

Computer-Aided Parameter Extraction of Behavioral RF Inductor Models

Elissaveta Gadjeva

Abstract – An extraction procedure is developed to obtain the parameters of behavioral RF planar inductor models. The procedure is based on measured data for the S -parameters. The computer realization is performed using macrodefinitions in the graphical analyzer *Cadence Probe*. The description of the behavioral computer model in accordance with the *PSpice* input language is given. The simulation results using the extracted model parameters confirm the validity of the computer model and the proposed extraction procedure.

Keywords – RF spiral inductors, Parameter extraction, PSpice simulator.

I. INTRODUCTION

Spiral inductors are important passive components applied in LC tank, VCO, etc. The accurate computer RF inductor models are a key problem in the simulation of RF circuits. Recently, two basic types of on-chip spiral inductor models are widely used – wide-band models with frequency-independent model parameters [1], [2], as well as models with geometry-dependent model parameters and frequency-dependent series resistance, taking into account the skin effect [3], [4], [5]. Computer approaches to modeling, parameter extraction and optimization of planar inductors using Matlab are developed in [6]. An approach is proposed in [7] to create accurate RF spiral inductor model, valid in the frequency range up to the resonance frequency f_{so} of the inductor. The proposed behavioral model is based on analytical approximation of the inductance, depending on the spiral track width and turn number.

In the present paper, a computer-aided extraction procedure is developed for obtaining the parameters of the behavioral model proposed in [7]. The procedure is based on measured data for the S -parameters. The computer realization of the extraction procedure is performed using macrodefinitions in the graphical analyzer *Cadence Probe*. The description of the behavioral computer model in accordance with the *PSpice* input language is given. The comparison between the simulation results and the measured data confirms the validity of the developed extraction procedure, as well as the high accuracy of the computer behavioral model.

II. PARAMETER EXTRACTION PROCEDURE

Based on the measured S -parameters of the RF spiral inductor, the two-port Y -parameters can be obtained. The

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inductance value $L_{sm}(f)$, corresponding to the measured data, is calculated using the following expression:

$$L_{sm}(f) = \frac{\text{Im}\left(\frac{1}{Y_{11}}\right)}{2\pi f}. \quad (1)$$

$L_{sm}(f)$ can be approximated by the following expression [7]:

$$L_s(f) = L_{s1}(f) + L_{s2}(f) + L_{s3}(f), \quad (2)$$

where

$$L_{s1}(f) = A_1 + B_1 \cdot f, \quad (3)$$

$$L_{s2}(f) = C \cdot \log_{10} \left(\frac{10^{\frac{f-f_1}{c_1}}}{1 + 10^{\frac{f-f_1}{c_1}}} \right), \quad (4)$$

$$L_{s3}(f) = D \cdot \log_{10} \left(1 + 10^{\frac{f-f_2}{d_1}} \right). \quad (5)$$

The frequency range $[f_{min}, f_{max}]$ is divided in three subintervals: $S_1 = [f_{min}, f_1]$, $S_2 = [f_1, f_2]$ and $S_3 = [f_2, f_{max}]$ (Fig. 1). The frequency f_{max} is near to the resonance frequency f_{so} . Each of the terms $L_{s1}(f)$, $L_{s2}(f)$ and $L_{s3}(f)$ dominates in the corresponding subinterval S_1 , S_2 and S_3 .

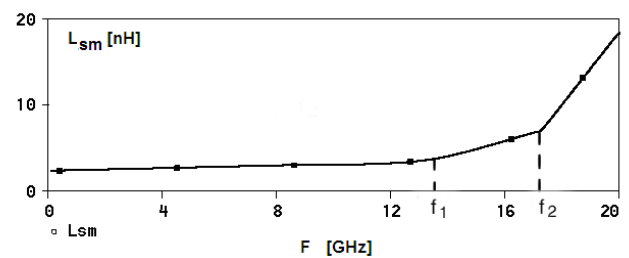


Fig. 1. Frequency-dependent inductance $L_{sm}(f)$

In order to realize the parameter extraction procedure, the equation (2) can be represented in the form:

$$L_s(f) = L_1(f) + L_2(f) + L_3(f), \quad (6)$$

where

$$L_1(f) = A + B \cdot f, \quad (7)$$

$$L_2(f) = -C \cdot \log_{10} \left(1 + 10^{\frac{f-f_1}{c_1}} \right), \quad (8)$$

$$L_3(f) = D \cdot \log_{10} \left(1 + 10^{\frac{f-f_2}{d_1}} \right), \quad (9)$$

$$A = A_1 - \frac{C \cdot f_1}{c_1} ; \quad B = B_1 + \frac{C}{c_1}. \quad (10)$$

The inductance $L_{sm}(f)$ (Fig. 1), corresponding to the measured two-port S -parameters, is used as input data for the realization of the extraction procedure. They are introduced tabularly using the graphical editor *Cadence Capture*.

The frequencies f_1 and f_2 are defined for the corresponding characteristic $L_{sm}(f)$. For the considered case shown in Fig. 1 (spiral inductor with trace width $w = 6\mu\text{m}$ and number of turns $n = 5$) [7] $f_1 = 13.4506$ GHz and $f_2 = 17.281$ GHz.

The proposed extraction procedure consists of the following steps:

1. Converting the measured two-port S -parameters to the two-port Y -parameters

The measured two-port S -parameters S_{ij} $i, j = 1, 2$, are converted to the two-port Y -parameters Y_{ij} $i, j = 1, 2$, needed for the parameter extraction procedure.

$$Y_{11} = \frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{\Delta}, \quad (11)$$

$$Y_{22} = \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{\Delta}, \quad (12)$$

$$Y_{12} = \frac{-2S_{12}}{\Delta} ; \quad Y_{21} = \frac{-2S_{21}}{\Delta}, \quad (13)$$

where

$$\Delta = ((1 + S_{11})(1 + S_{22}) - S_{12}S_{21})R_o.$$

$R_o = 50 \Omega$ is the characteristic impedance.

2. Determination of the inductance value $L_{sm}(f)$

The inductance value $L_{sm}(f)$, corresponding to the measurement data, is obtained using (1).

3. Determination of the coefficient A

The term $L_1(f)$ in (6) dominates in the subinterval S_1 :

$$L_s(f) \approx L_1(f) \quad (14)$$

and

$$A = L_s(f) - \frac{dL_s(f)}{df} f. \quad (15)$$

The subinterval S_1 is defined using the enable function E_{f_1} in the form:

$$E_{f_1} = \begin{cases} 1 & \text{for } f < f_1 \\ 0 & \text{for } f \geq f_1 \end{cases} \quad (16)$$

and A is calculated in the subinterval S_1 using the expression

$$\max \left[E_{f_1} \cdot \left(L_s(f) - \frac{dL_s(f)}{df} \cdot f \right) \right]. \quad (17)$$

The following macrodefinitions in the graphical analyzer *Cadence Probe* are used for the calculation of A according to (17):

```
ena_F1 = 0.5*(sgn(F1-F)+1)
A = max(ena_F1*(Lsm-d(Lsm)*F))
```

The function $\text{sgn}()$ gives the sign of the argument, F is the frequency and Lsm is the inductance value, corresponding to the measured values of the S -parameters.

The result for A is shown in Fig. 2.

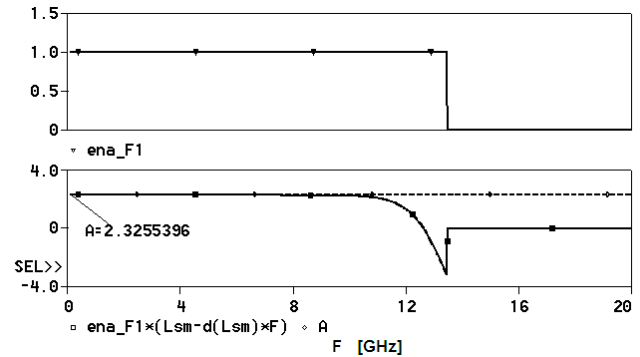


Fig. 2. Determination of the coefficient A

4. Determination of the coefficient B

Using (7) and (14), the coefficient B is obtained in the form:

$$B \approx \frac{dL_s(f)}{df} \quad (18)$$

The realization in the graphical analyzer *Probe* is illustrated in Fig. 3.

