Investigation of second-order digital filter structures having low sensitivity to parasitic effects

Maria Nenova[1]

Abstract –**The errors occurring after the process of representation of digital filter coefficients with a finite word length registers are sometimes crucial for the communication equipment. In the present paper an investigation and analysis of structures having improved low sensitivity characteristics to parasitic effects is performed. The different types of criterion for sensitivity measure are presented. Then according to them is performed analysis of a low sensitivity structures. The impact of the pole distribution after quantization is shown for different number of bits.**

Keywords – **sensitivity, quantization effects, IIR (infinite impulse response) low sensitivity structure.**

I. Introduction

The evaluation of sensitivity is the main factor to measure the effects occurring after the process of quantization. The analogue or digital circuit's characteristics depend on the values of their components. The characteristics sometimes are different from the specified value, because the elements are not ideal. They can also vary over time and environmental conditions.

The sensitivity is one of the most important features investigated when digital filters are implemented. If the digital filter structure is sensitive to parasitic effects then it worsens the characteristics.

The researchers aim for the past few decades is the reducing of quantization errors. One of the first structures published in the literature [1] with low pass-band sensitivity is based on all-pass sections. The desire in general is the structure having low sensitivity also in the band-stop. One more section with improved low-sensitivity is discussed in [2] which show very low sensitivity for transfer function poles.

Usually the infinite impulse response (IIR) filters are very sensitive to the quantization effects. The recursive also show large round off noise. One of the big problems is the behavior of the filters with poles located near the z=1. In this area the parasitic effects become more severe.

The authors in [3] developed a new second-order digital filter structure with good low sensitivity properties. Their section has a low sensitivity for values for the specific pole radius [0.8 – 0.99].

In [4] are presented three different structures which in the present paper are investigated and compared according to the effects arising after the quantization process. All those structures realize different transfer function (TF).

The first one called BQ1 is based on the lattice structure. The structure does not realize polynomial LP and HP output. Later the same author presented one more second-order section - BQ2 good for realizations in the low frequency band, which realizes all possible second-order transfer functions. A third section called BQ3 presented in [5] also has good sensitivity properties and provides full number of outputs.

The adaptive realization of the first two structures is presented in [6] and the third one is in [7].

During the years many authors have shown interesting and very useful implementations of such types of low sensitivity filters is presented in [8], [9]. In [8] is presented the design and implementation of allpass filter sections with low sensitivity near the area of z=1 with high accuracy in the process of design.

The application of low sensitivity second-order filter sections is proposed in [9], where the authors uses such type filters in the sound model for tracing or extraction of sound sources.

II. Sensitivity characteristics – general overview

The sensitivity of the frequency characteristics – magnitude and phase response, group delay time are functions of the frequency, which must be considered when comparing different schemes. The sensitivity is different in the transition band and in the passband of one and the same structure.

This work considers infinite impulse response (IIR) digital filters with poles near the unit circle and deals with coefficient sensitivity as a criterion for comparing filter structures with each other.

Magnitude and phase sensitivity functions have to be considered in a reasonably wide range when different recursive digital filter realizations are compared to each other.

The worst-case sensitivity (WS) is also called maximum sensitivity can be evaluated according to [10] as:

$$WS_{a_i}^{H(e^{jw})} = \sum_i \left| S_{a_i}^{H(e^{jw})} \right|, \qquad (1)$$

In the WS (1) criterion are taken into account only the absolute values of the partial sensitivities. This type of sensitivity of the three sections is investigated in this paper.

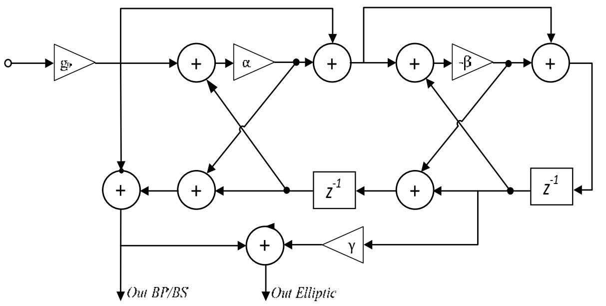
III. Second-order structures

[1]Maria Nenova is with the Faculty of Telecommunications at Technical University of Sofia, 8 Kl. Ohridski Blvd, Sofia 1000, Bulgaria, E-mail: mvn@tu-sofia.bg.

