# Effectiveness of Reed-Solomon and Convolutional Codes used in Digital Video Broadcasting

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*Abstract* – **This article presents the results of a study on the noise immunity of radio channels formed by QPSK, 8PSK, 16QAM, 32QAM and 64QAM, while using Reed-Solomon and convolutional codes. We have derived the dependency of the error probability at the decoder output on the code parameters and the error probability in the communication channel. We have analyzed the possibilities of increasing the radio channel noise immunity when using concatenated coding with Reed-Solomon and convolutional code.**

*Keywords* – **RS codes, convolutional codes, concatenated codes, BER, $E_b/N_0$.**

## I. INTRODUCTION

In the contemporary digital video broadcasting (DVB) systems it is necessary to provide Quasi-Error-Free (QEF) reception while the values of the parameter $E_b/N_0$ are relatively low and the encoding and decoding equipment is not very complex. Quasi-Error-Free means less than one uncorrected error event per hour, corresponding to $\mathrm{BER} = 10^{-10}$ to $10^{-11}$ at the input of the MPEG-2/4 demultiplexer.

Satellite and terrestrial DVB systems are particularly affected by power limitations, therefore, ruggedness against noise and interference, shall be the main design objective, rather than spectrum efficiency. To achieve high power efficiency without excessively penalizing the spectrum efficiency, the System shall use noise resistant types of modulation and effective channel codes. Very good results are achieved when Reed-Solomon and convolutional codes are combined. The convolutional code must be able to be configured flexibly, allowing the optimization of the system performance for a given satellite or terrestrial channel.

To achieve the appropriate level of error protection required for cable transmission of digital data, a FEC based on Reed-Solomon encoding shall be used. In contrast to the satellite and terrestrial DVB systems no convolutional coding shall be applied to cable transmission.

The aim of this paper is to study the impact of the parameters of Reed-Solomon, convolutional and concatenated codes on the noise immunity of radio channels formed by PSK and QAM methods.

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## II. ERROR PROBABILITY AFTER RS DECODING

The Reed-Solomon code belongs to the group of block codes, which demand a preliminary splitting of the information symbols into packets of a $K$ symbols length. Each symbol contains $n$ number of bits, where n can be every positive number with a value higher than 2. If $N$ denotes the total number of the encoded symbols in a packet and $T$ denotes the number of repairable symbol errors, the RS code can be represented basically as follows: RS($N, K, T$). In the most commonly used RS codes $N = 2^n - 1$, $K = 2^n - 1 - 2T$, and $2T = N - K$ is the number of the error protection symbols, or checksum.

The symbol error probability after RS decoding $P_s$ is related to the symbol error probability in the communication channel $p_s$ by the following dependency [1] :

$$P_s = \frac{1}{N}\sum_{i=\mathrm{T}+1}^{N} i \cdot \binom{N}{i} \cdot p_s^i \cdot (1-p_s)^{N-i} \tag{1}$$

The parameter $p_s$ depends on the energy-per-bit to noise power density radio and the type of modulation. In order to define its values, we can use the following dependencies [6]:

- when transmitting M-PSK signals

$$p_s = erfc\left[\left(\sqrt{\left(\log_2 M\right) \cdot \frac{E_b}{N_0}}\right) \cdot \sin\left(\frac{\pi}{M}\right)\right] \tag{2}$$

- when transmitting M-QAM signals

$$p_s \approx 2\left(1 - \frac{1}{\sqrt{M}}\right) \cdot erfc\left[\sqrt{\frac{3 \cdot \log_2 M}{2 \cdot (M-1)} \cdot \frac{E_b}{N_0}}\right], \tag{3}$$

when $m = 10\log_2 M$ is an even number and

$$p_s \le 1 - \left[1 - erfc\left(\sqrt{\frac{3 \cdot \log_2 M}{2 \cdot (M-1)} \cdot \frac{E_b}{N_0}}\right)\right]^2, \tag{4}$$

when $m$ is an odd number.

