Synchronization in Radio Communication Systems with Pseudo Random Restructuring Operation

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Abstract – **On the basis of Markov theory for optimal nonlinear filtration a problem is set and being researched for the estimation and maintenance of autonomous synchronization in the system for radio communication among remote moving objects.**

Keywords – **radio communication systems, synchronization.**

The analysis of systems using pseudo-mode with frequency hopping has specific characteristics determined by the need for joint evaluation of discrete and continuous processes. Estimation error of filtration can be carried out in accordance with methods of Markov theory [1].

If applied common description of the useful signal in the form of known function of a discrete data parameter $d(t)$ and random delay $\tau(t)$ can be written as follows:

$$s(t,dt) = s[(t-\tau(t)), d(t-\tau(t))] \quad (1)$$

The transmitted signal in the analyzed case of the proposed work is the sum of n elementary signals, each of which with duration of T_s seconds, i.e. $s(t)=\sum_{k=0}^{\infty} s_k(t-kT_s)$.

Then in the presence of random delays, adopted useful signal is:

$$s(t,\tau(t)) = s(t-\tau(t)) = \sum_{k=0}^{\infty} s_k(t-kT_s-\tau(t)).$$

The values of the information parameter of the respective clock intervals T_s form a simple homogeneous Markov chain d_k, $k=0,1,...$ with n states. The accidental time delay, which is a consequence of the relative movement between the receiver and transmitter, in the general case can be considered as a first component of a diffusion Markov process $\lambda(t)$, i.e. $\tau(t)=\lambda_1(t)$. In the theory of nonlinear filtering [1], process $\lambda(t)$ satisfies the system stochastic differential equations containing a' priori information about the signal:

$$\frac{\partial\lambda_i(t)}{\partial(t)} = K_i(\lambda,t)+n_i(t), \quad i=\overline{1,m} \quad (2)$$

Here $\lambda_i=\lambda_i(t)$ are components of multidimensional Markov random vector $\lambda(t)$, $n_i(t)$ is independent of the white noise.

A' priori probabilistic density $W_{pr}(\lambda,t)$ of random vector $\lambda(t)$ is described by the equation of Fokker-Planck-Kolmogorov:

$$\frac{\partial W_{pr}(\lambda,t)}{\partial t} = -\sum_{i=1}^{m}\frac{\partial}{\partial\lambda}\left[K_i(\lambda,t)W_{pr}(\lambda,t)\right] + \frac{1}{4}\sum_{i,j=1}^{m}\frac{\partial^2}{\partial\lambda_i\partial\lambda_j}\left[N_{ij}W_{pr}(\lambda,t)\right] \equiv L_{pr}W_{pr}(\lambda,t), \quad (3)$$

where K_i is a deterministic function (transmission coefficient).

Equation (3) characterizes the behaviour of the probabilistic density $W_{pr}(\lambda,t)$ at any point in time. All available information on the parameters of the useful signal is contained in the final a' posteriori probabilistic density $W(\lambda_t,t)=W_{pr}(\lambda_t,t\,|\,r_o^i)$ of the vector $\lambda(t)$, which satisfies the following integro differential equation:

$$\frac{\partial W(\lambda_t,t)}{\partial t} = L_{pr}W(\lambda_t,t)+F(\lambda_t,t) - \langle F(\lambda_t,t)\rangle W(\lambda_t,t), \quad (4)$$

wherein:

$$F(\lambda_t,t) = \frac{1}{N_0}\left[2r(t)s(t,\lambda)-s^2(t,\lambda)\right],$$

$$\langle F(\lambda_t,t)\rangle = \int_{-\infty}^{\infty} F(\lambda_t,t)W(\lambda_t,t)d\lambda_t,$$

r_0^t is adopted to implement interval $[0,t]$, N_0 – one-sided spectral density of white noise.