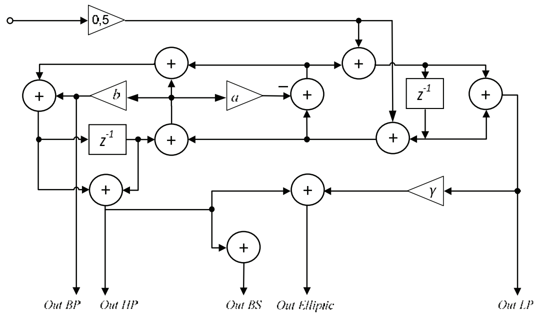
Figure 1 - a) b) c) shows three examined in this paper biquadratic sections. They realize on their outputs full number of transfer function - LP, HP, BP and BS. The section BQ1 realizes elliptic LP/HP TF and for the BP and BS the equations are [4]:

$$H_{BP}(z) = \frac{0.5(1-\alpha)(1-z^{-2})}{1-\beta(1+\alpha)z^{-1}+z^{-2}} \tag{2}$$

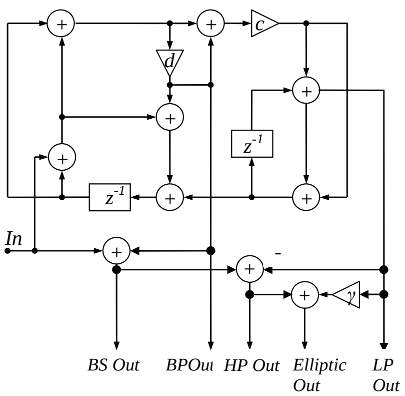
$$H_{BS}(z) = \frac{(1+\alpha)(1-2\beta z^{-1}+z^{-2})}{1-\beta(1+\alpha)z^{-1}+z^{-2}} \tag{3}$$



(a)BQ1



(b) BQ2



(c) BQ3

Fig. 1. Biquad sections

The BQ1 is based on lattice structure Fig.1. It is presented in [11], and in [12] the authors approved that this biquad is not universal, because of the lack of polynomial low and high frequency outputs.

This is the main reason for development of the next two sections – BQ2 и BQ3 [11], [12], [13] и [5]. Those structures are the basis for cascade realizations of identical second-order structures.

The BQ2 – Fig. 1 b) presents a full amount of possible realizations of transfer functions [4].

$$H_{BP}(z) = \frac{-0.5b(1-z^{-2})}{1+(-2+b+2a)z^{-1}+(1-b)z^{-2}};$$
$$H_{BS} = \frac{0.5(2-b)\left[1+\frac{-2+2a+b}{2-b}z^{-1}+z^{-2}\right]}{1+(-2+b+2a)z^{-1}+(1-b)z^{-2}} \tag{4}$$

The BQ3 section releases on the outputs all possible types of second-order TFs. In addition, as one of its biggest advantages over other implementations considered is the realization of Band Pass and Notch outputs independently (4).

This feature of the section gives possibility of extraction or elimination of different frequencies.

The structure depicted on Fig.1c). has only two multipliers denoted with *d* and *c* (the small number of multipliers leads to decreasing in parasitic effects, arising due to the quantization of the coefficients). The BP and BS TFs are:

$$H_{BS}(z) = \frac{(1-d)\left[1-2(1-2c)z^{-1}+z^{-2}\right]}{1+(-2+4c+2d-4cd)z^{-1}+(1-2d)z^{-2}} \tag{5}$$

$$H_{BP}(z) = \frac{d(1-z^{-2})}{1+(-2+4c+2d-4cd)z^{-1}+(1-2d)z^{-2}} \tag{6}$$

All tree structures have almost the same complexity, because they have two delays and two multipliers for the polynomial TFs and three for the elliptic one. The adders are 8 for BQ1 and 10 for BQ2 and BQ3. One of the biggest advantages of the three investigated structures is the possibility of independent tuning of the pole radii r_p, the angle of the poles Θ_z, Θ_p and zeros of the elliptic TF, and the central frequency and bandpass of the BP and BS TFs [4].

Another advantage of BQ1 and BQ3 is the tuning of r_p, which is performed by coefficients α for BQ1 and d for BQ3, will not affect Θ_z. Setting the radius of the poles is mostly used in bandpass and notch realizations and less frequently in elliptical ones.

The turning into adaptive of these three sections is desirable to be implement without any changes in the radius of the poles, because this can lead to possible instability.

IV. Pole location density

An indirect criterion for the sensitivity evaluation of a transfer function in a particular frequency band is the pole location density. In the corresponding area of the unit circle for a given word-length is different the amount of possible pole locations.

The poles in the denominator of the TF have the possibility to take place only in a limited number of points within the unit circle. This is due to the position of the coefficients of the TF and the limitation in the registers in which the multipliers are stored. This two limits leads to non-desirable effects. It is important to be noted that as the number of the possible pole locations increases, as uniformly they are distributed. The quantization error of the TF coefficients will become smaller when the distribution is uniform. If the size of the register increases then higher pole density will be observed. The pole location for the BQ3 is presented in simulation results.