The values of the complementary function of the error are calculated by the formula

$$erfc(x) \approx \frac{1}{x\sqrt{\pi}} \cdot \exp\left(-x^2\right). \tag{5}$$

In order to evaluate the quality of the received digital information, we usually use the parameter bit error probability, respectively BER. The relation between bit error probability ($p_b$) and symbol error probability ($p_s$) in the communication channel is given by the expression

$$p_b = \frac{p_s}{\log_2 M} \tag{6}$$

After the RS decoder the dependency between bit error probability $P_b$ and symbol error probability $P_s$ is the following

$$P_b = \frac{2^{n-1}}{2^n - 1} P_s \, . \tag{7}$$

In the graphic dependencies represented below, instead of the bit error probability we have used its statistical evaluation BER.

## III. Impact of the Parameters of the RS Code on the Radio Channel Noise Immunity

The effectiveness of the Reed-Solomon code is higher when the interference duration is much lower than the packet duration. In order to provide this condition, it is necessary to increase the total number of encoded symbols in the packet. It is easy to prove that noise immunity becomes higher when the packet size becomes greater. This is evident from the curve family shown in Fig. 1 where the code rate is constant 0.92, while the size of the packets changes from 64 symbols to 512 symbols.

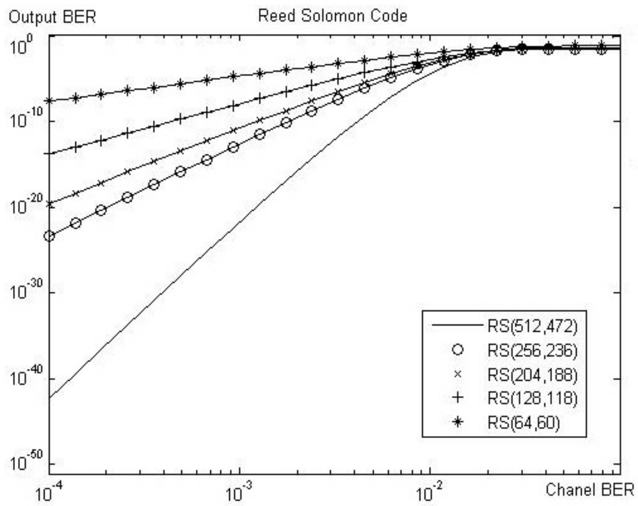


Fig. 1. Output BER versus packets length

The impact of the RS code rate on the noise immunity of the radio channel can be evaluated by using the dependencies shown in Fig. 2. They apply in the case where the size of the packet is constant ($N = 204$) and the number of information symbols changes from $K$ = 180 to $K$ = 196. It is evident that when the checksum increases, the radio channel noise immunity becomes better, but this is accompanied by a shortening of the channel effective bandwidth, which leads to a lowering of the information rate.

Fig. 3 shows results from a simulation study of the impact of the RS code rate on the noise immunity of radio channels

