TLM Method with Z-transforms - Efficient Tool for Dispersive Anisotropic Structures Modelling

Miloš Kostić, Nebojša Dončov, Zoran Stanković

Abstract – **An efficient approach for modelling of dispersive anisotropic structures is considered in this paper. The approach uses the Transmission Line Matrix (TLM) method based on Z-transforms to account for dispersive properties of anisotropic structures in the time-domain. In-house developed TLM code has been used to implement this approach and apply it on carbon-fibre composite as anisotropic conductive material. The accuracy and efficiency of the presented approach are first illustrated on the simplified case when carbon fibre anisotropic electric conductivity is treated as frequency independent parameter through comparison with analytically available solution. Then, the Drude model is employed to account for dispersive behaviour of conductivity and to consider its impact of reflection and transmission properties of carbon fibre sample material.**

Keywords – **Dispersive anisotropic materials, TLM method, Z-transform, Drude mode.**

I. Introduction

Differential time-domain numerical techniques are common tools for modelling of complex electromagnetic (EM) structures at high frequencies. Among them, the most popular are the finite difference time domain (FD-TD) method [1] and transmission line matrix (TLM) method [2]. Although very similar, as they are both based on space and time discretizations, TLM approach offers crucial advantage in certain complex problems. In TLM, the electric (E) and magnetic (M) fields are co-located in space and time (at the centre of the discretization cell), while in Yee's FDTD algorithm [1] the E-fields are on the edges and H-fields are at the centre of the cell. Therefore, TLM algorithm is more suitable for modelling of anisotropic and bi-anisotropic materials and the mesh layout is much simpler than in FDTD method. In addition, there is not need to perform field averaging and temporal interpolation in order to determine fields on cell boundaries.

Both methods in their conventional use allow for modelling of materials with frequency independent EM properties. However, as they basically operate in the time-domain they are perfectly suited for time-harmonic and transient simulation of frequency dependent structures for direct analysis of their dispersive behaviour. Several techniques have been already developed and implemented in FDTD method to incorporate frequency dispersion. Some of these techniques are detailed and referenced in [3] regarding dispersive metamaterials structures. One variation of the TLM method which is naturally suited to the description of arbitrary time-dependent responses is based on Z-transforms [4]. This approach has been successful in the development of numerical schemes for the time-domain treatment of variety of frequency-dependent materials, i.e. linear isotropic, linear anisotropic, bi-isotropic, nonlinear and quantum media [5-9]. In recent work, the TLM technique with Z transforms has been successfully applied to the time-domain simulation of dispersive metamaterials and graded refractive index metamaterials [10-11].

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In this paper, an in-house developed TLM code has been used to implement TLM method with Z-transforms for the purpose of efficient modelling of carbon-fibre composite used in aircraft constructions. The embedded carbon fibres cause the materials to have a high electric conductivity in the direction of the fibres, i.e. conductivity of these materials is anisotropic [4]. The accuracy of the presented method is first illustrated on the simplified case when carbon fibre anisotropic electric conductivity is treated as frequency independent parameter through comparison with analytically available solution. Then, the Drude model is employed to account for dispersive behaviour of electric conductivity and to consider its impact of reflection and transmission properties of carbon fibre sample material.

II. Anisotropic Materials Modelling by using TLM Method with Z-transforms

Using the notation for the fields, current and flux densities and corresponding constitutive relations for the electric and magnetic current and flux densities, Maxwell's curl equations can be expressed in the time-domain in compact form as [4,5]:

$$\begin{bmatrix} \nabla\times\underline{H} \\ -\nabla\times\underline{E} \end{bmatrix} - \begin{bmatrix} \underline{J_{ef}} \\ \underline{J_{mf}} \end{bmatrix} = \frac{\partial}{\partial t}\begin{bmatrix} \varepsilon_0\underline{E} \\ \mu_0\underline{H} \end{bmatrix} + \begin{bmatrix} \underline{\underline{\sigma_e}}*\underline{E} \\ \underline{\underline{\sigma_m}}*\underline{H} \end{bmatrix} + \frac{\partial}{\partial t}\begin{bmatrix} \varepsilon_0\underline{\underline{\chi_e}} & \underline{\underline{\xi_r}}/c \\ \underline{\underline{\zeta_r}}/c & \mu_0\underline{\underline{\chi_m}} \end{bmatrix} * \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix}, \tag{1}$$