Equation (4) describes the evolution of a' posteriori probabilistic density. At the initial moment of time, a' posteriori density coincides with the a' priori. In the process of monitoring the implementation $r(t)$ is accumulating information about the filtered parameters and a' posteriori probabilistic density is concentrated in the vicinity of the assessed values of the parameters of the useful signal. Solving equation (4), and its modeling, is a complex task. Therefore, for practical purposes is assumed, that the a' posteriori density $W(\lambda_t,t)$ at sufficiently high signal/noise ratio is close to normal. Then it is enough to estimate the value $\lambda_i^*(t)\equiv\lambda_i^*$ of the components of a vector λ_t and cumulants $h_{ij}(t)=h_{ij}$ (Gaussian approximation in the theory of nonlinear filtration [1]), satisfying the following equations:

$$\frac{d\lambda_i^*}{dt} = K_i(\lambda^*)+\sum_{j=1}^{m}h_{ij}\frac{\partial F(\lambda^*,t)}{\partial\lambda_j^*} \quad (5)$$

$$\frac{dh_{ij}}{dt} = \frac{1}{2}N_{ij}+\sum_{\mu=1}^{m}\left[\frac{\partial K_i(\lambda^*)}{\partial\lambda_\mu^*}h_{\mu j}+\frac{\partial K_i(\lambda^*)}{\partial\lambda_\mu^*}h_{i\mu}\right] + \sum_{\mu,\nu=1}^{m}h_{i\mu}h_{\nu j}\frac{\partial^2 F(\lambda^*,t)}{\partial\lambda_\mu^*\partial\lambda_\nu^*} \quad (6)$$

Equations (2) and (6) are equations for quasi optimal (quasi-linear) filtration, in accordance with which it can construct a device for filtration. These devices ensure minimum errors in filtration, characterized by the dispersions

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$\sigma_a^2 = h_{aa}(t)$ and the correlation moments $h_{ij}(t)$. In this case $\frac{dh_{ij}}{dt} = 0$ and by the function $F(\lambda^*, t)$ is passing to its average value at the time $F(\lambda^*)$. As a result, the equations (5), (6) can be written in the form:

$$\frac{\partial \lambda_i^*}{\partial t} = K_i(\lambda^*) + \sum_{j=1}^{m} \bar{h}_{ij} F_{ij} F_j(\lambda^*, t) \quad (7)$$

$$\frac{1}{2} N_{ij} + \sum_{\mu=1}^{m} \left[\frac{\partial K_i(\lambda^*)}{\partial \lambda_\mu^*} \bar{h}_{\mu j} + \frac{\partial K_i(\lambda^*)}{\partial \lambda_\mu^*} \bar{h}_{i\mu} \right] + \sum_{\mu,\nu=1}^{m} \bar{F}_{\mu\nu}(\lambda^*) \bar{h}_{i\mu} \bar{h}_{\nu j} = 0 \quad (8)$$

Here: $F_j(\lambda^*, t) = \frac{\partial F(\lambda^*, t)}{\partial \lambda_j^*}$, $\bar{F}_{\mu\nu}(\lambda^*) = \frac{\partial^2 \bar{F}(\lambda^*)}{\partial \lambda_\mu^* \partial \lambda_\nu}$,

$\sigma_j^2 = h_{ij}$ is independent of t stationary value of the a' posteriori dispersion of estimation of parameter $\lambda_j(t)$; h_{ij} characterized the degree of correlation of the parameter estimates, respectively to $\lambda_i(t)$ and $\lambda_j(t)$ at steady-state.

In order compensation of the delay $\tau(t)$ in the transmission medium of the signal $s(t)$, the same should be broadcast with overtaken in time $y(t)$, i.e. be of the form: $s_y(t) = s[t + y(t)]$. In case of delay $\tau(t)$ the useful signal at the input of the receiver will be:

$$s_y[t - \tau(t)] = s_y[t - \tau(t) + y(t - \tau(t))] \quad (9)$$

Problem whose solution is the purpose of this study is to determine the value of $y(t)$, at which is providing minimum mean square value of the offset $\varepsilon(\tau)$ in the time of reception of the signals at the input of the receiver in case of accidental delay $\tau(t)$, i.e.:

$$\varepsilon(t) = \tau(t) - y[t - \tau(t)] \quad (10)$$

For the determination of $y(t)$ can be used all the current information about the random delay, which is contained in the realized oscillation $r(t)$ for the interval $[0, t]$ at the input of receiver, whereupon this oscillation is the sum of the useful signal and the noise:

$$r(t) = s_y[t - \tau(t)] + n(t).$$

The signal emitted by the transmitter in random moment of time t_0, enters at the input of the receiver in a channel with a random delay time point t_1, so that the equality obviously is met: $t_0 = t_1 - \tau(t)$. The raised problem can be reduced, so that based on the monitoring of implementation $r(t)$ until the time of transmission of the signal $r_0^{t_0} = \{r(t), 0 \le t \le t_0\}$ to determine the overtaking $y(t_0)$, which is providing minimum mean square value of the offset $\varepsilon(t_1)$ of the signal, taken at the moment of time t_1. As is known, the optimal mean square assessment coincides with the conditional mathematical expectation, i.e.:

$$y(t_0) = M\left\{\tau(t_1) \middle| r_o^{t_o}\right\} = \int_{-\infty}^{\infty} \tau p_1(\tau | t_0) d\tau,\ p_1(\tau | t_0) = p\left\{\tau(t_1) \middle| r_o^{t_o}\right\} \quad (11)$$

is a' posteriori, i.e. the conditional at monitoring of implementation $r_0^{t_0}$ density of the probabilities of the random process $\tau(t)$ at the moment t_1. At a fixed time of signal broadcasting t_0, the time of its occurrence at the input of the receiver t_1, determining according to equality (10) is random. To avoid examining of the process in random moments of time may be introduced process:

$$\tau_1(t_0) = \tau(t_1). \quad (12)$$

From (10) it follows that:

$$\tau_1(t_0) = \tau[t_0 + \tau(t_1)] = \tau[t_0 + \tau_1(t_0)]. \quad (13)$$

So $p_1(\tau / t_0)$ is the current a posteriori probabilistic density of the process $\tau_1(t)$: $p_1(\tau | t_0) = p\left\{\tau(t_1) \middle| r_o^{t_0}\right\} = p\left\{\tau_1(t_0) \middle| r_0^{t_0}\right\}$.

In its physical sense $\tau_1(t)$ is the magnitude of delaying of emitted signal at the moment t.

Ratio (12) is a transcendental equation on the basis of which it is possible to determine the $\tau_1(t)$ at given process. From formula (13) can be obtained an equation defining the relationship between $p_1(\tau | t)$ and $p(\tau, l | t) = p\left\{\tau(t + l) \middle| r_o^{t_0}\right\}$, i.e. with a' posteriori probabilistic density of the random delay at some point of the time $\tau(t + l)$. When l considering as a random variable with probabilistic density $p(l)$, and $\tau(t + l)$ as a function of this value, then on the basis of (12) is satisfied:

$$p\left\{\tau_1(t) = \tau \middle| r_0^{t_0}\right\} = \int_{-\infty}^{\infty} p\left\{\tau(t + l) = \tau \middle| r_0^{t_0}\right\} p(l) dl. \quad (14)$$

From equation (3.13) follows that: $l = \tau_1(t)$, i.e. $p(l) = p_1\{l | t\}$, from where follows the ratio determining $p_1(\tau, t)$ at a set probabilistic density $p(\tau, l | t)$:

$$p_1(\tau / t) = \int_{-\infty}^{\infty} p(\tau, l | t) p_1(l | t) dl. \quad (15)$$

Equation (15) connects the probabilistic characteristics of the process $\tau_1(t)$ with the characteristics of process $\tau(t)$. The algorithm for calculation of the $p(\tau, l | t)$, follows from the results of the theory of optimal nonlinear filtration. The accidental delay may accepts non-negative values, i.e. $\tau_1(t) \ge 0$, $p_1(t | \tau) = 0$ to $\tau < 0$. Therefore, in equation (15) is only used $p(\tau, l | t)$ to $l \ge 0$, i.e. only the extrapolated density of probability. Monitoring $r(t)$ is determined by the formula (9).

Therefore, the determination of the a' posteriori probability density $p(\tau, l | t)$ based on the monitoring of r^{t_0} is solvable task of Markov theory for optimal nonlinear filtration.

References

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