# Trajectory Tracking Control for the Slew Motion of a Dragline Excavator

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*Abstract* – This paper proposes a trajectory tracking controller for a dragline excavator. First, a dynamic model of the excavator suitable for feedback control is developed. A desired trajectory for the slew motion is generated using fifth order polynomial function. After linearization of the nonlinear dynamic model, a linear feedback control is proposed. Simulation results illustrate the effectiveness of the proposed controller.

*Keywords* – Dragline excavator, Dynamic modelling, Trajectory planning, Feedback control.

## I. INTRODUCTION

Dragline excavators are heavy machines, widely used in the mining industry to remove overburden in open-pit coal mining.

The interest in automatic control of draglines has been increasing in recent years. This is because the advantages that the automatic control offers over a manual control include greater efficiency, operator's convenience and possibility of periodic break for repose. While the problem of automatic control of cranes has attracted a great deal of attention during the last decade [1,2], to our knowledge, the problem of controlling dragline excavators has seldom been addressed in the literature [3,4,5]. The goal is to transport the payload for a given period of time and, in the same time, to reduce the bucket swing angle. The Lagrange formalism is often used for derivation of different types of mechanical devices [6,7,8,9].

In this paper, we propose a simplified control strategy based on linearization of the dynamic model combined with trajectory tracking and linear feedback control law. The organization of the paper is as follows: In Section II, a dynamic model of the dragline excavator suitable for feedback control applications is derived. In Section III, a linear control law is designed. Section IV contains simulation results. Conclusions are presented in Section V.

### II. DYNAMIC MODEL

A schematic view of the dragline excavator is shown in Fig. 1. In order to derive a dynamic model suitable for control applications, we make the following assumptions: the bucket and the payload are considered as a point mass, the mass and stiffness of the drag and hoist ropes are neglected. In this case,

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<sup>2</sup>Plamen Petrov is with the Faculty of Mechanical Engineering, Technical University of Sofia, 8, Kl. Ohridski str., 1797 Sofia, Bulgaria (E-mail: ppetrov@tu-sofia.bg). the system has two degree of freedom and the associate generalized coordinates are

$$q = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T \in \Re^2 \tag{1}$$

where  $\theta_1$  is the slew angle, which represents the rotation of the house and the boom structures about the vertical  $z_0$  axis;  $\theta_2$  is the swing angle of the bucket, which represents the angle between the vertical plane passing through the boom axis and the plane passing through the boom axis and the bucket (Fig. 1).



Fig. 1. Schematic view of a dragline excavator

In the present paper, 4x4 matrices of homogenous transformation are used. An inertial coordinate system  $O_0x_0y_0z_0$  is assigned in the work space where the  $z_0$  axis is in the vertical direction. The  $z_1$  axis of the rotating together with the house structure  $O_1x_1y_1z_1$  coordinate system coincides with the  $z_0$  axis and the slew angle  $\theta_1$  is measured between  $x_0$  and  $x_1$  axes. The  $z_2$  axis is directed along the boom axis and the origin of the coordinate system  $O_2x_2y_2z_2$  is put at the point of the reduction of the mass of the boom A. This point is received from the intersection of the boom axis. The origin of coordinate system  $O_2x_2y_2z_2$  and axis  $z_2$  coincides with axis  $z_3$ . The swing angle  $\theta_2$  is measured between  $x_2$  and  $x_3$  axes. The parameters  $L_x$  and  $L_z$  are the distances from the point  $O_0$  to the 🖧 iCEST 2013

point  $O_2$ . The corresponding homogeneous transformation matrices which define the relative position and orientation between the adjacent coordinate systems are:

$$A_{01} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0\\ \sin\theta_1 & \cos\theta_1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}, A_{12} = \begin{bmatrix} \sin(\alpha) & 0 & \cos(\alpha) & L_X\\ 0 & 1 & 0 & 0\\ -\cos(\alpha) & 0 & \sin(\alpha) & L_Z\\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If we suppose that swing angle  $\theta_2$  is small than

$$A_{23} = \begin{bmatrix} 1 & -\theta_2 & 0 & 0 \\ \theta_2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$
 (2)