where $\underline{E}$ and $\underline{H}$ are electric and magnetic field vectors, $\underline{J_{ef}}$ and $\underline{J_{mf}}$ are the free electric and magnetic current density vectors, ε_0 and μ_0 are free-space permittivity and permeability, c is the speed of light in free-space and operator * denotes the time-domain convolution. The matrices $\underline{\underline{\sigma_e}}$ and $\underline{\underline{\sigma_m}}$ are the electric and magnetic conductivity matrices of the following form:

$$\underline{\underline{\sigma_e}} = \begin{bmatrix} \sigma_e^{xx} & \sigma_e^{xy} & \sigma_e^{xz} \\ \sigma_e^{yx} & \sigma_e^{yy} & \sigma_e^{yz} \\ \sigma_e^{zx} & \sigma_e^{zy} & \sigma_e^{zz} \end{bmatrix},\ \underline{\underline{\sigma_m}} = \begin{bmatrix} \sigma_m^{xx} & \sigma_m^{xy} & \sigma_m^{xz} \\ \sigma_m^{yx} & \sigma_m^{yy} & \sigma_m^{yz} \\ \sigma_m^{zx} & \sigma_m^{zy} & \sigma_m^{zz} \end{bmatrix}. \quad (2)$$

The matrices $\underline{\underline{\chi_e}}$ and $\underline{\underline{\chi_m}}$ are of similar form and they describe the electric and magnetic susceptibilities, respectively, while the matrices $\underline{\underline{\xi_r}}$ and $\underline{\underline{\zeta_r}}$ are dimensionless describing the magneto-electric coupling.

The curl terms of Eq.(1) can be expand in Cartesian coordinates and express as a functions of voltage pulses of appropriate transmission lines of TLM cell (Fig.1) [4] by using the field circuit equivalence for the electric and magnetic fields of the form:

$$E_i = -V_i / \Delta\ell,\ H_i = -i_i / (\Delta\ell\, \eta_0),\ i \in (x, y, z), \quad (3)$$

where V_i and i_i are equivalent voltage and current in the cell centre along i-th coordinate axis, $\Delta\ell$ is the size of TLM cell ($\Delta x = \Delta y = \Delta z = \Delta\ell$) and η_0 is the intrinsic impedance of free-space.

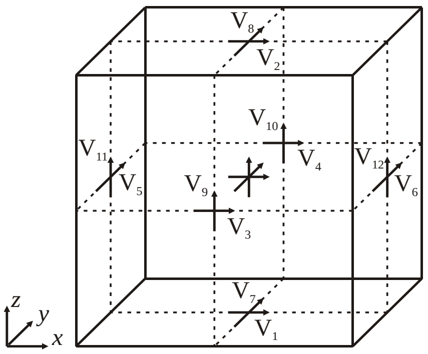

Fig. 1. TLM cell

After that, Eq.(1) can be expressed, after converting to travelling wave format, as [4]:

$$2\begin{bmatrix} (V_1^i + V_2^i + V_3^i + V_4^i) \\ (V_5^i + V_6^i + V_7^i + V_8^i) \\ (V_9^i + V_{10}^i + V_{11}^i + V_{12}^i) \\ -(V_7^i - V_8^i - V_9^i + V_{10}^i) \\ -(V_{11}^i - V_{12}^i - V_1^i + V_2^i) \\ -(V_3^i - V_4^i - V_5^i + V_6^i) \end{bmatrix} - \begin{bmatrix} i_{efx} \\ i_{efy} \\ i_{efz} \\ V_{mfx} \\ V_{mfy} \\ V_{mfz} \end{bmatrix} = 4\begin{bmatrix} V_x \\ V_y \\ V_z \\ i_x \\ i_y \\ i_z \end{bmatrix} + \quad , \quad (4)$$