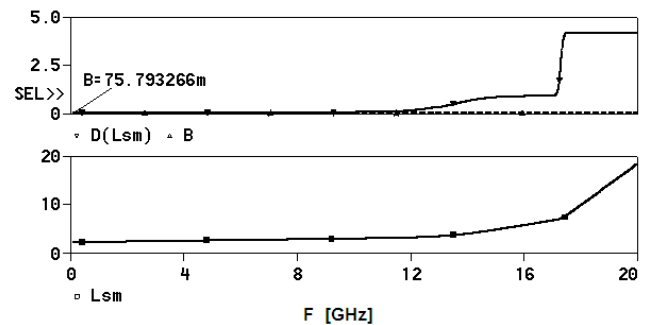


Fig. 3. Determination of the coefficient B

The following macrodefinition in *Probe* is used for the calculation according to (18):

```
B = min(D(Lsm))
```

where $D()$ is the function for differentiation in *Probe*.

The result is shown in Fig. 3.

5. Determination of the coefficient C

The coefficient C is obtained from (8) for $f=f_1$ where the term $L_3(f)$ is neglected:

$$C \approx -\max \left[E_{f_1} \cdot \frac{L_s(f) - A - B \cdot f}{\log_{10}(2)} \right]. \quad (19)$$

The following macrodefinitions in *Probe* are used for the calculation of C according to (19):

```
ena_F1 = 0.5*(sgn(F1-F)+1)
C=-max(ena_F1*(Lsm-A-B*F)/log10(2))
```

The result is shown in Fig. 4.

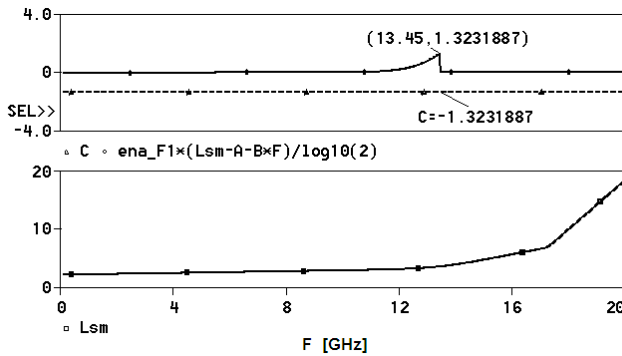


Fig. 4. Determination of the coefficient C

6. Determination of the coefficient c_1

The coefficient c_1 is obtained in the interval S_2 , where the term $L_3(f)$ is neglected. c_1 is calculated for the frequency $f_{12}=(f_1+f_2)/2$ in the interval S_2 .

$$c_1 = (f - f_1) \left(\log_{10} \left(10^{-\frac{L_a(f)}{C}} - 1 \right) \right)^{-1}, \quad (20)$$

where

$$L_a(f) = L_s(f) - A - B \cdot f. \quad (21)$$

c_1 is obtained in *Probe* using the expression:

$$\max \left[E_{f_{12}} (f - f_1) \left(\log_{10} \left(10^{-\frac{L_a(f)}{C}} - 1 \right) \right)^{-1} \right], \quad (22)$$

where

$$E_{f_{12}} = \begin{cases} 1 & \text{for } f < f_{12} \\ 0 & \text{for } f \geq f_{12} \end{cases}. \quad (23)$$

The computer realization in *Probe* has the form:

```
F12=0.5*(F1+F2)
C1=max((F-F1)/(log10(pwr(10,(Lsm-A-B*F)/(-C))-1))*ena_F12)
```

