Further results on integer and non-integer order PID control of robotic system

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Abstract –**This paper presents the new algorithms of integer and fractional order PID control based on genetic algorithms in the position control of a 3 DOF`s robotic system driven by DC motors. Also, we proposes a robust fractional-order PD^{α} sliding mode control of a given robotic system.**

Keywords –**Fractional PID controller, control, robot, DC motor, optimal settings**

I. INTRODUCTION

Fractional calculus (FC) is a mathematical topic with more than 300 years old history, but its application to physics and engineering has been reported only in the recent years. The fractional integro-differential operators are a generalization of integration and derivation to non-integer order (fractional) operators, [1],[2].As we know,due to its functional simplicity and performance robustness, the PID controllers are still used for many industrial applications.On the other hand, fractional calculus has the potential to accomplish what integer-order calculus cannot. In most cases, our objective of using fractional calculus is to apply the fractional order controller to enhance the system control performance i.e. better disturbance rejection ratios and less sensitivity to plant parameter variations compared to the traditional controllers. The fractional $PI^{\beta}D^{\alpha}$ controller,[2] the CRONE controllers, [3] and the fractional lead-lag compensator,[4] are some of the well-known fractional order controllers. Three definitions are generally used for the fractional differintegral. First is the Grunwald definition, [2] suitable for numerical calculation given as:

$$ {}^{GL}D_a^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\alpha}{j} f(t-jh), \quad (1)$$

where a, t are the limits of operator and [x] means the integer part of x. The left Riemann-Liouville (RL) definition of fractional derivative is given by

$$ {}_a^{RL}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad (2)$$

for $(n-1 \le \alpha < n)$ where $\Gamma(.)$ is the well known Euler's gamma function.

$$ {}_a^{RL}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \quad (2)$$

Also, there is another definition of left fractional derivative introduced by Caputo, [1],[2] as follows:

$$ {}_a^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad n-1<\alpha<n, (3)$$

Caputo and Riemann-Liouville formulation coincide when the initial conditions are zero. In this paper, we suggest and obtain a new optimal algorithms of fractional order PID control based on genetic algorithms (GA) [5] in the control of robotic system driven by DC motors. GA is a stochastic global adaptive search optimization technique based on the mechanisms of natural selection. The objective of this work is to find out optimal settings based on genetic algorithms for a integer and fractional $PI^{\beta}D^{\alpha}$ controller in order to fulfill different design specifications for the closed-loop system, taking advantage of the fractional orders, α and β. Also, a sliding-mode controller (SMC) is a powerful tool to robustly control incompletely modeled or uncertain systems [6] which has many attractive features such as fast response, good transient response and asymptotic stability. However, an SMC has some disadvantages related to well-known chattering in the system due to the discontinuous control action are neglected high order control plant dynamics, actuator dynamics, sensor noise, etc. Recently, a fractional-order sliding mode control technique by Monje et al. [7] has been successfully applied for a robot manipulator. In this paper, we suggest and obtain a chattering-free fractional PD^{α} sliding-mode controller in the control of a robotic system driven by DC motors.

II. MAIN RESULTS: NON-INTEGER CONTROL OF A ROBOTIC SYSTEM WITH DC MOTORS

A. Model of robotic system with DC motors

Here, we are interested in GA based fractional PID control of a robotic system (RS) with DC motors. RS is considered as an open linkage consisting of $n+1$ rigid bodies $[V_i]$ interconnected by n one-degree-of–freedom joints formed kinematical pairs of the fifth class, where the RS possesses n degrees of freedom, $(q) = (q^1,q^2,\ldots,q^n)^T$. Specially, the Rodriguez` method,[8], is proposed for modelling kinematics and dynamics of the RS. The geometry of the system has been defined by unit vectors $\vec{e}_i$, $i=1,2,\ldots,j,\ldots,n$ as well as vectors