$$\begin{bmatrix} \underline{\underline{g_e}} & \\ & \underline{\underline{r_m}} \end{bmatrix} * \begin{bmatrix} V_x \\ V_y \\ V_z \\ i_x \\ i_y \\ i_z \end{bmatrix} + 2\frac{\partial}{\partial T}\begin{bmatrix} \underline{\underline{\chi_e}} & \underline{\underline{\xi_r}} \\ \underline{\underline{\zeta_r}} & \underline{\underline{\chi_m}} \end{bmatrix} * \begin{bmatrix} V_x \\ V_y \\ V_z \\ i_x \\ i_y \\ i_z \end{bmatrix}$$

where $\frac{\partial}{\partial T} = \frac{\Delta\ell}{2c}\frac{\partial}{\partial t}$, $\underline{\underline{g_e}} = \underline{\underline{\sigma_e}}\,\Delta\ell\,\eta_0$ and $\underline{\underline{r_m}} = \underline{\underline{\sigma_m}}\,\Delta\ell / \eta_0$. Superscript i is used to denote incident wave quantities. Eq.(4) can be also written in compact notation [4]:

$$2\begin{bmatrix} \underline{V}^r \\ -\underline{i}^r \end{bmatrix} = 4\begin{bmatrix} \underline{V} \\ \underline{i} \end{bmatrix} + \begin{bmatrix} \underline{\underline{g_e}} & \\ & \underline{\underline{r_m}} \end{bmatrix} * \begin{bmatrix} \underline{V} \\ \underline{i} \end{bmatrix} + 2\frac{\partial}{\partial T}\begin{bmatrix} \underline{\underline{\chi_e}} & \underline{\underline{\xi_r}} \\ \underline{\underline{\zeta_r}} & \underline{\underline{\chi_m}} \end{bmatrix} * \begin{bmatrix} \underline{V} \\ \underline{i} \end{bmatrix}, \quad (5)$$

or in matrix notation:

$$2\underline{F}^r = 4\underline{F} + \underline{\underline{\sigma}}(T) * \underline{F} + 2\frac{\partial}{\partial T}\left[\underline{\underline{M}}(T) * \underline{F}\right], \quad (6)$$

where $\underline{\underline{\sigma}}(T)$ is the general conductivity matrix used to describe electric conductivity and magnetic resistivity terms and $\underline{\underline{M}}(T)$ is the general material matrix describing electric, magnetic and/or magneto-electric effects. Both matrices may contain time-dependent elements.

Discretizing the normalized time-derivative operator in Eq.(6) by using the bilinear Z-transform as:

$$\partial / \partial T \to 2(1 - z^{-1}) / (1 + z^{-1}), \quad (7)$$

and taking partial fraction expansions of matrices $\underline{\underline{\sigma}}(z)$ and $\underline{\underline{M}}(z)$, containing causal time-dependent elements, as [4,5]:

$$(1 + z^{-1})\underline{\underline{\sigma}}(z) = \underline{\underline{\sigma_0}} + z^{-1}\left[\underline{\underline{\sigma_1}} + \underline{\underline{\sigma_2}}(z)\right], \quad (8)$$

$$(1 + z^{-1})\underline{\underline{M}}(z) = \underline{\underline{M_0}} + z^{-1}\left[\underline{\underline{M_1}} + \underline{\underline{M_2}}(z)\right], \quad (9)$$

give the following equation:

$$\underline{F} = \underline{\underline{T}}\,[2\underline{F}^r + z^{-1}\underline{S}], \quad (10)$$

where $\underline{\underline{T}} = [\underline{\underline{4}} + \underline{\underline{\sigma_0}} + 4\underline{\underline{M_0}}]^{-1}$, $\underline{S} = 2\underline{F}^r + \underline{\underline{K}}\,F - \underline{S_\sigma} + \underline{S_M}$, $\underline{\underline{K}} = -[\underline{\underline{4}} + \underline{\underline{\sigma_1}} - 4\underline{\underline{M_1}}]$, $\underline{S_\sigma} = \underline{\underline{\sigma_2}}(z)\,\underline{F}$ and $\underline{S_M} = \underline{\underline{4M_2}}(z)\,\underline{F}$.