Using the transformation matrices (2), the coordinates of the points A and B with respect to  $O_0 x_0 y_0 z_0$  are

$$\begin{aligned} x_{A} &= \cos(\theta_{1})L_{X} \\ y_{A} &= \sin(\theta_{1})L_{X} \\ z_{A} &= L_{Z} \end{aligned} \tag{3}$$
$$\begin{aligned} x_{B} &= \cos(\theta_{1})L_{X} + (\cos(\theta_{1})\sin(\alpha) - \theta_{2}\sin(\theta_{1}))L \\ y_{B} &= \sin(\theta_{1})L_{X} + (\sin(\theta_{1})\sin(\alpha) + \theta_{2}\cos(\theta_{1}))L \\ z_{B} &= L_{Z} - L\cos(\alpha). \end{aligned}$$

The dynamic equations of motion of the dragline are derived using Lagrange formalism

$$\frac{d}{dt}\frac{\partial L_a}{\partial \dot{q}_i} - \frac{\partial L_a}{\partial q_i} + \frac{\partial \Phi}{\partial \dot{q}_i} = Q_i, \qquad i = 1, 2 \qquad (4)$$

where the Lagrangian  $L_a$  represents the difference between the kinetic and potential energy of the system;  $\Phi$  is the Rayleigh dissipation function;  $Q_i$  are the generalized forces associated with the generalized coordinates

The kinetic energy of the system comprises three components – the kinetic energies of the masses  $m_A$  and  $m_B$  and the kinetic energy of the rotating house. The full kinetic energy of the system is obtained as follows:

$$T = \frac{m_B}{4} \begin{pmatrix} 2\dot{\theta}_1^2 L_X^2 + 4LL_X \left( \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_1^2 \sin(\alpha) \right) + \\ + \left( (2\dot{\theta}_2^2 + \dot{\theta}_1 \dot{\theta}_2 4 \sin\alpha + (2\theta_2^2 - \cos(2\alpha) + 1)\dot{\theta}_1^2) L^2 \right) \end{pmatrix} + \\ + \frac{\dot{\theta}_1^2}{2} \left( J + m_A L_X^2 \right)$$
(5)

where  $\alpha$  is the angle between the boom and the horizontal plane; *L* is the length of line between points *A* and *B*; *m<sub>A</sub>* is the reduced to the point *A* mass of the boom; *m<sub>B</sub>* is the mass of the dragline bucket and payload; *J* is the mass moment of inertia of the rotated house.

The potential energy of the system is given as

$$U = m_B g \left( L_Z - L \cos(\alpha) \cos(\theta_2) \right).$$
(6)

In the present paper, we consider that the dissipation of the energy is present only in the rotating mechanism, supporting rotating house and is due to the resistive forces, which are proportional to the velocity. The Rayleigh dissipation function is defined by the following equation

$$\Phi = \frac{1}{2} b_1 \dot{\theta}_1^2 \tag{7}$$

where  $b_1$  is viscous damping coefficient, associated with coordinate  $\theta_1$ .

Using equations (3), (4), (5), (6) and (7), the dynamic equations of motion of the dragline are obtained in the form

$$D\ddot{q} + C\dot{q} + G = Q \tag{8}$$

After linearization, the matrices in (8) take the form:

$$D = \begin{bmatrix} (L_X^2 + 2\sin(\alpha)LL_X + (1 - \cos(2\alpha))L^2)m_B + m_A L_X^2 + J & m_B LK \\ m_B LK & m_B L^2 \end{bmatrix},$$
  
$$C = \begin{bmatrix} b_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ m_B g L \theta_2 \cos(\alpha) \end{bmatrix}, \quad Q = \begin{bmatrix} M \\ 0 \end{bmatrix}$$
(9)

 $K = L_X + L\sin(\alpha)$ 

where *M* is the control moment, acting on the rotating house. *Remark 1*: It should be noted that the matrix *D* is positive definite and the matrix  $1/2\dot{D} - C$  is skew-symmetric.

