mateo, California, University of Illinois at Urbana-Champaign, Stephanie Forrest, ed., Morgan Kauffman Publishers, (1993) pp. 416-423.

[6] Gal T., "A General Method for Determining the Set of All Efficient Solutions to a Linear Vectormaximum Problem", *European Journal of Operational Research*, Vol. 1, No 5, (1977) pp. 307 – 322.

[7] Goldberg D. E., *Genetic Algorithms in Search, Optimization and Machine Learning*, (Addison Wesley Reading, Mass, 1989).

[8] Guliashki V., H. Toshev, C. Korsemov, "Survey of Evolutionary Algorithms Used in Multiobjective Optimization", *Problems of Engineering Cybernetics and Robotics*, Vol. 60, (2009), pp. 42-54.

[9] Guliashki V., L. Kirilov, "SPEA-Based Method for MCDM Convex Integer Problems", *Cybernetics and Information Technologies – Journal of Bulgarian Academy of Sciences*, Vol.9, No4, (2009) pp.93-101,

[10] Guliashki V., L. Kirilov, K. Genova, "An evolutionary algorithm for integer multicriteria optimization (EVALIMCO)", In: *Proceedings of the 10-th International FLINS Conference, Uncertainty Modeling in Knowledge Engineering and Decision Making*, (Editors Prof. Cengiz Kahraman, Prof. Etienne E. Kerre, Prof. Faik Tunc Bozbura), 26-29.August, 2012, Istanbul, Turkey. pp. 118-123.

[11] Isermann, H., "The Enumeration of the Set of All Efficient Solutions for a Linear Multiple Objective Program", *Operational Research Quarterly*, Vol. 28, No 3, (1977) pp. 711-725.

[12] Korhonen, P., Wallenius, J., VIG – A Visual and Dynamic Decision Support System for Multiple Objective Linear Programming, *Multiple Criteria Decision Making and Risk Analysis Using Microcomputers*, Edited by B. Karpak, S. Zionts, Springer-Verlag, Berlin Heidelberg, (1989) pp. 251-281.

[13] Miettinen K., L. Kirilov, "Interactive Reference Direction Approach Using Implicit Parametrization for Nonlinear Multiobjective Optimization", *Journal of Multi-Criteria Decision Analysis*, vol.13, (2005) pp. 115-123.

[14] Miettinen K., Neittaanmäki P., Mäkelä M.M., Périaux J., „*Evolutionary Algorithms in Engineering and Computer Science*", John Wiley & Sons, Ltd., (Chichester, Weinheim, New York, Brisbane, Singapore, Toronto, 1999)

[15] Nakayama H., Sensitivity and Trade-Off Analysis in Multiobjective Programming, Methodology and Software for Interactive Decision Support, Edited by A. Lewandowski, I. Stanchev, Lecture Notes in Economics and Mathematical Systems 337, Springer-Verlag, (1989) pp. 86-93.

[16] Narula S.C., Kirilov L., Vassilev V., Reference Direction Approach for Solving Multiple Objective Nonlinear Programming Problems, *IEEE Transactions on Systems, Man, and Cybernetics*, vol.24, No5, (1994) pp. 804-806.

[17] Vassilev V., Genova K., Vassileva M., Narula S., "Classification-based Method of Linear Multicriteria Optimization", *International Journal on Information Theories and Applications*, vol.10, No 3, (2003), 266-270.

[18] Vassilev V., Narula S. and Gouljaski V., "An Interactive Reference Directions Algorithm for Solving Multiobjective Convex Nonlinear Integer Programming Problems", *International Transactions in Operational Research*, 8, (2001) pp. 367-380.

[19] Vassileva M., "An Optimizationally Motivated Interactive Method for Solving a Class of Discrete Multicriteria Choice Problem", Cybrnetics and Information Thechnologies, Vol. 1, № 2, (2001), pp. 70-77.

[20] Yu, P. L. and M. Zeleny, "The Set of All Non-Dominated Solutions in Linear Cases and a Multicriteria Simplex Method", *Journal of Mathematical Analysis and Applications*, Vol. 49, No 2, (1975) pp. 430-468.