There are many models that can be used to describe the frequency dependence of electric conductivity and magnetic resistivity, electric and magnetic susceptibilities and parameters expressing magneto-electric effects (e.g. Debye, Lorentz, Drude model, etc). By using any of available Z-transforms such as exponential or bilinear Z-transforms, it is possible to transfer this dependence in the time-domain, i.e. to obtain functions $\sigma(z)$ and $M(z)$ and, after taking partial fraction expansions given by Eqs.(8-9), to incorporate this model into algorithm of the TLM method with Z-transforms.

Combination of the Drude model for electric and magnetic conductivity and bilinear Z-transform was used in [10] to allow for the direct time-domain modelling of left-handed metamaterials. In a case of material with anisotropic electric conductivity of dispersive behaviour according to the Drude model:

$$\sigma_e(s) = \sigma_{e0} / (1 + s\tau_e), \quad (11)$$

or:

$$g_e(s) = \sigma_e(s)\eta_0 \Delta l = g_{ec} / (1 + s\tau_e), \quad (12)$$

for normalized electric conductivity, the elements of matrices $\underline{\underline{\sigma_0}}$, $\underline{\underline{\sigma_1}}$ and $\underline{\underline{\sigma_2}}(z)$, using equations derived in [10], are:

$$g_{e0}^{ij} = g_{ec}^{ij} / B_{ce}^{ij}, \quad (13)$$

$$g_{e1}^{ij} = g_{e0}^{ij}(2 + a_{ce}^{ij}), \quad (14)$$

$$g_{e2}^{ij} = z^{-1} b_{ce}^{ij} / (1 - z^{-1} a_{ce}^{ij}), \quad (15)$$

respectively, where $ij \in (xx, xy, xz, yx, yy, yz, zx, zy, zz)$, $g_{ec}^{ij} = \sigma_{e0}^{ij}\eta_0 \Delta l$, $A_{ce}^{ij} = 2\tau_e^{ij} / \Delta t - 1$, $B_{ce}^{ij} = 2\tau_e^{ij} / \Delta t + 1$, $a_{ce}^{ij} = A_{ce}^{ij} / B_{ce}^{ij}$ and $b_{ce}^{ij} = g_{e0}^{ij}(1 + 2a_{ce}^{ij} + a_{ce}^{ij} * a_{ce}^{ij})$.

III. Numerical Results

In-house developed TLM code has been used to implement TLM method with Z-transforms for the purpose of efficient modelling of carbon-fibre composite used in aircraft constructions. The embedded carbon fibres cause the materials to have a high conductivity in the direction of the fibres, i.e. conductivity of these materials is anisotropic [4]. Geometry of the problem is shown in Fig. 2. The composite material is represented with three layers of material having an isotropic relative permittivity of $\varepsilon_{rI} = \varepsilon_{rII} = \varepsilon_{rIII} = 43$ and thickness $d_I = d_{II} = d_{III} = 3.75$ mm.

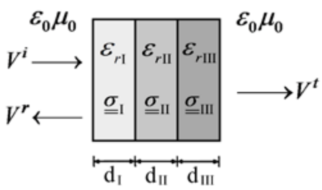

Fig. 2. Geometry of a carbon composite material

First it is assumed that carbon fibre anisotropic electric conductivity is frequency independent parameter, i.e. that parameter τ_e in Eq.(19) is such that $s\tau_e << 1$ so that:

$$\underline{\underline{\sigma_0}}_I = \underline{\underline{\sigma_1}}_I = \begin{bmatrix} g_{ecI}^{yy} & 0 \\ 0 & 0 \end{bmatrix}, \tag{16}$$