### **III. FEEDBACK CONTROL DESIGN**

In this paper, we consider the problem of position control of the dragline excavator. The goal is to transport the bucket by slew motion of the boom and to reduce the swing angle of the bucket. The desired trajectory for the slew motion of the boom is proposed in the form of a fifth order polynomial:

$$\theta_1^d(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$
(10)

where the coefficients  $a_0...a_5$  are determined by the initial and end conditions.

 $u = \tau - \ddot{\theta}_1^{d}$ 

We make the following change of coordinate

$$e_{\theta_1} = \theta_1 - \theta_1^d \tag{11}$$

(12)

and input

where

$$\tau = \frac{m_B g \theta_2 K \cos(\alpha) + M - b_1 \dot{\theta}_1}{m_A L_x^2 + J}$$
(13)

Finally, using (11)-(13), after some work, the dynamic equations of the dragline excavator can be written in error coordinate as

$$\dot{e} = A_e e + b_e u$$
.

where

$$A_{e} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{g\cos(\alpha)}{L} & 0 \end{bmatrix}, \ b_{e} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\frac{K}{L} \end{bmatrix}$$

We assume that  $e = \begin{bmatrix} e_{\dot{q}} & e_{\dot{q}} & \theta_2 & \dot{\theta}_2 \end{bmatrix}^T \in \Re^4$  are measurable.

$$T = \begin{bmatrix} \frac{L}{g\cos(\alpha)} & 0 & \frac{L^2}{Kg\cos(\alpha)} & 0 \\ 0 & \frac{L}{g\cos(\alpha)} & 0 & \frac{L^2}{Kg\cos(\alpha)} \\ 0 & 0 & -\frac{L}{K} & 0 \\ 0 & 0 & 0 & -\frac{L}{K} \end{bmatrix}$$

and

$$A_c = TA_e T^{-1} , \ b_c = Tb_e .$$

Then, the system is stabilized by linear feedback of the form

$$u = -kx_c \tag{17}$$

by using the pole placement method, where  $k = [k_1, k_2, k_3, k_4]$ and the gains  $k_i$  (i = 1, 2, 3, 4) are positive numbers.

### **IV. SIMULATION RESULTS**

Several simulations using MATLAB were carried out in order to illustrate the performance of the proposed controller. The desired trajectory of the boom slew motion is given by (10) and coefficients for the desired angle  $\pi/2$  rad and final time of 22s are  $a_0 = a_1 = a_2 = 0$ ,  $a_3 = 1.475 \cdot 10^{-3}$ ,  $a_4 = -1.006 \cdot 10^{-4}$ ,  $a_5=1.829.10^{-6}$ . The dragline excavator is tested with a reduced mass of the boom  $m_A = 150.10^3$  kg, mass of the bucket and payload  $m_B=50.10^3$  kg, distances  $L_X=40$  m and L=15 m, mass moment of inertia of the rotating house  $J=37.10^6$  kg.m<sup>2</sup>, damping coefficients  $b_1 = 150.10^3$  N.m.s,  $b_2 = 2.10^3$  N.m.s. In the first simulation, from Fig. 2, we can see the evolution in time of the swing angle  $\theta_2$  during the rotation of the boom. Fig. 3, presents the evolution in time of the movement of the boom  $\theta_1(t)$  according to desired trajectory  $\theta_1^{d}(t)$ . Fig. 4 presents the tracking error  $e_{\theta_1}$ . The results of the simulations confirm the validity of the proposed controller.

(16)

The system (14) is controllable, since

$$rank \left[ b_e \mid A_e b_e \mid A_e^2 b_e \mid A_e^3 b_e \right] = 4$$
(15)

The system (14) is transformed in control canonical form by using the transformations

 $x_c = Te$ 

where

(14)



Fig. 2. Time history of the swing angle of the bucket



Fig. 3. Time history of the boom displacement (red line), desired trajectory (blue line)



# V. CONCLUSION

In this paper, a trajectory tracking controller for a 2-DOF dragline excavator has been proposed. A dynamic model of the dragline was developed by using the Lagrange formalism.. A desired trajectory for the boom rotation was generated using a fifth order polynomial. Linear feedback was proposed for the linearized control system. Simulation results confirm the validity of the proposed controller.

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