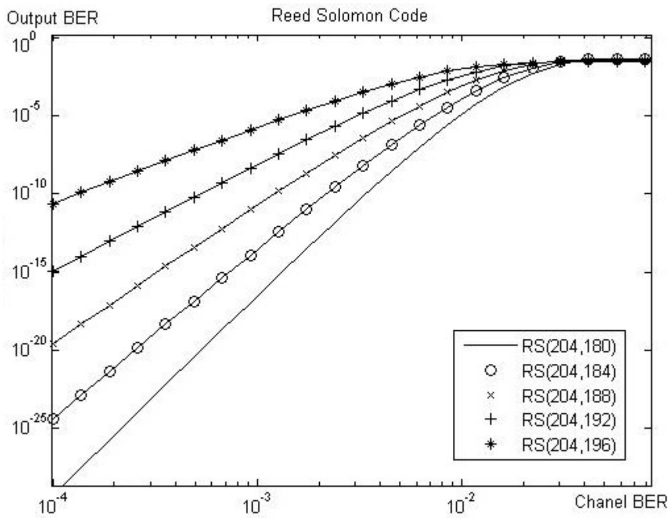


Fig. 2. Output BER versus code rate

formed by different manipulations. In the study we have taken that $N = 204$, and the maximum acceptable BER value is $10^{-11}$. It is evident that there are three characteristic fields. In the first one ($R_{\mathrm{RS}} \leq 0.4$) the increasing of the code rate leads to higher radio channel noise immunity, in the second one ($0.4 \leq R_{\mathrm{RS}} \leq 0.7$) the change of the code rate has no essential impact on the noise immunity, and in the third one ($R_{\mathrm{RS}} \geq 0.7$) when the code rate is increased, the radio channel noise

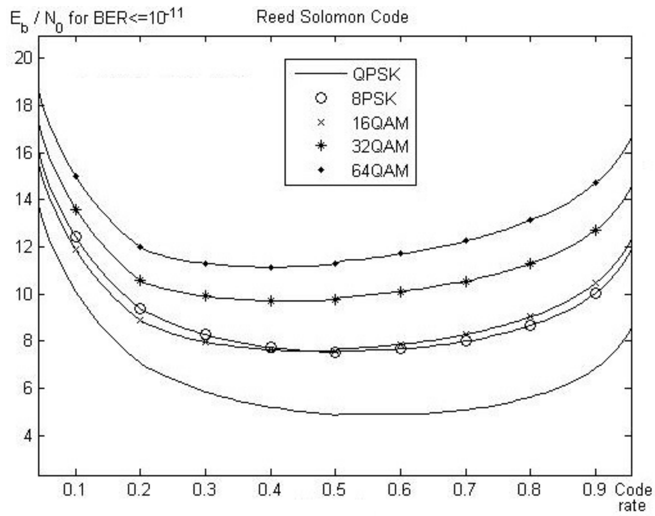


Fig. 3. Noise immunity of radio channels formed by QPSK, 8PSK, 16QAM, 32QAM and 64QAM with different RS code rates

immunity becomes lower.

Two factors have impact on the radio channel noise immunity. The first one is the checksum size: when it increases, the radio channel noise immunity becomes higher. The second factor is the bit transmission energy: it becomes lower when the checksum increases and, as a result, the demodulator makes more errors. When code rates are low, the second factor has greater impact, which results in noise immunity deterioration. When code rates are high, the first factor has greater impact and noise immunity becomes lower as a result of the smaller checksum. In the middle section of the graphics $(0.4 \le R_{RS} \le 0.7)$ the two factors have commensurable impact and, respectively, the noise immunity remains relatively constant.

The dependencies shown in Fig. 3 make it possible to define the optimal RS code rates when using different methods for radio channel forming. In order to do this, two factors have to be taken into account: the radio channel noise immunity and the bandwidth effectiveness, which decreases when the code rate becomes lower.

## IV. RADIO CHANNEL NOISE IMMUNITY WHEN THE CONVOLUTIONAL CODE PARAMETERS CHANGE

The convolutional encoding devices have memory, and their output is not only the function of the current symbol but also of the previous $l-1$ input symbols (the length of each symbol is n bits), where $l$ is the constraint length. In digital TV broadcasting, punctured convolutional codes based on a rate 1/2 convolutional code with constraint length $l = 7$ have become established. The code generator polynomials of these codes are as follows:

$$G_x = 1 + x^1 + x^2 + x^3 + x^6 = 171_{\mathrm{OCT}}\ ,$$
$$G_y = 1 + x^2 + x^3 + x^5 + x^6 = 133_{\mathrm{OCT}}\ . \qquad (8)$$

In satellite and terrestrial DVB systems are used convolutional codes with rate $R_C$ = 1/2, 2/3, 3/4, 5/6 and 7/8.