$$\underline{\underline{\sigma_0}}_{II} = \underline{\underline{\sigma_1}}_{II} = \begin{bmatrix} g_{ecII}^{yy} & g_{ecII}^{yz} \\ g_{ecII}^{zy} & g_{ecII}^{zz} \end{bmatrix}, \tag{17}$$

$$\underline{\underline{\sigma_0}}_{III} = \underline{\underline{\sigma_1}}_{III} = \begin{bmatrix} 0 & 0 \\ 0 & g_{ecIII}^{zz} \end{bmatrix}, \tag{18}$$

where $g_{ecI}^{yy} = g_{ecIII}^{zz} = 12\eta_0 \Delta\ell$,

$g_{ecII}^{yy} = g_{ecII}^{yz} = g_{ecII}^{zy} = g_{ecII}^{zz} = 8.5\eta_0 \Delta\ell$ and $\underline{\underline{\sigma_2}}(z)$ is zero matrix for all three layers. Space-step used for the simulation was $\Delta\ell = 93.75$ μm.

The magnitude of the frequency-domain reflection Rij and transmission Tij coefficients (where first index corresponds to the component of the reflected and transmitted fields while the second index expresses the polarization of the incident wave) are shown in Figs.3-6. Good agreement between TLM results (marked with red and blue solid lines) and analytic results (marked with red and blue plus and cross symbols) for frequency independent anisotropic conductivity carbon composite material can be observed in the considered frequency range up to 10 GHz.

Then, the parameters of the Drude model in Eq.(12) are chosen so that anisotropic normalized electric conductivity varies with frequency in such way that at central frequency of 5 GHz, its real part is equal to the value given for frequency independent case for each layer. A significant impact of dispersive behaviour of conductivity on reflection and transmission properties of carbon fibre sample material can be observed marked in Figs.3-6 with dashed red and blue lines.