V. Simulation Results

When a quantization of the coefficients is performed, the coefficients of the TF become with a finite and shorter word length. Then the poles of the TF can be positioned on a certain finite number of places. If a binary word with length *B* in bits is implemented, then in each quadrant of the unit circle will result [14]:

$$L_{\max} = (2^{B} - 1)^{2} \qquad (7)$$

Number of poles for a TF of second-order with coefficients in the denominator $a_1 \neq 0$ and $a_2 \neq 0$. The number of the positions increases as the error due to the quantization decreases, instead of the non uniform placement.

Digital filters give the possibility of changes in the pole density for a specific frequency area instead of another area. The number of poles remains constant.

The poles sensitivities of the BQ1, BQ2 and BQ3 are compared.

On Fig. 2 can be observed the result of simulations performed on digital filter structure BQ3. The density of poles situated near the unit circle increases with the increase of number of bits for quantization.

The control of pole location is often employed technique in variable and adaptive filtering. In the areas where the poles are densest can be performed precise tuning of the frequency. In this type of areas the sensitivity is low.

The analysis of sensitivity is important because structures having higher density and more uniform distribution of poles for a given transfer function and the same word-length can be compared. The low sensitivity leads to structures represented with shorter word-length. This will also decrease the calculation complexity.

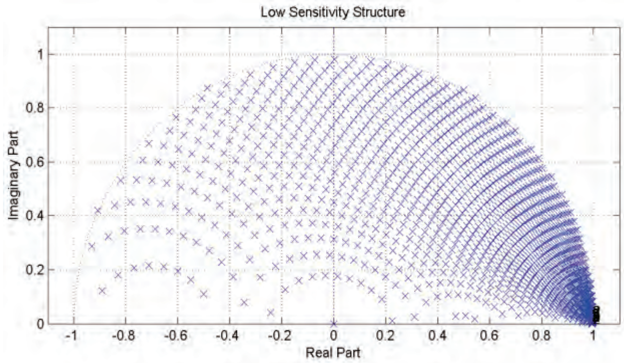


Fig. 2. Possible pole location density of BQ3

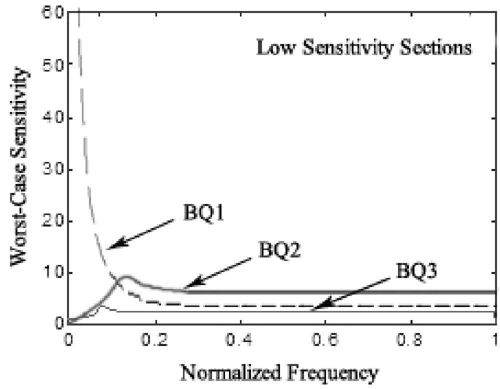


Fig. 3. Worst-case sensitivity investigation of three different low frequency structures

Fig. 3 shows the results of the experiments for three different structures suitable for use in the low frequency band. It is investigated the sensitivity of the multiplier coefficients. A feature which is similar for the three structures is the minimized sensitivity to non-desirable effects. All digital structures – BQ1, BQ2 and BQ3 are of second order. The BQ1 and BQ2 are low frequency digital filter structures.

The overall sensitivity to all multiplier coefficients is evaluated using the worst-case sensitivity.

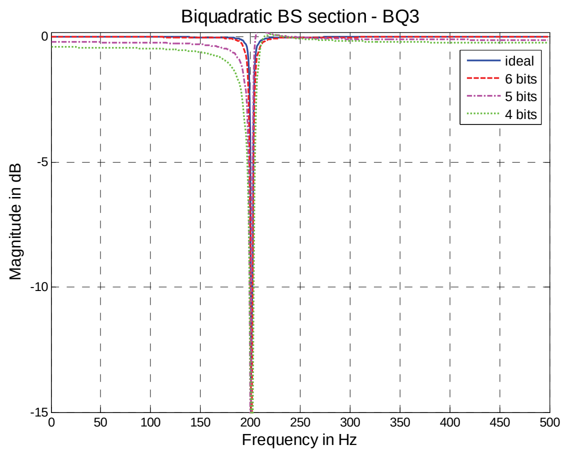


Fig. 4. Sensitivity investigation of low sensitivity structure

On Fig. 4 are the curves of simulation results after the experiments for quantization with different number of bits (b=4,5,6 bits) performed on the structure BQ3.

In order to be compared with the ideal characteristic it is also presented. It is obvious that with increase of the number of bits it gets closer to the ideal one. Even the quantizing is performed with les bits the characteristics are very close to the desirable.