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$\vec{\rho}_i$ and $\vec{\rho}_{ii}$ and the parameters $\xi_i, \bar{\xi}_i = 1-\xi_i$ denote parameters for recognizing joints, $\xi_i = 1-prismatic,\ 0-revolute$. Here, equations of motion of the RS can be expressed in the identical covariant form as follows

$$\sum_{\alpha=1}^{n} a_{\alpha i}(q)\ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \Gamma_{\alpha\beta,i}(q)\dot{q}^{\alpha}\dot{q}^{\beta} = Q_i \quad i=1,2,...,n. \quad (4)$$

where coefficients $a_{\alpha\beta}$ are covariant coordinates of basic metric tensor $\left[a_{\alpha\beta}\right] \in R^{n\times n}$ and $\Gamma_{\alpha\beta,\gamma}\ \alpha,\beta,\gamma = 1,2,...,n$ presents Christoffel symbols of first kind and Q_i generalized forces.

Here, it is used RS with 3 DOF's ,Fig. 1, driven by 3 DC motors.

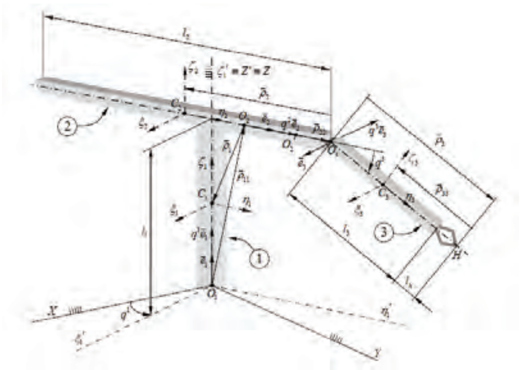

Fig.1 Robotic system with 3 DOF's

The next equation describes the given circuit of DC motor

$$R_i i_i(t) + L_i \frac{di_i(t)}{dt} + ems_i(t) = u_{vi}(t),\ i=1,2,3 \quad (5)$$

where R_i, L_i, i_i and u_{vi} are respectively resistance, inductivity, electrical current and voltage. Electromotive force is $ems_i(t) = k_e dq_m/dt$ where $k_e = const$ and $q_m(t)$ is generalized coordinate of a DC motor as well as N_i than is $q_{mi}(t) = N_i q_i(t),\ i=1,2,3,\ N_i$ degree of reduction. It is assumed that $Q_i^u(t) = N_i k_m i_i(t)$ where $k_m = const$ is the torque constant. If the equation of RS is combined with (6) next equation can be written and taking into assumption that $L \approx 0$ we obtain

$$R\left[NK_m\right]^{-1}\left(A(q)\ddot{q} + C(q,\dot{q})\right) + K_e N\dot{q} = u_v(t), \quad (6)$$

or in state space, $x_p = \left[q_1\ q_2\ q_3\right]^T, x_v = \left[\dot{q}_1\ \dot{q}_2\ \dot{q}_3\right]^T$, as follows:

$$\dot{x} = \begin{bmatrix}\dot{x}_p \\ \dot{x}_v\end{bmatrix} = \begin{bmatrix}x_v \\ -A^{\bullet-1}(x_p)\left(C(x)+Fx_v\right)\end{bmatrix} + \begin{bmatrix}0_{3\times3} \\ -A^{\bullet-1}(x_p)\end{bmatrix}\tau(t) \quad (7)$$

$$y = h(x) = x_p \quad (8)$$

where are $\quad F = NK_m R^{-1}K_e N, \quad \tau = NK_m R^{-1}u_v \quad (9)$

B. GA-based optimal fractional PID control

B.1 Fractional order PID controller- $PI^{\beta}D^{\alpha}$

Fractional order PID controller (FOPID) is the generalization of a standard (integer-order) PID (IOPID) controller, whereas its output is a linear combination of the input and the fractional integer/derivative of the input. Recently, published results of FOPID [2], [4], [9] indicate that the use of a FOPID controller can improve both the stability and performance robustness of feedback control systems. However, FOPID itself is an infinite dimensional linear filter and the tuning rules of FOPID controllers are much more complex in compared classical PID controllers. Unlike conventional PID controller, there is no systematic and rigor design or tuning method existing for FOPID controller. The time equation of the FOPID controller is given by:

$$u(t) = K_p e(t) + K_{d\ 0}D_t^{\alpha} e(t) + K_{i\ 0}D_t^{-\beta} e(t) \quad (10)$$

