Optimal experiment for determination of the thermo physical properties on materials with low thermal conductivity

Zore Angelevski[1], Cvete Dimitrieska[2], Silvana Angelevska[3] and Ivo Kuzmanov[4]

Abstract – **This paper determinate an optimal experiment for estimation of the thermo physical properties of materials with low thermal conductivity. Using the differencing on the so called out of dimensions temperature on the thermal conductivity, and with differencing on the volume of the thermal capacity, a fully defined experiment is defined. By using of the optimal experiment on the thermo physical properties, the calculated values are very close to the real one.**

Keywords – **Optimal experiment, Determine sensitivity, Sensitivity coefficient.**

I. Introduction

The experiment is a confirmation of the mathematical model. Using determinate number of experiments, the values are completely the same one from the mathematical model, although the same one represents the physical occurrence. It is noticed that with changing of the location of the measurement sensors the results from parameters that are get are different. With an aim to get more specific and accurate values of the parameters of the given conditions, in the science there is an special branch which creates a ways for planned experiment. The basic aim of this optimal experiment is to get an smallest number of performed experiments, with a high level of accuracy of the parameters (values).

The first discussion about the optimal experiments are given by Legendre (1806) and Gauss (1809), with a familiar method of the smallest quadrates. R.C. Pfaht and B.J. Mitchel (1969) continued with their work and they gave a method for same time determination on the thermo physical properties. Complex book for determination on the parameters and optimal experiment is "Parameter estimation in engineering and science" by J.V. Beck and K.J. Arnold [1] from 1977. It's contribute to engineering problems, especially with transfer of heat problems, is an base for further more improvements in grading the parameters on known or unknown systems.

[1]Zore Angelevski is with the Faculty of Technical Science at the University "St. Kliment Ohridski" in Bitola, Ivo Lola Ribar nn, Bitola 7000, Republic Macedonia.

[2]Cvete Dimitrieska is with the Faculty of Technical Science at the University "St. Kliment Ohridski" in Bitola, Ivo Lola Ribar nn, Bitola 7000, Republic Macedonia.

[3]Silvana Angelevska is with the Faculty of Technical Science at the University "St. Kliment Ohridski" in Bitola, Ivo Lola Ribar nn, Bitola 7000, Republic Macedonia.

[4]Ivo Kuzmanov is with the Faculty of Technical Science at the University "St. Kliment Ohridski" in Bitola, Ivo Lola Ribar nn, Bitola 7000, Republic Macedonia, E-mail: ivo_kuzmanov@yahoo.com

II. Mathematical Model

The partial differential formula for heat transfer is given by the following

$$\lambda \frac{\partial^2 T}{\partial x^2} = c \frac{\partial T}{\partial t}$$

With boundary conditions:

$$q = -\lambda \left.\frac{\partial T}{\partial x}\right|_{x=0}, \left.\frac{\partial T}{\partial x}\right|_{x=L} = 0, T(x,0) = T_i \qquad (1)$$

With differencing of the previously defined formulas (1), regarding the heat conductivity, you can get:

$$\lambda \frac{\partial^2 X_1}{\partial x^2} + \frac{\partial^2 T}{\partial x^2} = c \frac{\partial X_1}{\partial t}$$

With boundary conditions:

$$0 = \lambda \left.\frac{\partial X_1}{\partial x}\right|_{x=0} + \left.\frac{\partial T}{\partial x}\right|_{x=0}, \left.\frac{\partial X_1}{\partial x}\right|_{x=L} = 0, \quad X_1(x,0) = 0$$

Where:

$$X_1 = \frac{\partial T}{\partial \lambda} \qquad (2)$$

X_1 is the sensitivity coefficient (heat conductivity)

With differencing of the formulas (1) taking in consideration the volume of the heat capacity you can get:

$$\lambda \frac{\partial^2 X_2}{\partial x^2} = c \frac{\partial X_2}{\partial t} + \frac{\partial T}{\partial t}$$

With boundary conditions:

$$\left.\frac{\partial X_2}{\partial x}\right|_{x=0} = 0, \qquad \left.\frac{\partial X_2}{\partial x}\right|_{x=L} = 0, \quad X_2(x,0) = 0$$

Where:

$$X_2 = \frac{\partial T}{\partial c} \qquad (3)$$

X_2 is the sensitivity coefficient (heat conductivity)