The BQ3 structure is suitable for noise cancelation in the low frequency band, because it presents very narrow and sharp magnitude response.

VI. Conclusion

Frequency-dependent sensitivities allow different digital filter realizations to be compared to each other in a wide frequency range. For this reason, worst-case sensitivity (1) is considered in this work.

An overview of the various types of evaluations of sensitivity of structures suitable for implementation with digital filters is presented. The impact of the change in the number of quantization bits with which are quantized the coefficients in different type realizations is shown. The use of low-sensitive to parasitic effects structures leads to improvements in the realization. The sections that have been investigated were in the worst-case sensitivity. All analyzed structures are suitable for the low-frequency band and the BQ3 section as expected has shown the lowest sensitivity characteristics with comparison to the other structures. All experiments were conducted in one and the same poles location and the conditions were equal.

All three investigated here sections are suitable for cascade realizations of identical sub filters.

The section BQ3 has shown the lowest sensitivity according to the graphical results and has also presented very good properties during the process of quantization.

All the sections presented here can be also realized as adaptive digital filters.

References

[1] Agarwal, Ramesh C., Burrus, C. Sidney, "New recursive digital filter structures having very low sensitivity and roundoff noise", IEEE Transactions on Circuits and Systems, Volume: 22 , Issue: 12, Page(s): 921 – 927, Dec 1975.

[2] Nishihara, A. and K. Sugahara, A synthesis of digital filters with minimum pole sensitivity, Trans. Of the IEICE of Japan, vol. E65-A, No. 5, pp. 234-240, May 1982.

[3] Jovanovic D.G. and Mitra S.K. A new low sensitivity second–order bandpass digital filter structure. Electronics Letters, 38 (16):858–860. August 2002.

[4] Soyanov, G., "A comparative study of variable biquadratic digital filter sections sensitivities and tuning accuracy", Proc. Int. Conference "Telecom2002", pp. 674-681, 2002.

[5] Stoyanov G., " A new digital biquadratic section with pole and zero frequencies tuned by a single parameter", Proc. 37thInternational Sc. Conf. on Information, Comunication and Energy Systems, "ICEST'2002', Nis, Yugoslavia, Oct. 2-4, 2002.

[6] Z. Nikolova, G. Stoyanov, G. Iliev, V. Pulkov, "Complex Coefficient IIR Digital Filters", Chapter 9 of "Digital filters", Fausto Pedro García Márquez (Ed.), ISBN: 978-953-307-190-9, Intech, pp. 209–239, Apr. 2011.

[7] Maria Nenova, Georgi Stoyanov, Georgi Iliev, "Investigation of least-squares algorithm based adaptive digital filter section for detection/ suppression of a single sinusoidal signal", National Conference with Foreign Participation Telecom'2005, Varna, Bulgaria, pp. 370-375, October 6-7, 2005

[8] G. Stoyanov, K. Nikolova, High-accuracy design and implementation of allpass-based Hilbert transformers and fractional delay filters, Proc. 5th International Symp. IGNOIE&SOIM (System Const. Global-Oriented Inf. Electronics), Sendai, Japan, Vol. 1, pp. 418-423, Feb. 22-24, 2012.

[9] Pleshkova-Bekiarska Sn., Al. Bekiarski, Sound Propagation Model for Sound Source Localization in Area of Observation of an Audio Robot, International Journal of Neural Networks and Application, 2(1), Januare-June 2009, International Science Press, India, pp.1-4.

[10] Sugino N. and A. Nishihara, Frequency-Domain Simulator of Digital Networks from the Structural Description, IEEE Transactions of the IEICE, Vol. E-73, No.11, November 1990.

[11] Stoyanov G. and Uzunov I., " A new variable digital second-order elliptic low-pass filter section", Proc. 9th Sc. Conference "Electronics'2000", Sozopol, Bulgaria, Vol. 1, pp. 21-26, Sept. 20-22, 2000.

[12] Stoyanov G., I. Uzunov and M. Kawamata, "Deign of variable IIR digital filters using equal subfilters", Proc. 2000 IEEE Inetrnational Symp. Intelligent Signal Processing and Communications Systems (ISPACS'2000), Honolulu, Hawaii, vol. 1, pp. 141-146, Nov. 5-8, 2000.

[13] Stoyanov G., I. Uzunov and M. Kawamata, "Design and realization of variable IIR digital filters as a cascade of identical sub-filters", IEICE Trans. Fundamentals, vol. E84-A, No. 8, pp. 1831-1839, August 2001.

[14] Proakis, J. and D. Manolakis, *Digital signal processing, algorithms and applications*, Third edition, Prentice-Hall, 1996