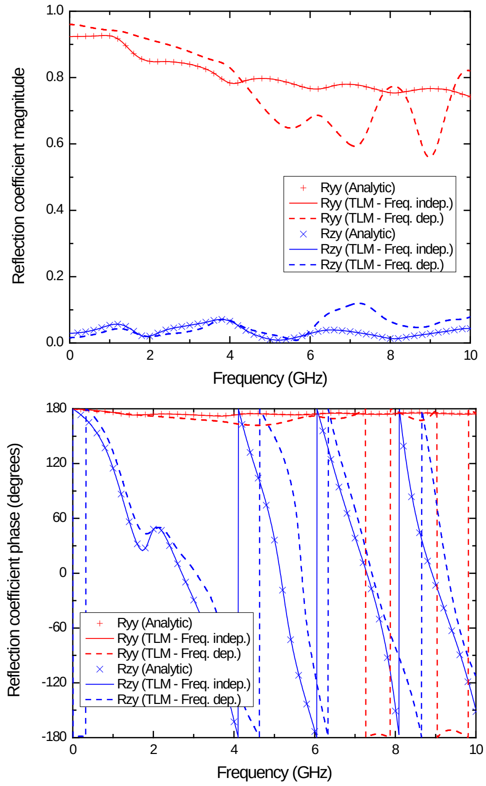

Fig. 3. Magnitude and phase of reflection coefficients *Ryy* and *Rzy*

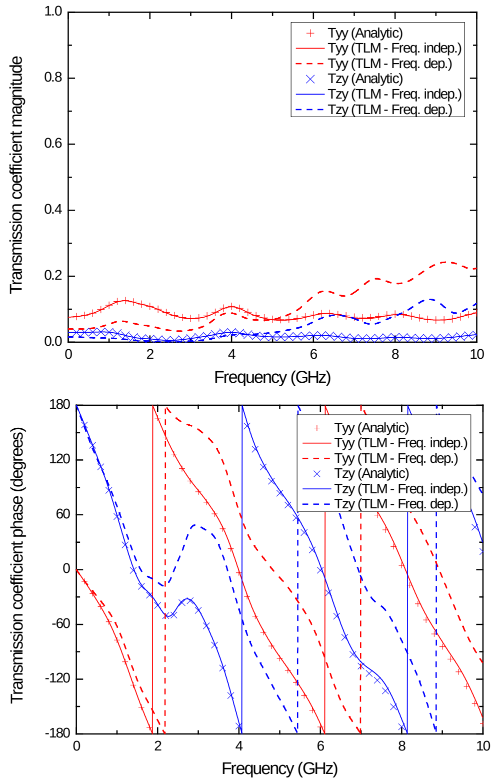

Fig. 4. Magnitude and phase of transmission coefficients *Tyy* and *Tzy*

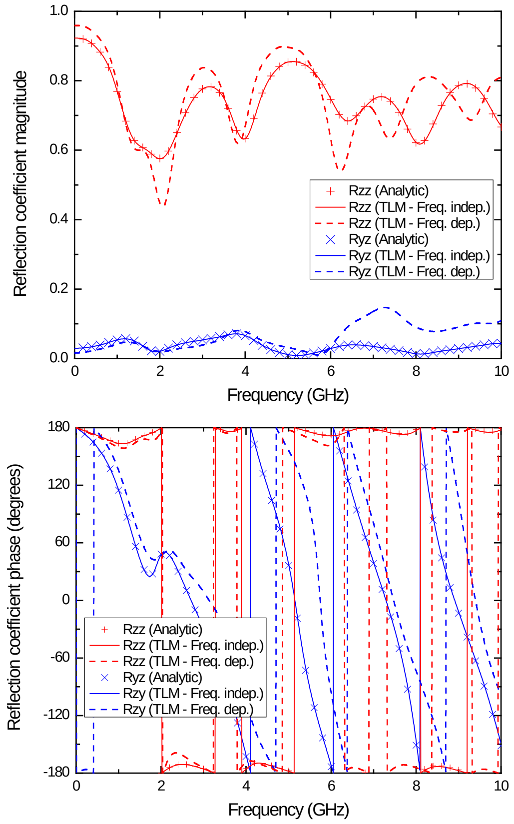

Fig. 5. Magnitude and phase of reflection coefficients *Rzz* and *Ryz*

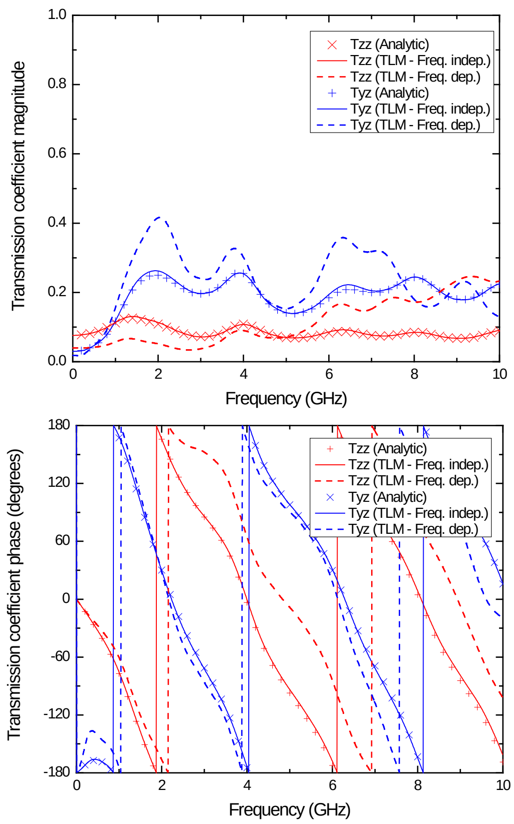

Fig. 6. Magnitude and phase of transmission coefficients *Tzz* and *Tyz*

IV. Conclusion

The approach based on TLM method with Z-transforms is used in this paper to account for dispersive properties of carbon-fibre composite as anisotropic electric conductive material. In-house developed TLM code has been used to implement this approach. The accuracy and efficiency of the presented method are illustrated on the cases of frequency independent and frequency dependent anisotropic electric conductivities when in later case the Drude model is employed to account for dispersive behaviour of electric conductivity and to consider its impact of reflection and transmission properties of carbon fibre sample material.

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