In most of the cases, the parameters that are characteristic for the material, you can know the values of the heat transfer and the specific heat capacity of the material. For models with two parameters, the determinant D with specific parameters is given by the following formula:

$$D = C_{11}C_{22} - C_{12}^2 \quad (4)$$

The criteria for the optimal experiment (4) on models with two parameters is given with the following formula:

$$\max D = \left| C_{11}C_{22} - C_{12}^2 \right| \quad (5)$$

The criteria for optimal experiment with two parameters applies to models which are linear and nonlinear dependent from the parameters, as well as for models which are represented with partial differential formulas. The criteria for optimal experiment are given in unconditional form of the following formula:

$$\max D^+ = C_{11}^+ C_{22}^+ - (C_{12}^+)^2 \quad (6)$$

For accurate and safety during the experiment, you should put as more as you can measurement sensors on different locations:

$$C_{ij}^+ = \frac{1}{(u_{\max}^+)^2} \frac{1}{m t_n^+} \sum_{k=1}^{m} \int_0^{t_n^+} X_i^+(x_k^+, t^+) X_j^+(x_k^+, t^+) dt^+ \quad (7)$$

Where m is the number of the measurement sensors, x_k is the location of the measurement sensor and $u_{\max}^+$ is the maximal no dimensional value on the dependent variable during the experiment.

From materials with small value of heat conductivity, for the experiment it is chosen composite material. It is considered is a case where the temperature on the surface doesn't reach more that 550 ^oC and a heat flux of 20000 Wm^{-2}. The partial differential formula (1) in no dimensional form is given by the following formula:

$$\frac{\partial^2 T^+}{\partial x^{+2}} = \frac{\partial T^+}{\partial t^+}, \quad 0 < x^+ < 1, \quad t^+ > 0 \quad (8)$$

Where the boundary conditions in no dimensional form are:

$$-\frac{\partial T^+}{\partial x^+} = \begin{cases} +1 & 0 < t^+ \le t_h^+ \\ 0 & t_h^+ < t^+ \le t_n^+ \end{cases} \quad x^+ = 0$$

$$T^+ = 0, \quad x^+ = 1, \quad t^+ > 0$$

$$T^+ = 0, \quad 0 \le x^+ \le 1, \quad t^+ = 0 \quad (9)$$

The analytical solution from the formula (8) and the boundary conditions (9) are given by the following formula, for the time when the heat flux is active:

$$T^+(x^+, t^+) = \left(1 - x^+\right) - 2\sum_{n=1}^{\infty} \frac{1}{k^2} \cos(kx^+) e^{-k^2 t^+}, \quad (10)$$

$$0 < t^+ \le t_h^+$$

Where

$$T^+ = \frac{T - T_o}{q\frac{L}{\lambda}}, \quad t^+ = \frac{at}{L^2}, \quad t_h^+ = \frac{at_h}{L^2}, \quad x^+ = \frac{x}{L}, \quad k = \frac{(2n-1)\pi}{2} \quad (11)$$

The change on the no dimensional temperature according to the formula (10) for the experiment time, is given into figure 1. The time interval where the experiment is active, in the beginning should give an fully review on the change of the temperature.

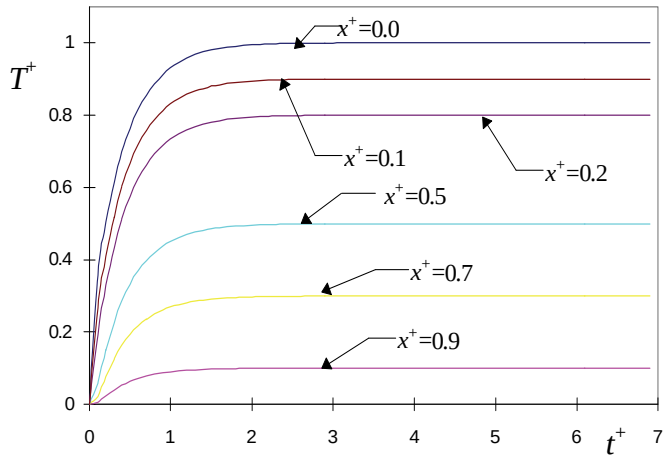

Fig. 1

From figure 1 you can see that the temperature is rising sharply, and considering the no dimensional time from t^+>2.5 is the same. The temperature is the highest on the surface where the heat flux is active, and the same one is gradually reduced with increasing of the x^+.

III. Sensitivity coefficient

With differencing of the formulas (10), considering heat conductivity, coefficient of sensitivity considering the heat conductivity, given by the following formula is given:

$$X_1^+ = \frac{\lambda}{q\frac{L}{\lambda}} \frac{\partial T}{\partial \lambda} = -(1 - x^+) + 2\sum_{n=1}^{\infty} \frac{1}{k^2} \cos(kx^+) e^{-k^2 t^+} (1 + k^2 t^+)$$

For $0 < t^+ \le t_h^+$ (12)

The change of the coefficient of the sensitivity, considering heat conductivity (12), graphically is given into figure 2.