7. Determination of the coefficient D

Using (9) for $f=f_2$

$$D \approx \frac{L_3(f)}{\log_{10}(2)}, \quad (24)$$

where

$$L_3(f) = L_s(f) - A - B \cdot f + C \log_{10} \left(1 + 10^{\frac{f-f_1}{c_1}} \right). \quad (25)$$

D is obtained in *Probe* using the expression:

$$\max \left[E_{f_2} \frac{L_3(f)}{\log_{10}(2)} \right], \quad (26)$$

where

$$E_{f_2} = \begin{cases} 1 & \text{for } f < f_2 \\ 0 & \text{for } f \geq f_2 \end{cases}. \quad (27)$$

The realization in *Probe* has the form:

```
ena_F2 = 0.5*(sgn(F2-F)+1)
L3F = Lsm-A-B*F+C*LOG10(1+pwr(10,(F-F1)/C1))
DD = max(L3F/LOG10(2)*ena_F2)
```

The result is shown in Fig. 5.

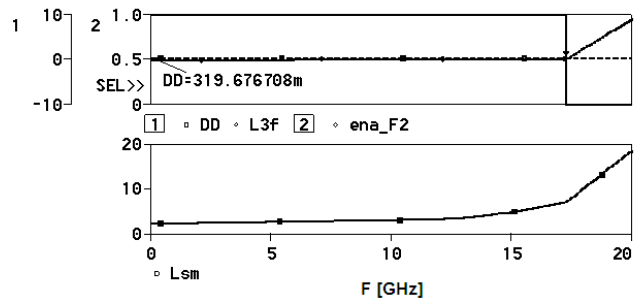


Fig. 5. Determination of the coefficient D

8. Determination of the coefficient d_1

The coefficient d_1 is obtained using (5) in the interval S_3 for the frequency $f_{23}=(f_2+f_{\max})/2$:

$$d_1 = (f - f_2) \left(\log_{10} \left(10^{-\frac{L_3(f)}{D}} - 1 \right) \right)^{-1}. \quad (28)$$

d_1 is obtained in *Probe* using the expression:

$$\max \left[E_{f_{23}} (f - f_2) \left(\log_{10} \left(10^{-\frac{L_3(f)}{D}} - 1 \right) \right)^{-1} \right], \quad (29)$$

where

$$E_{f_{23}} = \begin{cases} 0 & \text{for } f < f_{23} \\ 1 & \text{for } f \geq f_{23} \end{cases}. \quad (30)$$

The realization in *Probe* has the form:

$$\begin{aligned} F23 &= (F2 + \text{MAX}(F))/2 \\ D1 &= \max(\text{ena_F23} * (F - F2) / \text{LOG10}(\text{pwr}(10, L3F/D) - 1)) \\ \text{ena_F23} &= 0.5 * (\text{sgn}(F - F23) + 1) \end{aligned}$$

Input data for the extraction procedure are the inductance values $L_{sm}(f)$ of the spiral inductor. For the considered example, the trace width is $w = 6\mu\text{m}$, the number of turns is $n = 5$, $f_1 = 13.4506\text{ GHz}$ and $f_2 = 17.281\text{ GHz}$ [7]. The extracted model parameters obtained using the developed extraction procedure are compared with the model parameters obtained in [7]. The results are presented in Table 1.

TABLE I
EXTRACTED MODEL PARAMETERS

Model parameters	Extracted values	Extracted values in [7]
A_1		-9.546
B_1		0.9584
A	2.32554	2.3255
B	0.075793	0.0758
C	-1.32319	-1.3239
D	0.319677	0.3251
c_1	1.49915	1.5
d_1	0.098333	0.1

The simulation results for the inductor model with extracted parameters match very closely to the measurement data given in [7].

In accordance with the possibilities of the input language of the *PSpice* simulator [8], the inductance value of the inductor cannot be described as a frequency-dependent function. For this purpose, the *PSpice* frequency-dependent behavioral inductor model, based on expression (6), is realized using voltage controlled current source (VCCS) of GLAPLACE type as shown in Fig. 6.