For practical digital realization, the derivative part in s-domain has to be complemented by the first order filter

$$G_{FOPID}(s) = K_p\left(1 + \frac{1}{s^{\beta}T_i} + \frac{T_d s^{\alpha}}{(T_d/N)s+1}\right), \quad (11)$$

The parameters are: gain $K_p, K_d,\ K_i$, noninteger order of derivative α and integrator β, as well as the integral time constant, $T_i = K_p/K_i$, and the derivative $T_d = K_d/K_p$.

B.2 Optimal tuning FOPID using GA

In this paper, we propose using GA for determine the optimal parameters fractional order PID controllers. In real coding implementation, each chromosome is encoded as a vector of real numbers, of the same lengths as the solution vector. According to control objectives, five parameters $K_p, K_d, K_i, \alpha, \beta$ are required to be designed in these settings. Next, optimality criterion which involves besides steady state error e, i.e IAE, integral of absolute magnitude of the error, overshoot P_o, as well as settling time T_s is introduced

$$J = \left|P_0\right| + T_s + \int|e|dt \to min \quad (12)$$

All the GA parameters are arranged as follows: population size: $N=100$; crossover probability: $p_c = 0.75$;-mutation probability: $p_m = p_{m0}\min\left(1, l/g\right)$, $p_{m0} = 0.1$-initial mutation probability, $l = 25$- generation threshold, $g-$ current number of generation, generation gap $gr = 0.35$.Remainder stochastic sampling with replacement as selection method is used.

C. Simulations and discussion

Both the FOPID and the IOPID controllers are designed based on the proposed GA. Here, vector has the FOPID parameters the ranges of *FOPID* parameters are selected as

$$K_p \in [10,200],\ K_i \in [0,100],\ K_d \in [10,200], \alpha \in (0.2,1], \beta \in [0,1], \quad (13)$$

TABLE I
THE OPTIMAL PARAMETERS OF THE FOPID,IOPID CONTROLLER BASED ON GA

controller		K_p	K_i	K_d	β	α	J_{opt}
PID	1.	199	2	24	-	-	0.98651
	2.	212	2	26	-	-	0.84875
	3.	246	1	28	-	-	0.68718
FOPID	1.	199	2	24	0.020	0.965	0.69887
	2.	212	2	26	0.145	0.933	0.72954
	3.	246	1	28	0.135	0.932	0.56187

In Table 1. they are presented the optimal parameters of the FOPID as well as IOPID controller using GA.