One of the most common and applied methods for convolutional code decoding is the Viterbi algorithm. The upper limit of bit error probability (BER) after Viterbi decoding can be defined by the formula [4]:

$$P_b = \frac{1}{k_0} \cdot \sum_{d=d_f}^{\infty} w(d) \cdot p(d) \qquad (9)$$

where $w(d)$ is the number of paths of Hamming distance $d$ from the all-zero path, $p(d)$ – the probability of choosing an incorrect path, which is different from the correct path in terms of $d$ positions, and $d_f$ – the free distance of the used code.

The values of $w(d)$ and $d_f$ are given [2], [4] and [5], and for the derivation of $p(d)$ are used the expressions from (2) to (6), in which the argument of the error complementary function is substituted by $x^* = x \cdot d \cdot R_C$. As a result, the following formulae for the calculation of $p(d)$ are derived:

- when QPSK signals are transmitted

$$p(d) \approx 0.5 \cdot erfc\left(\sqrt{R_C \cdot d \cdot E_b / N_0}\right) \qquad (10)$$

- when 8PSK signals are transmitted

$$p(d) \approx 0.33 \cdot erfc\left(0.66\sqrt{R_C \cdot d \cdot E_b / N_0}\right) \qquad (11)$$

- when 16QAM signals are transmitted

$$p(d) \approx 0.38 \cdot erfc\left(0.63\sqrt{R_C \cdot d \cdot E_b / N_0}\right) \qquad (12)$$

- when 32QAM signals are transmitted

$$p(d) \approx 0.2\left\{1 - \left[1 - erfc\left(0.49\sqrt{R_C \cdot d \cdot E_b / N_0}\right)\right]^2\right\} \qquad (13)$$

- when 64QAM signals are transmitted

$$p(d) \approx 0.29 \cdot erfc\left(0.38\sqrt{R_C \cdot d \cdot E_b / N_0}\right) \qquad (14)$$

The impact of the code rate on the radio channel noise immunity can be observed in Fig. 4. It shows a curve family for different convolutional code rates with generator polynomials $G_x$=171 and $G_y$=133. It is evident that when the code rate is increased, the noise immunity decreases.

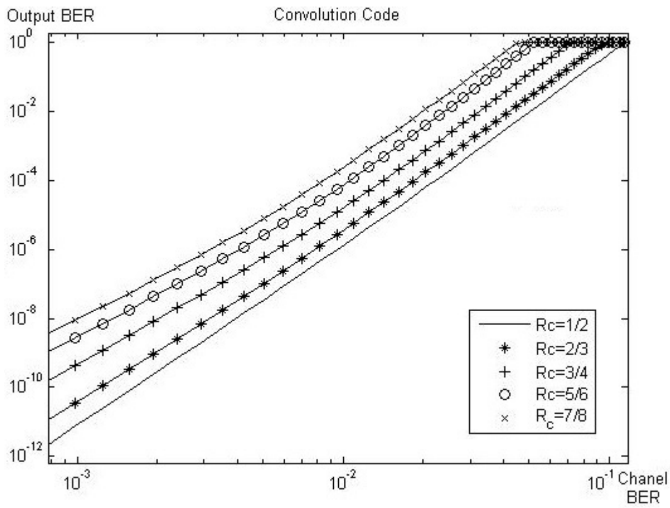


Fig. 4. Dependencies of BER after Viterbi decoding on the convolutional code rate

The results of this study show that when the probability of bit error occurrence in the communication channel is high, the convolutional code does not manage to correct the error bits and even increases their number. In the borderline case it may even occur that all bits after the decoder are error bits.

The impact of the code constraint length $l$ on the radio channel noise immunity can be analyzed by studying convolutional codes with different generator polynomials. Fig. 5 shows the results of such a study, which are relevant to convolutional codes with rate $R_C$ = 1/2. It is evident that when $l$ increases, the channel noise immunity becomes higher.

## V. EFFECTIVENESS EVALUATION OF DOUBLE-CASCADE RS/CONVOLUTIONAL ENCODING APPLICATION

The material we have considered up to here points to the fact that in order to make the radio channel noise immunity higher, it is necessary to increase the length of the packets undergoing RS encoding and the constraint length of the convolutional code. However, this leads to a complication of the encoding and decoding devices and in this respect the impact of the convolutional code constraint length is stronger.