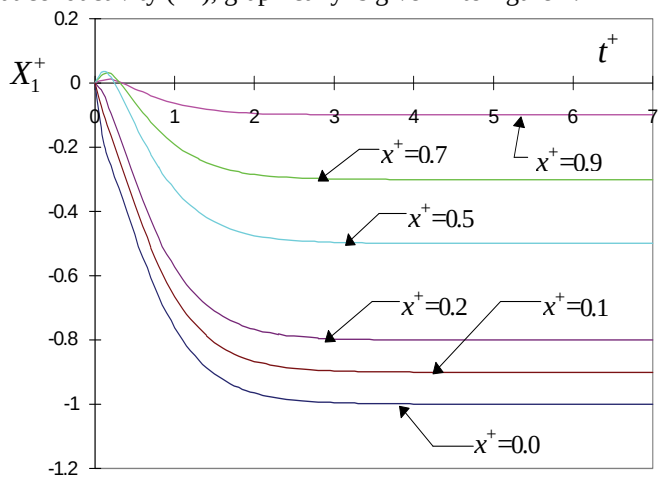

Fig. 2

From figure 2 you can see that the value of X_1^+ is the highest on the surface that is under active exposure from the heat flux, and the same one is lowest with the reduced value of the x^+.

With differencing of the formulas (10), considering the volume of the heat capacity, sensitivity coefficient of the volume heat conductivity, is given by the following formula:

$$X_2^+ = \frac{c}{q\frac{L}{\lambda}}\frac{\partial T}{\partial c} = -2\sum_{m=1}^{\infty}\cos(kx^+)t^+e^{-k^2t^+}$$

Where:

$$0 < t^+ \le t_h^+ \qquad (11)$$

The change of the coefficient of the sensitivity, considering the volume of the heat capacity (11), graphically is shown in figure 3 in addition.

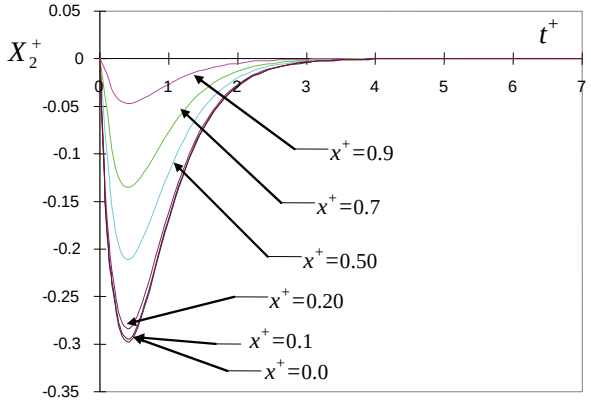

Fig. 3

The biggest value of the sensitivity coefficient, considering the volume of the heat capacity, is for x^+=0.0, and the same one it is reduced with highest value of the x^+. X_2^+ at the beginning rapidly is upping and for t^+=0.45 it gets it's highest value for all of the values of x^+, and then it's declining to the value of zero.

IV. Experiment planning

For deterring of the optimal experiment, all of the chosen possibilities for the number of the place of the thermo couples are taken under consideration.

With a final aim to see the change of the sensitivity determinate, the proposed plans for a thermo couple are given by the locations x^+=0.0; 0.1; 0.2; 0.5, 0.7; 0.9. For the proposed plans the change of the determinant is given into figure 4 in addition.

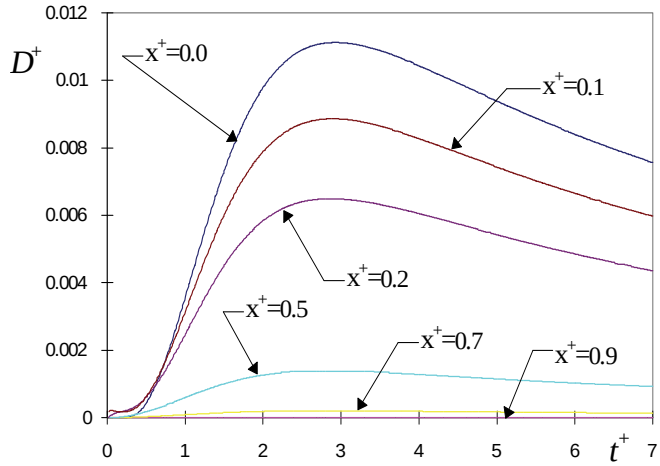

Fig. 4

The highest value of the sensitivity determinant is D^+= 0.01112374, and the same one is for the thermo couple x^+=0., for no dimensional time t^+=2.93. It's more that obvious than for the same time, D^+ has it's highest value.