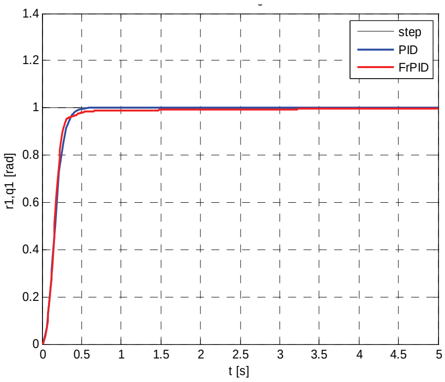

Fig. 2. The step responses of the $q_1(t)[rad]$

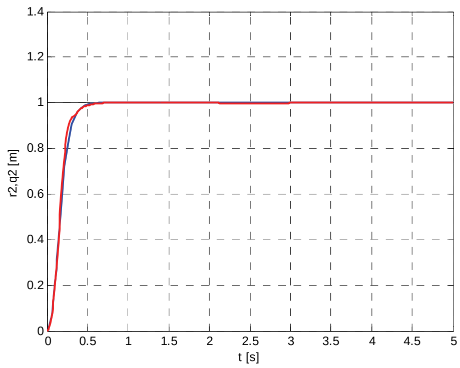

Fig.3 The step responses of the $q_2(t)[m]$

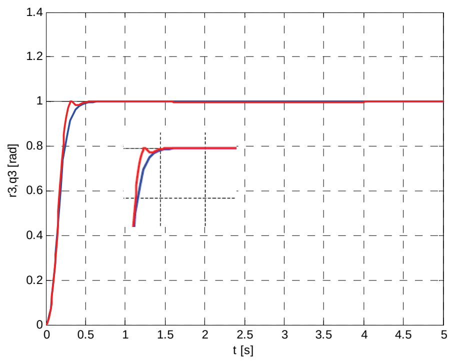

Fig.4 The step responses of the $q_3(t)[rad]$

In simulations they are compared step responses of these two optimal FOPID/IOPID controllers, presented in Figs.2-4.As can be seen from the Figs.2-4 and Table1, better performance for robot control can be achieved using FOPID.

D. Chattering-free sliding mode controller design based on the fractional order PD^{α} sliding surface

Also, we suggested chattering-free fractional PD$^{\alpha}$ sliding-mode controller in the control of a RS driven by DC motors. It is well-known that the sliding-mode control is used to obtain high-performance robust control nonsensitive to disturbances and parameter variations. For a nonlinear MIMO system represented in a so-called normal form

$$\dot{x} = f(x) + G(x)u \tag{15}$$

one general sliding mode control law is, [10]

$$u = -[\Lambda G(x)]^{-1}\Lambda[f(x) - \dot{x}_d] - [\Lambda G(x)]^{-1} Q\operatorname{sgn}(s) \tag{16}$$

consisting of a continuous and discontinuous control part where switching surfaces $\boldsymbol{s}$ are defined as $s = \Lambda(x - x_d)$, x_d being the vector of the desired states and the Q positive definite diagonal matrix. The elements of the matrix Λ are chosen so that the $\boldsymbol{i}$-th component of the sliding hypersurface has the structure

$$s_i = \left(\frac{d}{dt} + \lambda_i\right)^{(r_i - 1)}(x_i - x_{di}), \quad i = 1, 2, ..., n \tag{17}$$

where r_i is the order of the $\boldsymbol{i}$-th subsystem and $\lambda_i > 0$. More generally, considering Eq. (14) as a nominal (known) plant dynamics, we can write

$$\dot{x} = f(x) + \tilde{f}(x) + \left[G(x) + \tilde{G}(x)\right]u \tag{18}$$

where $\tilde{f}(x)$ and $\tilde{G}(x)$ represent uncertainties or unknown plant dynamics.Using the Lyapunov method one may conclude

$$\dot{s} = -PQ\operatorname{sgn}(s) + (P - I)\Lambda[\dot{x}_d - f(x)] + \Lambda\tilde{f}(x) \tag{19}$$

where $P := \Lambda(G + \tilde{G})(\Lambda G)^{-1}$. Regardless whether $\tilde{G} \neq 0$ and/or $\tilde{f} \neq 0$, with an appropriate choice of Q, we can obtain $s^T\dot{s} < 0$ for $\|s\| > 0$, and this result indicates that the error vector defined by the difference $x - x_d$ is attracted by the subspace characterized by $s = 0$ and moves toward the origin according to what is prescribed by $s = 0$, [10]. In most cases, this leads to good results but there are some disadvantages such as a *chattering* phenomenon. We suggested the application of the fractional sliding surface in order to decrease output signal oscillations. In this paper, it can be shown that, without a special tuning of Q for the perturbed plant case, model uncertainties can be successfully compensated using just the fractional order sliding surface and the values of Q suitable for the nominal plant. For a 3-DOF RS, a conventional sliding manifold is of the first order PD structure $s_i = d\tilde{x}_i/dt + \lambda_i\tilde{x}_i$, $i = 1, 2, 3$ where $\tilde{x}_i = x_i - x_{id}$ and here we propose a fractional PD^{α} structure as follows:

$$s_i = d^{\alpha}\tilde{x}_i/dt^{\alpha} + \lambda_i\tilde{x}_i, \; i = 1, 2, 3 \tag{20}$$