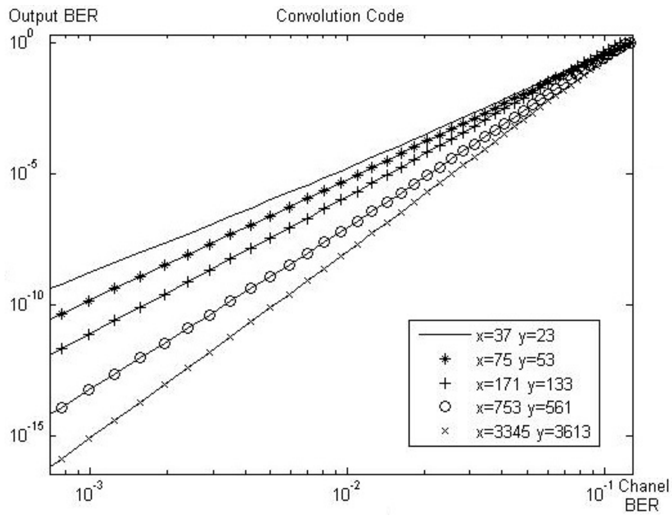


Fig. 5. Noise immunity of the radio channel for convolutional codes with different generator polynomials

Interesting for the engineering practice are the combinations of RS and convolutional codes, where the concatenated code rate remains constant. Such codes can be derived by combining convolutional codes with great constraint length and Reed-Solomon codes with small packet length or by combining convolutional codes with small constraint length and Reed-Solomon codes with great packet length. In this case the increase in the complexity of one device leads to the decrease in the complexity of the other one.

In Fig. 6 it is evident that when there is a great error probability in the communication channel, the choice of a convolutional code with great constraint length and small packets length of the Reed-Solomon code makes possible the achievement of better noise immunity. When there is a low probability of error in the communication channel, the result is better when a convolutional code with a small constraint length is combined with a Reed-Solomon code with great packets length.

When choosing the adequate concatenated codes, it is necessary to take several facts into consideration: the BER set at the receiver output, the bit error probability in the communication channel, the complexity of the encoding and decoding equipment.

## VI. Conclusion

In this paper we have presented results from simulation studies of the dependency of BER at the receiver output on the parameters of the RS and convolutional codes used, and these results are not only the basis of DVB systems but also of other telecommunication systems using these codes. The analytical and graphic dependencies make it possible, by using a set BER after channel decoder, type of modulation and carrier-to-noise ratio at the receiver input, to define the parameters of RS, convolution and concatenated codes.

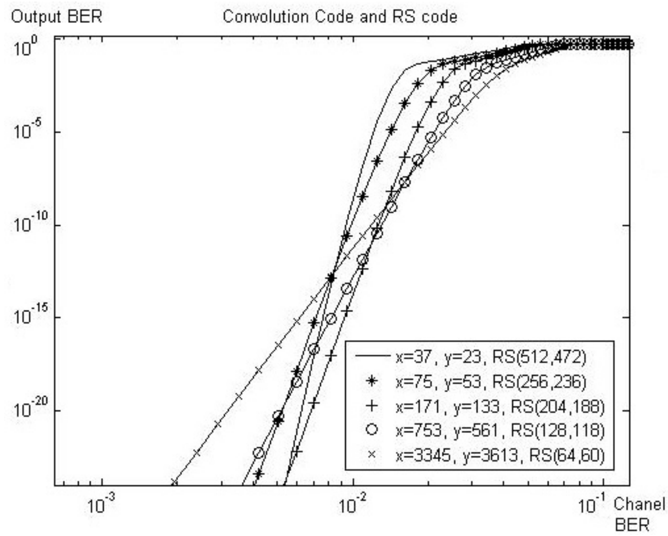


Fig. 6. Impact of packets length and constraint length on the noise immunity of radio channels formed by QPSK manipulation

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