Table I

Pl. num.	Number of thermo-couples	Location of the thermo-couples	t^+	Max D^+
1	1	0.0	2.93	0.0111237
2	2	0.0;0.05	2.92	0.0105791
3	3	0.;0.05;0.1	2.91	0.0100086
4	4	0.0'0.05'0.1;0.15	2.9	0.0094196
5	2	0.0;0.1	2.91	0.0100006
6	3	0.0;0.1;0.2	2.88	0.0088073

Analyzing the results, conclusion is that, the highest absolute values of the sensitivity coefficients are on the surface under the heath flux and the locations near to the same one. In table 1, are represented six plans with different number of thermocouples and their locations. The change of the sensitivity determinant for every plan is given into figure 5.

On figure 5, you can see that the plan number 1 has the highest value for sensitivity determinant. The maximal values of the sensitivity determinant for every plan and for the no dimensional times are given into figure 5, in addition of this paper.

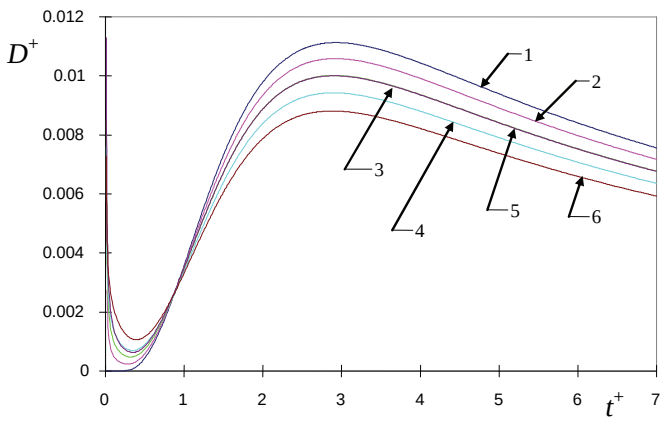

Fig.5

For the plan number 1, the sensitivity determinant has it's maximal value. This experiment is defined with:

One or more thermo couples on the surface exposed to thermo flux x^+=0.0.

Full time for the experiment is t_n^+=2.9.

The time when the heat flux is active is t_h^+=2.2.

The dimensions on the experiment tube are dependent by the following formula:

$$L = \frac{T - T_0}{0.996441 \cdot q} \frac{\lambda}{} \qquad (12)$$

Example. For determination on the thermo physical conditions of the composite material, from the literature, all of the values for the heat transfer are known, λ=0.30 W/mK, and the heat conductivity is a=2.5*10^{-7} m^2/s. The heat source gives as constant value of the heat flux of q=1.0*10^4 W/m^2. The temperature of the surface exposed to heat flux doesn't get no more that T-T_0 =300 ^{o}C. For this conditions, the value of the measurement tube is L=7.5 mm. For the calculated value, the time for the experiment is t_n=652.5 s, and the time of the heat flux (when the same one is active) is t_h=495 s.

The only condition during the experiment is to get an relevant number of measurements and data during the experiment. In this case, taking into consideration that the time of the experiment is t_n=652.5 s, the time frame for measurement can be bigger, but at the same time it will get an relevant number of data.

V. Conclusions

For determination of the true values of the given conditions for the parameters in the differential formulas, it's necessary to have an optimal experiment. The optimal experiment is getting by analysis of the sensitivity coefficient by the given parameters. All of the locations where the coefficients have the highest values are targets, and with their analysis a real plans for performance of the experiment are given. During the experiment the location and the number of measurement sensors are defined. With additional analysis the time of the experiment and the heat flux are given, and also by the boundary conditions, the dimensions on the measurement tube. This analysis is taken in non dimensional shape.

With this procedure, the experiment is fully defined, which means that with one experiment is sure that the values of the gated parameters are the real one for the given conditions.

References

[1] J.V. Beck, K.J. Arnold, "Parameter Estimation in Engineering and Science", John Wiley, 1977

[2] J.W. Beck, B. Blackwel, C.R. Clair, "Inverse Heat Conductivity", J Wiley, 1985

[3] V. V. Fedorov, "Theory of optimal experiments", Moscow, 1971

[4] Z. Angelevski, "Optimal experiment for determination of the thermo physical conditions on composite materials", PhD. dissertation, Bitola, 1998