Simulation results for the position control based on fractional PD^{α} sliding-mode control

Some experimental simulations were undertaken for $\alpha = 0.7, 0.8, 0.9, 0.95, 0.99$, and we have found that the best results are obtained with $\alpha = 0.95$, and the matrix $Q_{nom} = diag[5,5,5]$ as well as $\lambda = (5, 2.5, 2.5)^T$. To verify the robustness of the proposed fractional sliding/mode control we have applied the next parameters variation as follows:

$$\frac{\Delta m_i}{m_i}, i = 1,2,3 \sim 9.5\%, \quad \frac{\Delta K_i}{K_i} \sim 10\%, \quad \frac{\Delta J_i}{J_i} \sim 15\% \tag{21}$$

The simulation results are depicted in Figs.5 to 8, where the black lines ($h(t)$) are the desired trajectories. In particular, we present the comparison results for the second coordinate q_2 responses with the PD and fractional PD^{α} cases with all other conditions being the same, for the nominal object, Fig.6 and the perturbed object, Fig.8.

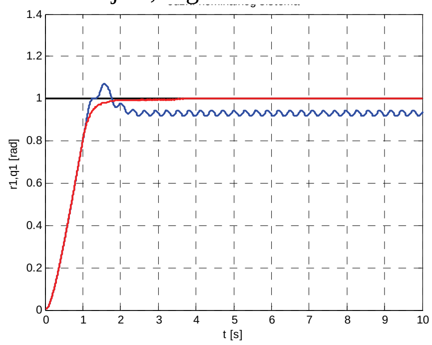

Fig.5. Stabilizing using the sliding mode control *PD* and the fractional PD^{α} - nominal case, $q_1(t)[rad]$

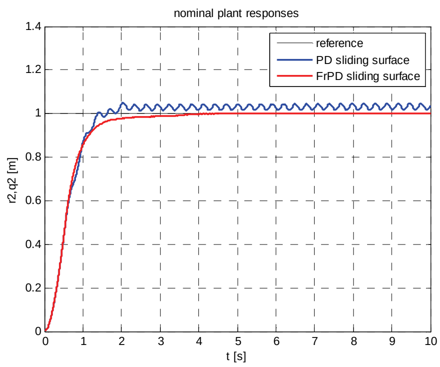

Fig 6. Stabilizing using the sliding mode control *PD* and the fractional PD^{α} - nominal case, $q_2(t)[m]$

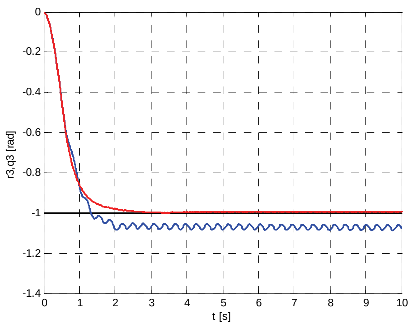

Fig 7. Stabilizing using the sliding mode control *PD* and the fractional PD^{α} - nominal case, $q_3(t)[rad]$

Perturbated case:(only q2)

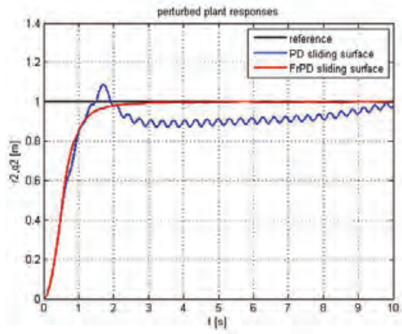

Fig. 8. Stabilizing using the sliding mode control *PD* and the fractional PD^{α} -perturbed case

III. Conclusion

From previous comparison we conclude that the optimal FOPID controller gives better performance for robot control as compared to optimal IOPID controller method. Also, it is shown that a sliding mode control with the fractional sliding surface is more robust to parameter perturbations and, what is most important to emphasize, the output oscillations are almost completely attenuated and the overall quality of the transient response is much better.

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