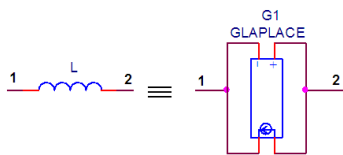


Fig. 6. *PSpice* behavioral inductor model

As the parameter “frequency” cannot be directly used in the input language of the *PSpice* simulator, it is defined using the module of the complex frequency s by the expression

$$f = \frac{M(j\omega)}{2\pi} = \frac{M(s)}{2\pi} \quad (31)$$

Using the extracted parameter values, the controlling coefficient of the VCCS G1 is in the form:

$$\frac{1/(s * (A + B * M(s) / \pi^2 + C * \log_{10}(\text{pwr}(10, (M(s) / \pi^2 - F1) / C1) / (1 + \text{pwr}(10, (M(s) / \pi^2 - F1) / C1)))) + D * \log_{10}(1 + \text{pwr}(10, (M(s) / \pi^2 - F2) / D1))}{}$$

where $\pi^2 = 2\pi$.

III. CONCLUSION

An extraction procedure has been developed to obtain the parameters of the behavioral RF planar inductor model. The procedure is based on measured *S*-parameters. It is realized using macrodefinitions in the graphical analyzer *Cadence Probe*. The *PSpice* description of the behavioral computer model is given. The simulation results using the extracted model parameters match very close to the measured data.

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REFERENCES

- [1] Gil, J., H. Shin, “A Simple Wide-Band On-Chip Inductor Model for Silicon-based RF Ics”, *IEEE Transactions on Microwave Theory and Techniques*, vol. 51, issue 9, pp. 2023–2028, Sept., 2003, ISSN: 0018-9480
- [2] Chen, Hao-Hui, Huai-Wen Zhang, Shyh-Jong Chung, Jen-Tsai Kuo, Tzung-Chi Wu, “Accurate Systematic Model-Parameter Extraction for On-Chip Spiral Inductors”, *IEEE Transactions on Electron Devices*, Lausanne, Switzerland, vol. 55, issue 11, pp. 3267–3273, Nov. 2008, ISSN: 0018-9383
- [3] Yue, C. P., C. Ryu, J. Lau, T. H. Lee, S. S. Wong, “A Physical Model for Planar Spiral Inductors on Silicon”, *Proc. IEEE Int. Electron Devices Meeting Tech. Dig.*, San Francisco, CA, Dec. 1996, pp. 155-158.
- [4] Mohan, S., M. Hershenson, S. Boyd, T.H. Lee, “Simple Accurate Expressions for Planar Spiral Inductances”, *IEEE Journal of Solid-State Circuits*, vol. 34, issue 10, pp.1419-1424, Oct. 1999, ISSN: 0018-9200
- [5] Ashby, K., B., W. C. Finley, J. J. Bastek, S. Moinian, I. A. Koullias, “High Q Inductors for Wireless Applications in a Complementary Silicon Bipolar Process”, in *Proc. Bipolar and BiCMOS Circuits and Technology Meeting*, Minneapolis, MN, USA, 1994, pp. 179-182, ISBN: 0-7803-2117-0
- [6] E. Gadjeva, V. Durev and M.Hristov, “*Matlab - Modelling, Programming and Simulations*”, Chapter 14: Analysis, model parameter extraction and optimization of planar inductors using MATLAB, Published by SCIYO, ISBN 978-953-307-125-1, www.sciyo.com, http://www.intechweb.org/
- [7] S.J. Pan, L.W. Li, W.Y. Yin, M.S. Leong, “Electrical Characteristics and Behavioral Model of Spiral Inductors On GaAs Substrate”, *Journal of Electromagnetic Waves and Applications*, *Journal of Electromagn. Waves and Appl.*, vol. 16, no. 4, 2002, pp. 443-455.
- [8] *PSpice A/D Reference Guide*, Cadence Design Systems, Inc., 2006.